



Control of hypersonic boundary-layer transition by suppressing fundamental resonance using surface heating

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Abstract

In this paper, we propose a new concept that delays laminar-turbulent transition in hypersonic boundary layers by stabilising fundamental resonance (FR), which is a key nonlinear mechanism when oblique perturbations undergo rapid growth supported by the finite-amplitude Mack modes. As a pioneering demonstration, a surface heating is applied exclusively during the nonlinear phase. Unlike traditional control methods that target the linear phase, the stabilising effect on secondary instability modes during FR is evident across various Reynolds numbers, wall temperatures and fundamental frequencies, as confirmed by direct numerical simulations and secondary instability analyses. To gain deeper insights into this control concept, an asymptotic model is developed, revealing that the suppression effect of FR is primarily influenced by modifications to the fundamental-mode profile, while mean-flow distortion has a comparatively modest yet opposing impact on this process. This research presents a promising approach to controlling transition considering the nonlinear evolution of boundary-layer perturbations, demonstrating advantages over conventional methods that are sensitive to frequency variations.

Keywords: *boundary layer stability, boundary layer control*

Nomenclature

Latin

x, y, z – Streamwise, wall-normal, and spanwise coordinates, respectively

t – Time

u, v, w – Velocity components in the x, y, z directions, respectively

p – Pressure

T – Temperature (fluid)

T_w – Wall temperature

T_{ad} – Adiabatic wall temperature

E – Total energy

C_f – Skin-friction coefficient

M – Mach number

Pr – Prandtl number

R – Reynolds number

Q – Second invariant of the velocity gradient tensor (Q -criterion for vortices)

Greek

γ – Ratio of the specific heats

ρ – Density

μ – Dynamic viscosity

ν – Kinematic viscosity

α, β – Streamwise and Spanwise wavenumber of the instability mode, respectively

ω – Frequency of the instability mode

$\bar{\epsilon}$ – Small perturbation amplitude parameter (defined as half of the physical disturbance amplitude since the conjugate part also contributes)

$\bar{\sigma}$ – Growth rate of the secondary instability (rescaled by $\bar{\epsilon}_{10}$)

$\bar{\sigma}_1$ – First-order change in growth rate due to heating

Θ – Non-dimensional heating intensity

Superscripts

$(\cdot)^*$ – Dimensional quantity

Subscripts

$(\cdot)_{mn}$ – The parameters associated with the mode (m, n)

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1. Introduction

Laminar-turbulent transition in hypersonic boundary layers appears as a crucial issue in the high-speed vehicle designs, as the skin friction and aerodynamic heating increase drastically during transition. Thus, delaying transition can substantially reduce drag and thermal loads, motivating various flow control strategies. Most conventional approaches attempt to damp the linear growth of unstable disturbances in the early laminar phase. Examples include passive or active techniques like surface suction, cooling, or porous coatings, which aim to stabilise the dominant instability waves. While such methods have demonstrated transition delay under certain conditions, their effectiveness can be strongly dependent on the disturbance frequency and flow parameters. In fact, small changes in the frequency content of the incoming perturbations may reverse the stabilising effect into a destabilising one [1], undermining the robustness of purely linear-phase control strategies.

In low-disturbance environments, hypersonic transition often follows the natural route, initiated by small-amplitude modal instabilities that grow exponentially (e.g. the Mack first- or second-mode waves in a flat-plate boundary layer). As these primary waves reach finite amplitude, nonlinear interactions become important. One crucial nonlinear mechanism is the fundamental resonance (FR) [2], wherein a finite-amplitude two-dimensional (planar) fundamental mode interacts with a pair of oblique waves at the same frequency. This resonant triad leads to rapid growth of the oblique modes and can precipitate the onset of turbulence. Two other classical nonlinear transition mechanisms are subharmonic resonance and oblique breakdown, but FR is often dominant in high-speed boundary layers when a strong planar second-mode wave is present. Notably, FR-driven transition exhibits a characteristic two-step rise in disturbance energy or skin-friction coefficient, corresponding to the initial amplification of oblique modes followed by their breakdown into turbulence. This signature has been observed in both simulations and experiments (e.g. [3, 4]).

Recent studies suggest that targeting the nonlinear stage of transition can yield effective control even when linear stability analysis predicts mixed outcomes. For instance, porous coatings in hypersonic flows were found to delay transition by suppressing the FR mechanism, despite the fact that these coatings can amplify certain linear instability modes [5, 6, 7]. These findings point to an alternative concept of laminar-flow control that transcends strategies focusing solely on linear growth. Instead of globally altering the base flow from the outset, one may intervene locally during the nonlinear growth phase to inhibit the resonant amplification of secondary perturbations.

In light of this concept, the present work explores a novel transition control strategy that operates exclusively during the nonlinear FR phase. We introduce a surface heating segment on the flat plate, activated downstream of the linear instability region, as a prototype control. Surface heating (a localised elevation of wall temperature) was chosen as a demonstration because wall temperature has known influences on high-speed stability [8, 9]. By applying heating only in the FR regime, we seek to directly damp the growth of resonant secondary waves without significantly perturbing the earlier linear development of the primary mode. We carry out direct numerical simulations (DNS) of a Mach 5.92 flat-plate boundary layer to evaluate how this nonlinear-phase heating affects transition. In addition, a secondary instability analysis (SIA) is performed on the controlled and uncontrolled flow states to quantify changes in secondary-mode growth rates. Finally, an asymptotic theoretical framework is developed to provide mechanistic insight and predictive capability for the observed control effect.

2. Physical Model and Governing Equations

We consider a hypersonic boundary layer developing on a flat plate with a sharp leading edge. The free-stream Mach number is $M = 5.92$, and the flow conditions are chosen to match a well-studied wind-tunnel case [10]. A Cartesian coordinate system (x, y, z) is used, where x is the streamwise direction along the plate, y is normal to the wall, and z is the spanwise direction. The origin ($x = 0$) is placed at a reference location in the laminar region. Lengths are non-dimensionalized by a characteristic boundary-layer thickness $\delta^* = \sqrt{\nu_\infty^* L^* / U_\infty^*}$, and velocities by the free-stream speed U_∞^* . Under this normalization, the flow is characterised by two dimensionless parameters: the Reynolds number $R = \sqrt{U_\infty^* L^* / \nu_\infty^*}$ and the Mach number $M = U_\infty^* / a_\infty^*$, where a_∞^* represents the sound speed of the free stream. The governing equations are the 3-D unsteady Navier-Stokes equations for a compressible fluid, which can

be expressed in the non-dimensional form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1a)$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u} + p \mathbf{I} - \boldsymbol{\tau}) = 0, \quad (1b)$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot ((\rho E + p) \mathbf{u} - \boldsymbol{\tau} \cdot \mathbf{u} - \mathbf{q}) = 0, \quad (1c)$$

The viscous stress tensor $\boldsymbol{\tau}$ and heat flux $\mathbf{q}$ are expressed as

$$\boldsymbol{\tau} = \frac{\mu}{R} \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right), \quad \mathbf{q} = \frac{\mu M^2}{(\gamma - 1) Pr R} \nabla T. \quad (2)$$

Here, $E = p/[\rho(\gamma - 1)] + \mathbf{u} \cdot \mathbf{u}/2$ denotes the total energy, $\mathbf{I}$ denotes the identity matrix, $\gamma = 1.4$ is the ratio of the specific heats, and $Pr = 0.72$ is the Prandtl number. The dimensionless dynamic viscous coefficient μ is assumed to satisfy the Sutherland's law. The equation of state for a perfect gas closes the system. We solve (1) using a high-order finite-difference DNS code. Sponge layers and characteristic outflow conditions are employed at the domain boundaries to avoid non-physical reflections.

The flow configuration is designed to induce transition via the FR mechanism. At a short distance downstream of the leading edge, small-amplitude disturbances are introduced into the laminar boundary layer to seed a primary planar Mack mode and its resonant oblique partners. Specifically, the inflow or upstream boundary is perturbed with a superposition of eigenmodes:

$$\Phi'(0, y, z, t) = \bar{\epsilon}_{10} \hat{\Phi}_{(1,0)}(y) e^{-i\omega_0 t} + \bar{\epsilon}_{11} \hat{\Phi}_{(1,1)}(y) e^{i(\beta_0 z - \omega_0 t)} + \bar{\epsilon}_{1-1} \hat{\Phi}_{(1,-1)}(y) e^{i(-\beta_0 z - \omega_0 t)} + c.c., \quad (3)$$

where $\Phi' = (\rho', u', v', w', T', p')$ represents the disturbance flow field, *c.c.* denotes the complex conjugate, and $\hat{\Phi}_{(m,n)}(y)$ denotes the eigenfunction of the linear stability mode with streamwise wavenumber $m\alpha_0$ and spanwise wavenumber $n\beta_0$. In (3), the (1, 0) term is the fundamental two-dimensional mode (Mack second mode in our case), and the (1, ± 1) terms are a pair of equal and opposite oblique modes of the same frequency ω_0 . The small parameters $\bar{\epsilon}_{10}$, $\bar{\epsilon}_{11}$, $\bar{\epsilon}_{1-1} \ll 1$ controls the initial disturbance amplitude. Considering the more unstable nature of 2-D second mode, the amplitude $\bar{\epsilon}_{10}$ is taken to be much greater than $\bar{\epsilon}_{1\pm 1}$ due to the historical accumulative effect. The fundamental frequency ω_0 is selected near the most amplified second-mode frequency for each flow condition (based on linear stability theory), to ensure a strong resonance. For the cases considered, ω_0 lies in the range 0.10 – 0.13 and β_0 is set to 0.1, yielding oblique waves with a shallow angle typical of Mack-mode secondary instabilities. The specific choice of flow parameters and disturbance amplitudes can be found in [11]

A surface heating segment is implemented to control the transition. For $x < x_0$, the flat plate is held at a uniform wall temperature $T_w = T_{\text{base}}$. At the streamwise location $x = x_0$, well into the nonlinear disturbance growth region, the wall temperature is raised to $T_w = T_{\text{base}}(1 + \Theta)$. Here Θ is a nondimensional measure of heating intensity. In practice, the temperature increase is applied smoothly over a short interval to avoid numerical discontinuities. In this study, we examine cases with $\Theta = 0$ (no control), Θ at a small nonzero value representing a moderate heating, and a larger Θ for a stronger heating, to assess the effect of control intensity. Two baseline wall-temperature conditions are considered: an adiabatic wall ($T_{\text{base}} = T_{\text{ad}}$, denoted 'T1') and a cooled wall ($T_{\text{base}} = 0.6T_{\text{ad}}$, denoted 'T2'). The latter represents a less unstable laminar profile (since cooling stabilises second-mode growth in the linear regime). For each T_w condition, we simulate a moderate Reynolds number case ($R = 2298$, denoted 'R1') and a higher Reynolds number case ($R = 10^4$, denoted 'R2') to verify the robustness of control at more extreme flow parameters. The heating segment begins at x_0 approximately midway through the laminar region (e.g. $x_0 \approx 500$ in the non-dimensional coordinates for case T1R1), which is just before the uncontrolled flow's nonlinear breakdown would normally accelerate.

3. Numerical Demonstration of Transition Control

In the uncontrolled case (no heating, $\Theta = 0$), the laminar boundary layer undergoes natural transition via FR. As shown in figure 1, C_f follows the laminar Blasius solution at the early stage. Around a certain

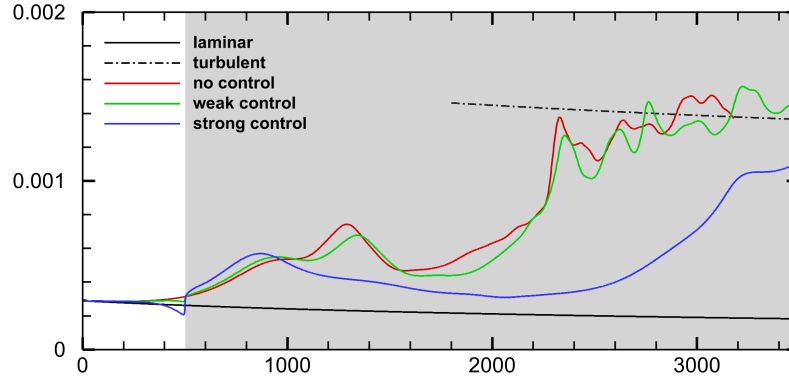


Fig 1. Streamwise evolution of the C_f curves obtained by DNS for case T1R1 with different heat intensity (no control $\Theta = 0$, weak control $\Theta = 0.1$, strong control $\Theta = 0.5$), where $\omega_0 = 0.104$ and $\beta_0 = 0.1$. The dashed and dash-dotted curves denote the empirical curves for the laminar and turbulent states, respectively.

location ($x \approx 150$), C_f begins to deviate upward due to the growth of the resonant oblique modes. A first sharp rise in C_f is observed as these secondary waves reach large amplitudes. Subsequently, there is a slight dip or plateau, followed by a second rapid increase around $x \approx 1300$ that leads to the full turbulent C_f level downstream. The two-stage increase in C_f is a well-known signature of the FR breakdown [3, 4, 12]. With the introduction of a surface heating segment starting in the nonlinear regime, the transition process is substantially altered. For a moderate heating intensity ($\Theta = 0.1$), the onset of the C_f rise is shifted downstream relative to the uncontrolled case. The growth of C_f is also less steep under control, indicating a milder amplification of disturbances. For a stronger heating ($\Theta = 0.5$), the delay in the C_f rise is even more pronounced; in some of our high-intensity heating cases, the second rapid increase of C_f does not occur within the computational domain, implying that transition has been pushed beyond the domain extent. These results confirm that surface heating applied during the FR phase can effectively postpone the transition to turbulence.

Flow-field visualizations provide further evidence of the control mechanism in figure 2. In the uncontrolled flow, DNS results show the development of spanwise periodic Λ -vortices in the late nonlinear stage, which eventually break down into small-scale turbulence. When surface heating is applied, the emergence of these Λ -vortices is delayed. Three-dimensional vortex-core visualizations (using the Q -criterion) indicate that for higher Θ , the Λ -structures remain weaker and persist farther downstream before breaking apart, compared to the no-control case where they appear stronger and earlier. All of these observations are consistent with a scenario in which the secondary instability growth (the FR of oblique modes) has been subdued by the heating, thereby delaying the onset of turbulence.

To quantify the effect of heating on the secondary instability, we perform a local secondary instability analysis (SIA) on the base flow profiles extracted from the DNS at selected streamwise locations, following the approach of [13, 14]. In practice, the DNS fields are spanwise-averaged to construct quasi-2D base states, on which the linear eigenvalue problem for oblique secondary disturbances is solved. As shown in figures 3 (a) and (b), the Fourier components (0,1) and (1,1) experience significant amplification during the nonlinear phase due to the resonance mechanism, and this growth rate is reduced by the application of surface heating. Notably, surface heating has a limited effect on the evolution of the fundamental modes for a given wall temperature and Reynolds number. Moreover, by varying Θ , an approximately linear decrease of the growth rate is observed in panels (c) and (d), indicating a first-order stabilisation effect, which will be further explained by the asymptotic analysis in the next section.

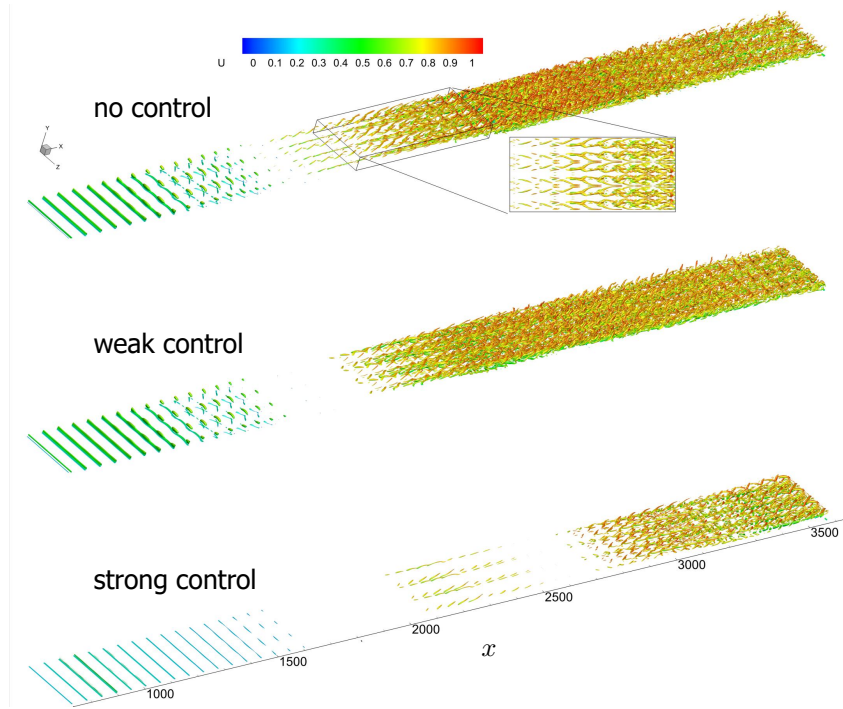


Fig 2. Visualization of flow structures using isosurface of Q -criterion ($Q = 5 \times 10^{-4}$) obtained from the DNS for case T1R1 with different heat intensity (no control $\Theta = 0$, weak control $\Theta = 0.1$, strong control $\Theta = 0.5$). The isosurface is coloured by the streamwise velocity magnitude.

4. Theoretical Analysis of the Control Mechanism

To gain deeper insight into how surface heating suppresses the FR, we develop an asymptotic theory for the nonlinear disturbance evolution under weak control. The FR is described by a triad consisting of the saturated planar fundamental mode $(1, 0)$, the streak mode $(0, 1)$, and the oblique pair $(1, \pm 1)$. The disturbance expansion follows the SIA framework, with the oblique wavenumber α_{11} taken real ($\alpha_{11} = \text{Re}(\alpha_{10})$) and amplitudes scaled so that $\bar{\epsilon}_{10} \gg \bar{\epsilon}_{01} \gg \bar{\epsilon}_{11}$.

4.1. Unheated resonance system

The fundamental mode satisfies Rayleigh equation with eigenvalue α_{10} . To capture finite- R effects, an $O(R^{-1/2})$ wall-layer analysis leads to a modified boundary condition linking wall-normal velocity and pressure at the wall. This correction substantially improves the accuracy of growth-rate predictions compared with the inviscid condition.

The coupled streak and oblique modes form a sixth-order linear system governing their mutual amplification. Wall-layer corrections of thickness $O((\bar{\epsilon}_{10}R)^{-1/3})$ provide improved boundary conditions that regularize the near-wall behaviour, yielding accurate predictions of the rescaled growth rate $\bar{\sigma}$. Further mathematical details of the asymptotic derivation can be found in [15].

4.2. Heating correction

Introducing a localised wall-temperature increase Θ modifies the growth rate. The correction of the growth rate is defined as

$$\bar{\sigma}_1 = \frac{\bar{\sigma}|_{\text{heated}} - \bar{\sigma}|_{\text{unheated}}}{\Theta}, \quad (4)$$

which can be decomposed into three contributions:

$$\bar{\sigma}_1 = \bar{\sigma}_{1,\text{CFW}} + \bar{\sigma}_{1,\text{CFP}} + \bar{\sigma}_{1,\text{MFD}}. \quad (5)$$

These terms correspond to corrections of the fundamental wavenumber (CFW), the fundamental eigenfunction (CFP), and the mean-flow distortion (MFD). The detailed mathematical expressions of each

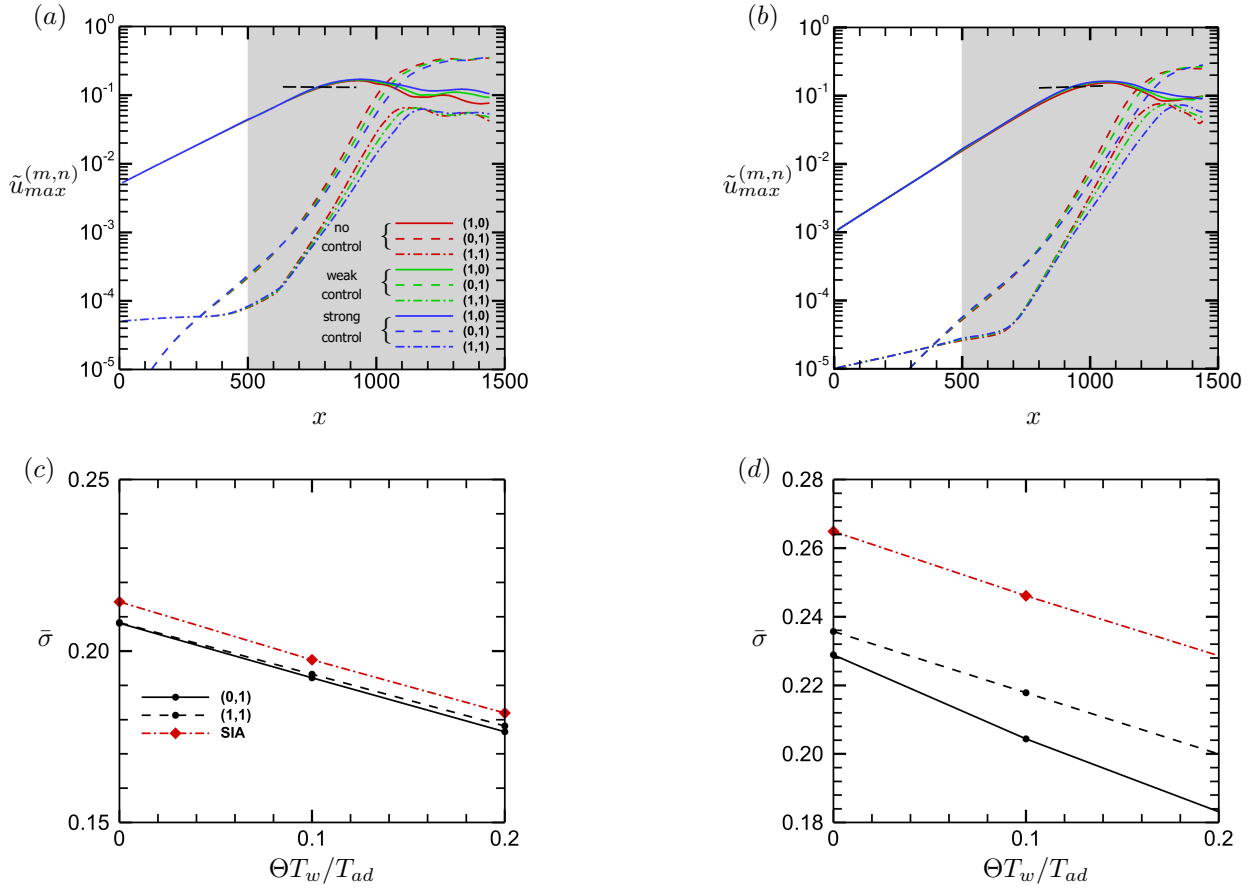


Fig 3. (a-b) Streamwise evolution of disturbance amplitudes $\tilde{u}_{max}^{(m,n)}(x)$ obtained by DNS for T1R2 and T2R2 with different heat intensity (no control $\Theta = 0$, weak control $\Theta = 0.1$, strong control $\Theta = 0.2$); dashed lines mark $\bar{\epsilon}_{10} = 0.065$. (c-d) At the streamwise location marked in (a) and (b), rescaled secondary growth rate $\bar{\sigma}$ vs heating intensity Θ , comparing (0, 1) and (1, 1) with SIA.

contribution can be found in [11]. These theoretical corrections provide a unified framework to quantify the influence of wall heating on the resonance growth rate. Figure 4 demonstrates that the asymptotic theory matched the SIA results for the growth rate $\bar{\sigma}$ and its first-order change $\bar{\sigma}_1$, especially as R increases. Notably, the growth-rate correction induced by surface heating maintains negative values, demonstrating its sustained suppression effect on the fundamental resonance process.

Figure 5 further resolves the heating effect into the three components introduced above. Consistent with the decomposition used here, the profile correction provides the primary suppressing contribution, whereas the mean-flow distortion exerts a promoting effect whose magnitude is slightly smaller than that of CFP; the wavenumber term is comparatively minor in the present parameter regime. Their superposition yields a net negative $\bar{\sigma}_1$ (overall suppression), and the magnitude of the combined suppression increases with R . These findings provide valuable insights for future control strategies, suggesting that approaches primarily targeting the fundamental mode eigenfunction while avoiding significant modifications to the mean-flow structure could lead to more effective laminar-flow control schemes.

5. Conclusions

We have investigated a new strategy for delaying transition in a hypersonic boundary layer by acting on the nonlinear instability mechanism known as fundamental resonance. Unlike traditional approaches that modify the flow during the linear growth phase of disturbances, our method applies a control measure—

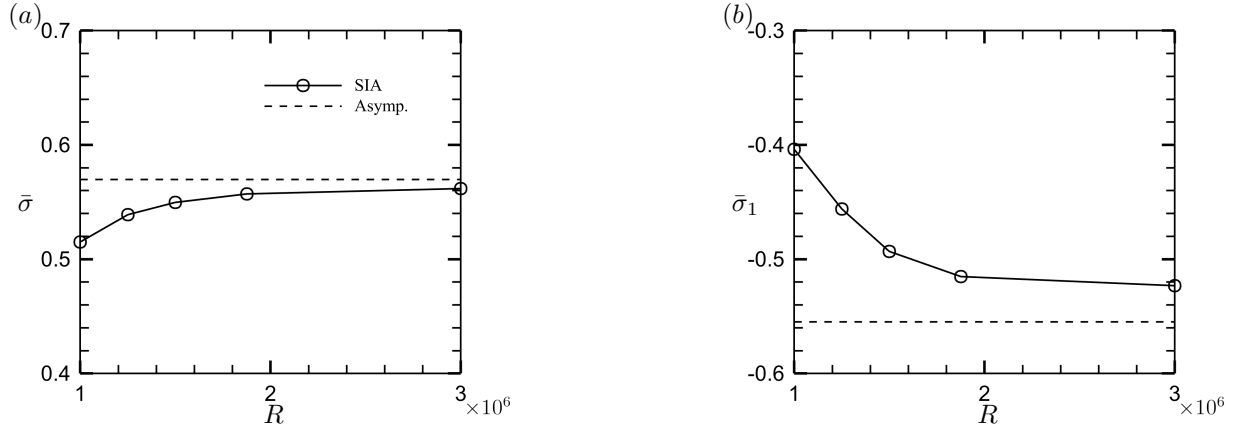


Fig 4. Dependence on R of the leading-order growth rate $\bar{\sigma}$ (a) and second-order growth rate $\bar{\sigma}_1$ (b) obtained by the SIA and asymptotic predictions for $T_w/T_{ad} = 1$, $\omega_0 = 0.120$, $\beta_0 = 0.1$ and $\bar{\epsilon}_{10}R \equiv 15000$.

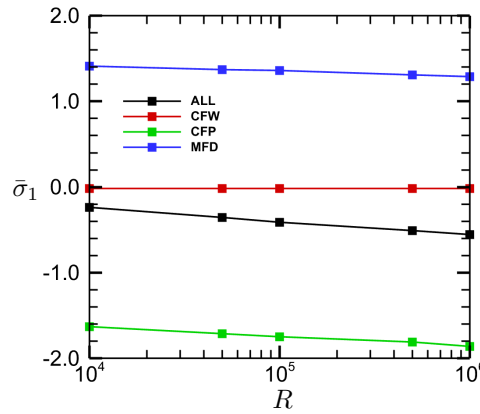


Fig 5. Variation of the contributing factors on the rescaled growth rate $\bar{\sigma}_1$. Results are shown for $\omega_0 = 0.120$, $\beta_0 = 0.1$ and $\bar{\epsilon}_{10} = 0.015$.

specifically, a localised surface heating patch—only during the late nonlinear stage where resonant secondary instabilities would otherwise rapidly amplify. Through direct simulations, we demonstrated that this nonlinear-phase control can substantially delay the onset of turbulence. In the Mach 5.92 flat-plate flow considered, activating a modest wall heating in the downstream portion of the laminar region led to a later transition location compared to the uncontrolled case. The delay was observed consistently across different wall temperature levels (adiabatic and cooled walls) and Reynolds numbers, indicating the robustness of the approach. The controlled cases showed a pronounced suppression of the growth of oblique instability modes and a postponed appearance of turbulent structures.

Secondary instability analysis confirmed that the surface heating has a stabilising effect on the resonant secondary (oblique) modes responsible for FR. Even a small increase in wall temperature in the nonlinear region produced a measurable decrease in the growth rate of these modes. The relationship between heating intensity and growth-rate reduction was found to be nearly linear for the range tested, simplifying the design of control amplitude for a desired transition delay.

An asymptotic theoretical analysis was developed to interpret the control mechanism. The theory revealed that the primary effect of the heating is to alter the eigenfunction of the fundamental Mack mode in a way that reduces its efficiency in driving the secondary instabilities. In contrast, the secondary

effect of heating—changing the mean flow—tends to slightly promote instability, but this is outweighed by the eigenfunction effect. Thus, the net impact of the heating is a reduction in the amplification rate of the secondary oblique waves. The asymptotic predictions captured the main features seen in DNS and SIA, including the approximately linear scaling of the stabilisation with heating intensity and the enhanced effectiveness of control at higher Reynolds numbers.

In summary, controlling the transition process by targeting the nonlinear FR stage appears to be a promising paradigm for hypersonic laminar flow control. This strategy offers a degree of robustness to changes in disturbance spectra, since it does not rely on cancelling or tuning out specific linear instability frequencies. The surface heating approach presented here is just one realization of this concept; other actuation methods (such as a localised suction/blowing strip or a small surface roughness designed to interfere with the resonance) could be envisioned to achieve a similar effect. The insights from this study suggest that an effective nonlinear-phase control should primarily target the disturbance eigenfunction while minimizing adverse mean-flow distortion. Such an approach could maximize the suppression of secondary instabilities like FR and thereby yield a greater delay in transition.

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