UV/IR Sodium PLIF for Hypersonic Flow Imaging: Diagnostic Development

Christopher J. Grunbok[1], Boris S. Leonov[2], Richard B. Miles[3]

Abstract

In this work, the authors present the development of a new approach to sodium planar laser-induced fluorescence (PLIF). The approach is a LIF scheme which excites the 3s-4p transition of sodium near 330 nm and detects fluorescence from the intermediate 3d-3p transition at 819 nm. The 3s-4p transition avoids optical thickness effects typically encountered with the sodium D-lines near 589 nm, and fluorescence from the 3d-3p transition is an intermediate excited state transition, which helps avoid self-trapping effects. This non-resonant scheme allows the use of simple long-pass filters to block both the 330-nm excitation laser light and the naturally occurring 589-nm emission from sodium in the targeted hypersonic and high enthalpy environments. This work demonstrates the use of this LIF scheme for planar imaging initially at a laser repetition rate of 30 Hz, and subsequently 250 kHz, in a benchtop sodium-seeded flame. The 250-kHz repetition rate is sufficient to capture dynamic phenomena in hypersonic and high enthalpy environments of interest, where sodium occurs nearly ubiquitously. Future work includes deployment of this approach in these environments to image flow fields within boundary or shear layers, post-shock environments, or combustion reaction/product zones.

Keywords: sodium, PLIF, hypersonic, high enthalpy

1. Introduction

Hypersonic and high enthalpy flows impose demanding requirements on the techniques used to measure critical properties of interest within the flow. High temperatures, microsecond timescales, and thermal nonequilibrium are a few among many challenging characteristics of these flows. As such, high-speed, nonintrusive diagnostics are required, and laser-based diagnostics have been developed to meet these requirements. They have been deployed extensively in hypersonic ground test facilities [1], and can capture important flow parameters including temperature, pressure, velocity, and species number density. Moreover, laser diagnostics can do so at high repetition rates, which are needed to capture the highly dynamic nature of these environments.

Many different laser diagnostic approaches have been developed, such as planar laser-induced fluorescence (PLIF) [2, 3] and coherent anti-Stokes Raman Scattering (CARS) [4], which continue to be matured further, or have their capabilities extended. Meanwhile, there are also ongoing efforts towards novel diagnostics such as Nitric-Oxide induced ionization Flow Tagging and Imaging (NiiFTI) [5]. The work presented here focuses on extending the capabilities of PLIF, which has been used for decades in hypersonic facilities. It can be used for measuring various properties mentioned above, and in two dimensions, which provides valuable insight into the flow field behavior. With high repetition rate lasers and fast sweeping of the laser sheet, three-dimensional data can be acquired. The development of optical parametric oscillators (OPOs) and dye lasers has enabled many different atomic and molecular species to be targeted for excitation with PLIF. Species such as acetone [6] can be seeded into low-enthalpy hypersonic facilities for subsequent excitation by the 4th harmonic of commercially available Nd:YAG lasers. When seeding is either undesirable or not possible, naturally occurring species are targeted instead. For example, in hypersonic flow, nitric oxide (NO) is often targeted [7]; in combustion studies, hydroxyl (OH) is frequently used [8].

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One other such naturally occurring species in both combustion and hypersonic environments is sodium, often as a contaminant. At high enough temperatures, sodium-containing compounds will dissociate, leaving sodium in its atomic form. The high temperature can also result in natural emission from the sodium atomic D-lines at 589 nm. As an example, an optical emission spectrum (OES) of the Texas A&M University Hypervelocity EXpansion Tunnel (HXT) during a high enthalpy test is shown in Fig. 1. The Na emission peak at 589 nm indicates the presence of sodium, and therefore the potential to use sodium as a diagnostic species in this hypersonic facility.

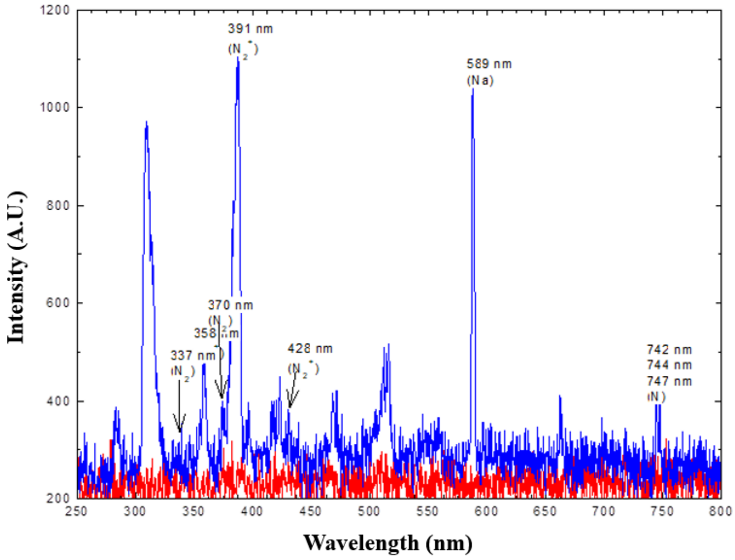

Fig 1. Optical Emission Spectra of HXT run. The sodium emission line occurs at 589 nm**.**

Other studies investigating combustion also observe the sodium emission line at 589 nm [9, 10], so sodium PLIF may be a useful technique in a variety of cases. If it does not occur naturally, sodium can be seeded into the flow and then excited with a laser at 589 nm. Miles et al. [2] used this method for the first demonstration of planar imaging in hypersonic flows, and performed velocity, pressure, and temperature measurements. Littleton et al. also seeded sodium in the University of Queensland's X1 expansion tube to perform flow tagging velocimetry [11]. Whether it occurs naturally or artificially, sodium has had a long history of use in 1D and 2D LIF applications for various types of measurements. Many such measurements rely on the sodium D-lines at 589 nm [12-18]. The sodium D-lines exhibit high-absorption cross sections, which is a primary factor in their usage because relatively modest laser pulse energies are sufficient to provide a strong signal. However, strong laser absorption, optical thickness, self-trapping, and background scattering from the 589 nm excitation laser limit this approach. In addition, the naturally occurring emission from the D-lines in the target environments competes with the fluorescence at the same wavelength. Despite the existence of line-reversal techniques and other methods, which yield successful results, eliminating these issues has potential payoffs because of the otherwise desirable characteristics of sodium.

The approach we present here is the use of the 3s-4p transition near 330 nm to excite atomic sodium, followed by detecting fluorescence at 819 nm from the intermediate 3d-3p transition [19, 20]. This is an approach first explored by Weiland et al. [10] in combustion environments; however planar imaging has not been previously explored. The ~50 times weaker absorption cross section of the 3s-4p transition avoids laser absorption loss and optical thickness effects. The fluorescence wavelength at 819 nm is spectrally remote from the excitation and natural emission wavelengths at 330 and 589 nm, respectively, and bypasses self-trapping issues because it is an intermediate upper-level transition. This is useful in the hypersonic environment because it simplifies filtering out the unwanted light from the excitation laser and the environment. In addition, the emission captured is from an atomic line, so it is narrowband and can be paired with a narrow linewidth spectral filter to improve the signal-to-noise ratio. The excitation laser can be swept in frequency to map out the absorption linewidth and spectral Doppler shifts, potentially enabling measurements of temperature, pressure and velocity.

In the energy diagram in Fig. 2, the excitation/de-excitation pathway considered is outlined by solid arrows. Following the 330-nm excitation, the 4p-3d transition at 9109 nm is assumed to be enhanced by stimulated emission and further assisted by a rapid energy exchange that becomes increasingly relevant at high temperatures. When $k_b T_{gas}$ exceeds the energy difference of the two states (~0.136 eV), collisional deexcitation becomes increasingly likely to occur. In the hypersonic environment, these temperatures are common [1, 21], and therefore the presence of 3d-3p fluorescence at 819 nm following 330-nm excitation is highly likely. The final transition to the ground state is the 3p-3s transition at 589 nm.

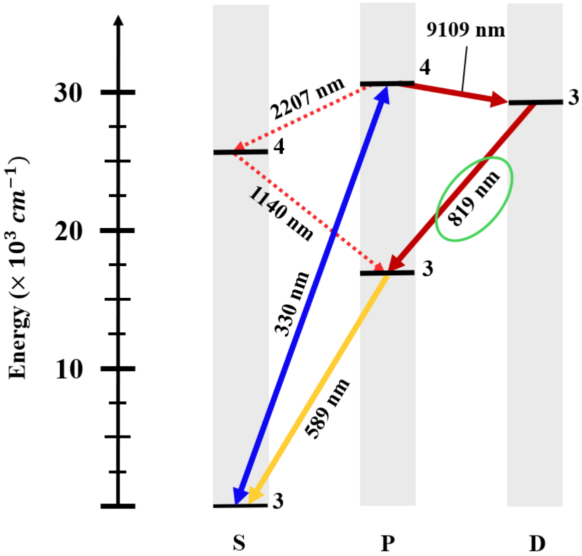

Fig 2. Energy diagram of sodium. Fluorescence wavelength measured is circled in green. Ref. [20].

Our work in Refs. [19] and [20] demonstrated planar imaging using this UV/IR approach to sodium LIF at 30 Hz [19] and 250 kHz [20]. 250-kHz is generally capable of resolving most dynamic phenomena in hypersonic flows [1], and so the latter demonstration strongly suggests the use of UV/IR Na PLIF for hypersonic flow visualization and imaging. In the case of unseeded flow, regions where high-enough temperatures exist to dissociate sodium from its molecular compounds – hypersonic shear layers, post-shock environments, and boundary layers – are potential imaging areas using this method. Combustion environments are also candidates for using this technique and are reflected in the previous work in which 330-nm excitation followed by 819-nm fluorescence has been applied. The work presented here discusses the development of the UV/IR Na PLIF method, and comments on future work investigating its utility as an effective approach for high repetition PLIF measurements in hypersonic environments.

2. Experimental Methods

The experimental setup utilized a custom-built OPO together with up-conversion mixing, shown in Fig. 3. The OPO is described in detail in Ref. [22]. The second harmonic (532 nm) of a narrow linewidth (injection seeded or burst mode) Nd:YAG laser is vertically polarized and is used to pump the OPO, which provides tunable laser radiation. The signal and idler beams of the OPO were tuned to 869 nm and 1370 nm, respectively. The signal beam at 869 nm was mixed with the residual 532-nm beam after it had passed through a polarization-rotating delay line to optimize pulse temporal overlap and rotate the pump polarization by 90 degrees. The two beams were mixed in a Type-I BBO mixing crystal, precut at $\theta = 36.8^{\circ}$ (12x12x7mm). The narrow linewidths of the OPO and the Nd:YAG laser enable precision tuning of the mixed output onto the 330-nm 3s-4p sodium transition. The pulse energies used ranged from 0.1 to 1 mJ per pulse. Pulse energies in this range are sufficient to induce saturation effects [10, 19, 20], depending on the laser sheet size. Hence, despite the reduced absorption strength of the 3s-4p transition in comparison to the 589-nm D-lines, appreciable LIF signal is still attainable.

It is noted that only quasi-saturation conditions can be said to be achieved here due to the spatial variation in the OPO beam profile and the pulse-to-pulse temporal behaviour. In some cases, beam profile corrections may be applied if the profile is monitored, though this is more frequently done in the weak excitation limit. The beam profile was not monitored in our previous works [19, 20], so no corrections were applied. Operation within a level of saturation of the LIF transition can alleviate the effects of shot-to-shot pulse energy fluctuations, which can be prominent in OPOs.

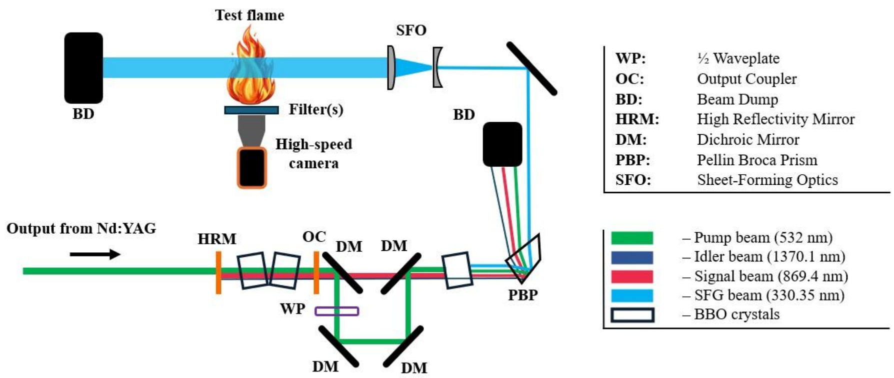

Fig 3. General setup for UV/IR Na PLIF.

The flame used in Refs. [19] and [20] was ethanol-based, into which NaCl salt and water were mixed. The NaCl was dissociated by the flame, providing atomic sodium necessary for excitation by the laser. While sodium often occurs naturally in combustion environments [12, 13], seeding was necessary here owing to the nature of the experimental setup. The use of UV/IR Na PLIF in hypersonic facilities requires somewhat further consideration on the method of introducing sodium into the flow. Atomic sodium may have varied contamination concentrations depending on the facility and the test conditions used, such as within the HXT. In some cases, this may hinder achieving adequate signal-to-noise ratios, despite relatively low sodium concentrations typically needed [10]. This scenario was encountered by Littleton et al. [11] where sodium seeding was necessary on the tunnel diaphragms for sufficient absorption within the test section, despite OES indicating sodium's natural appearance in the facility [23]. For boundary layer imaging, a similar approach may be necessary in which a sodium-containing solution is applied to the model surface and subsequently vaporized and dissociated by the high enthalpy air. This has the advantage of highlighting the flow in the boundary layer. Therefore, in some cases sodium seeding may be desirable regardless of natural occurrence in a hypersonic facility. Sodium also oxidizes in air, so the seeding method must be carefully considered.

The ability of the OPO to be run at high repetition rates enables high-repetition-rate Nd:YAG lasers and imaging equipment to be used. In Ref. [19], we used a 30-Hz Continuum Powerlite 9030 laser with a PCO.dicam C1 camera [19]. For the same flame, in Ref. [20], we used a QuasiMODO pulse-burst laser (Spectral Energies, LLC) operated at 250 kHz, with a Photron Pharsighted Camera (E9-100S) and a Lambert Instruments HiCATT Intensifier [20]. Both setups used a ZEISS Milvus 2/100M ZF.2 Lens. In the former case [19], a 695-nm long pass filter (SCHOTT RG695) was placed in front of the lens to block the laser and natural light. The flame was first observed without the excitation laser to determine the extent of background due to natural emission from the flame. It was determined that through the filter, negligible background emission was detected. Therefore, the PLIF images acquired with the laser excitation were assumed to consist almost entirely of the 819-nm fluorescence signal. In the high-repetition-rate case [20], two of the same 695-nm long pass filters were used, and the camera was operated at twice the laser repetition rate to capture background emission on alternate shots for time-accurate background subtraction. Background subtraction had a minimal effect on the original PLIF images taken of the flame, owing to the success of the filters. However, in hypersonic and high enthalpy flows, natural emission can be strong enough to be detected with the filters in place. Naturally occurring emission may also be emitted within the transmission range of the filters: for example, broad bandwidth nitrogen first positive emission overlapping the 819-nm spectral region may be present. This can be minimized using narrow linewidth spectral filters in future work, which pairs well with the narrow 3d-3p atomic fluorescence line. In high-enthalpy and hypersonic environments, background subtraction is sometimes performed to remove any natural emission that is detected [21] and to remove camera noise. Since these are our target environments, we continued to perform background subtraction here [20].

3. Results and Discussion

Figures 4 and 5 show resultant PLIF images acquired at 30 Hz and 250 kHz in a flame, respectively. In Fig. 4, the laser propagates from right to left, while in Fig. 5, the laser propagates from left to right. Flame structures are apparent, and since sodium typically recombines into other molecular compounds in flames and in air, sodium is not seen outside of the flame zone. The laser absorption across the flame volume is not distinguishable, owing to the oscillator strength of the 3s-4p transition being nearly two orders of magnitude smaller than that of the sodium D-lines, which commonly experience absorption issues. Further analysis of this is found in Ref. [20].

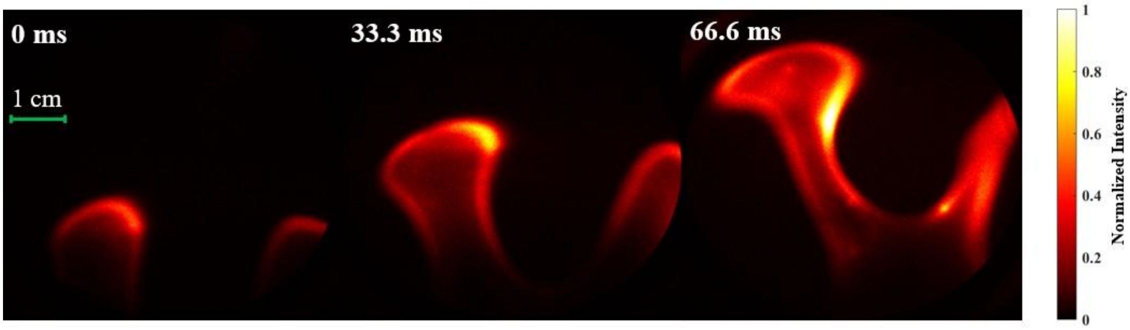

Fig 4. Sequential-shot PLIF images taken at 30 Hz from work in Ref. [19].

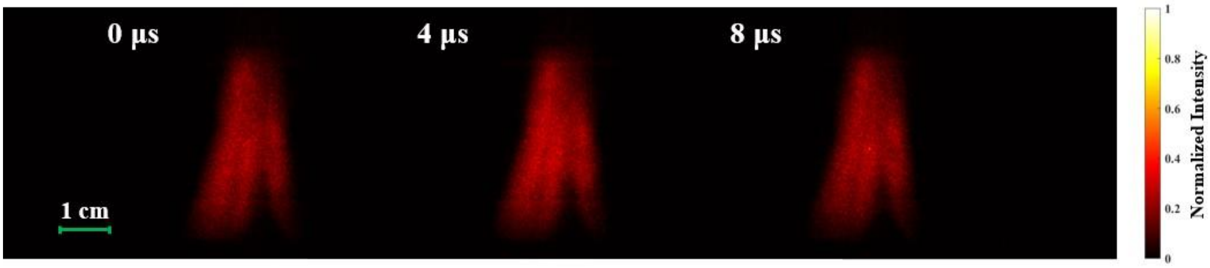

Fig 5. Sequential-shot PLIF images taken at 250 kHz from work in Ref. [20]. The flame structure is slow moving and appears to remain fixed over the entire burst train, so no variation is seen here.

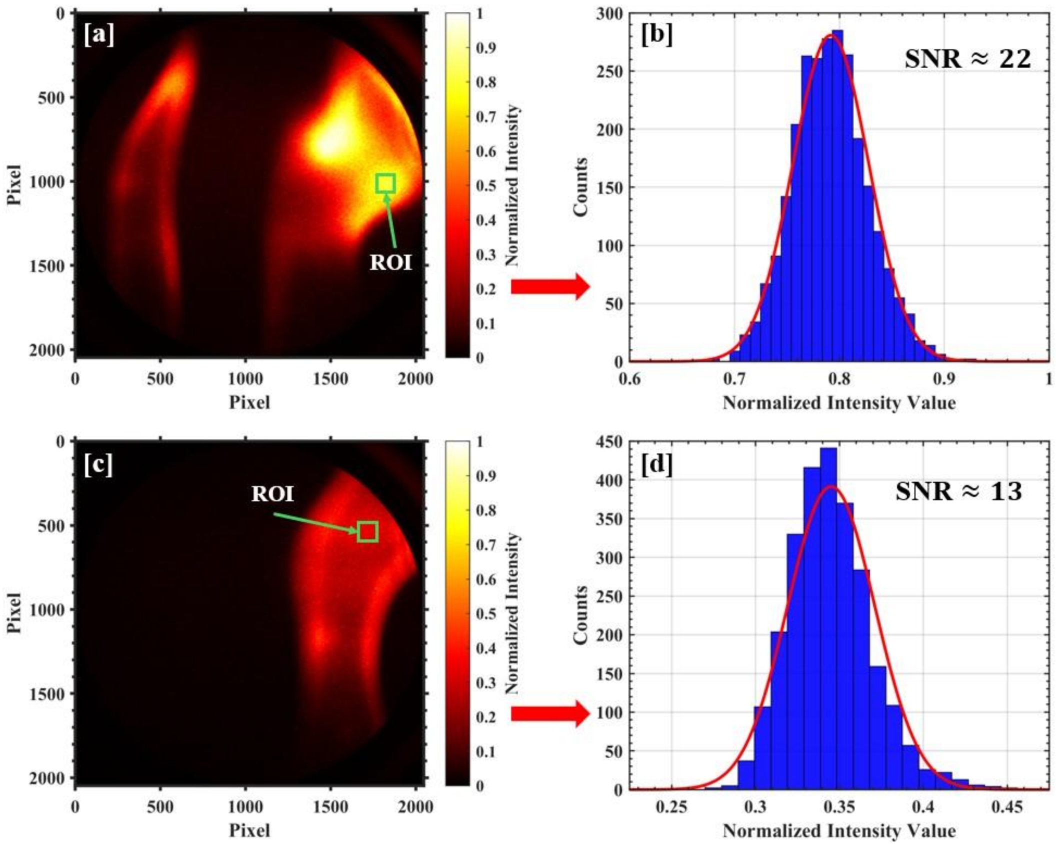

Fig 6. (a, c) PLIF images recorded in Ref. [19]; (b, d) ROI histogram and resultant SNR. Each ROI is 50x50 pixels, and each histogram has 30 bins.

Because this approach to Na PLIF is relatively new, characterization of this technique is important to provide insight into its practical utility. One important aspect is the attainable signal-to-noise ratio (SNR), as good SNRs are critical if quantitative measurements are desired. SNR here is defined by the average pixel value in a selected region of interest (ROI) divided by the standard deviation of the pixel values in the same region. The flame in Refs. [19] and [20] shows a high degree of variability in sodium distribution, as seen in Figs. 4 and 5. Defining SNR as I_{ave}/σ_{roi} renders it somewhat difficult to provide a single value of the SNR because choosing a uniform ROI that is also adequate in size is somewhat ad hoc for this flame. Even so, acceptable ROIs can still be found in some instances, examples of which are shown in Figs. 6 along with the corresponding SNR. Because of this variability, though, more uniform flames such as a Hencken or Bunsen burner may provide a comprehensive insight into expected SNRs. A calibrated reference cell may also be used.

Though not unique to the approach described herein, aspects of the imaging system, such as camera resolution (DPI and bit depth), can affect the SNR. When an intensifier is paired with a camera as was done in Ref. [20], the images can appear coarse, reducing the SNR even further – despite the measurement environment being the same as in Ref. [19]. Single-shot SNRs of 6 were obtained in the study in Ref. [20], details of which can be found there. The same challenge of image coarseness has also been discussed previously during NO PLIF experiments [24, 25]. One method of reducing its effects in fluid dynamics applications is through robust principal component analysis (RPCA), which is described in further detail in Ref. [26]. The SNR can be increased significantly [24], and so this technique could potentially be applied to future work using the UV/IR Na PLIF approach.

Monitoring the fluorescence with a reference cell simultaneously during experiments may be desirable to check for any effects of varying fluorescence due to energy fluctuations. It also may be used to monitor the laser wavelength overlap with the desired excitation transition [21]. A notable consideration in doing so is that the 819-nm fluorescence following 330-nm excitation has only been discussed for sodium-containing environments above 300 ℃ [10, 27, 28]. In the current excitation/detection scheme, it is assumed that if $k_b T_{gas}$ is comparable to the 4p-3d energy difference (0.136 eV ≈ 1600 K), then the 4p-3d down-transition is assisted; otherwise, this down-transition, and therefore the 3d-3p fluorescence, may not be as significant. This aspect requires further investigation because the 4p-3d transition is already enhanced by stimulated emission, so the 3d-3p fluorescence is potentially still observable in the absence of highly elevated temperatures. This would simplify the experimental set-up with respect to the use of a sodium reference cell, making quantitative measurements more viable.

4. Conclusion

We have presented an overview of previous work discussing a potential new approach to Sodium PLIF for use in hypersonic and high-enthalpy flows, having had success in combustion environments. The LIF scheme excites the 3s-4p transition of sodium and detects fluorescence from the intermediate 3d-3p transition, access to which is assumed to be assisted by rapid collisional de-excitation by the high temperature environments that this approach targets. This approach has been demonstrated in a tabletop flame at repetition rates up to 250 kHz, a repetition rate that is too high to capture dynamic phenomena in the flame presented, but which is required in order to resolve hypersonic flow properties properly. Sodium occurs often as a natural contaminant in these environments, which lends itself to using it as the trace species for LIF, though it should be assessed whether additional seeding is needed. Overall, this approach has shown the ability to circumvent some of the common issues associated with the sodium D-lines where self-trapping and laser absorption issues are reduced.

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5. References

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