HiSST: 4th International Conference on High-Speed Vehicle Science & Technology

22–26 September 2025, Tours, France

Surrogate model-based strategy to assess the influence of the atmospheric dispersions on the performance evaluation of a hypersonic vehicle

Mathieu Balesdent[1] *Loïc Brevault*[2] *Arnault Tremolet*[3]

Abstract

The design process of hypersonic vehicles requires trajectory optimization. For that purpose, the trajectory simulation involves an atmosphere model that is used to predict the surrounding air flow conditions all along the trajectory. These conditions are affected by variability depending on several parameters such as the season, the location on Earth, the altitude, *etc.* This paper presents a comprehensive approach to assess the influence of the dispersions in atmospheric states on the performance evaluation of hypersonic vehicles. The proposed method models atmospheric variability as a stochastic process and combines model order reduction and Gaussian process regression in order to build a surrogate model of the performance criterion as a function of the atmospheric dispersion. This surrogate is eventually used to perform uncertainty quantification studies to evaluate the robustness of the hypersonic vehicle performance with respect to the atmospheric variability.

Keywords: *Atmospheric conditions, uncertainty quantification, Gaussian-process, stochastic process*

1. Introduction

Evaluating the performance of a hypersonic vehicle often involves trajectory optimization, which entails optimizing the guidance of the vehicle while solving a system of ordinary differential equations (ODEs) representing the flight dynamics [6]. The performance of a hypersonic vehicle using air-breathing propulsion (turbojet, ramjet, scramjet) is directly affected by the air flow and the atmospheric conditions (*e.g.*, air density, air temperature) in the vicinity of the vehicle. These atmospheric conditions mainly vary according to the altitude along with the season, the location on Earth, the weather, *etc.* Classical models of atmosphere (*e.g.*, US Standard Atmosphere 1976 [9], International Standard Atmosphere [8]) provide atmospheric "profiles" of temperature, pressure, density as a function of the altitude (Figure 1) in the different layers of the atmosphere (*e.g.*, troposphere, stratosphere, mesosphere).

Designing hypersonic vehicles requires to simulate and optimize trajectories for a large diversity of case studies. The case studies parameters include different times and locations on Earth. However, the atmospheric conditions can greatly vary between the different seasons or the locations on Earth and suffer from aleatory uncertainties resulting from the weather conditions. Consequently, it is necessary to take into account these variations in order to ensure a reliable performance estimation for the whole flight conditions. A major challenge in accounting for variations of atmospheric conditions when estimating hypersonic vehicle performance lies in the probabilistic modeling of these uncertain atmospheric conditions and their propagation through the trajectory optimization.

The aim of this work is to propose a methodology to model atmospheric variability and to estimate its influence on the performance of a hypersonic vehicle. The proposed approach (Figure 2) employs a surrogate model-based strategy to carry out uncertainty propagation from the dispersion of atmospheric conditions through trajectory optimization, ultimately obtaining the dispersion of the vehicle performance. In the first step, the dispersion of atmospheric conditions (*e.g.*, temperature as a function of the altitude) is modeled as a stochastic process to account for the inherent variability of local temperature while considering its strong dependence on altitude. To reduce the dimensionality of this

[1] *DTIS, ONERA, Université Paris-Saclay, 91120 Palaiseau, France mathieu.balesdent@onera.fr*
[2] *DTIS, ONERA, Université Paris-Saclay, 91120 Palaiseau, France loic.brevault@onera.fr*
[3] *DTIS, ONERA, Université Paris-Saclay, 91120 Palaiseau, France arnault.tremolet@onera.fr*

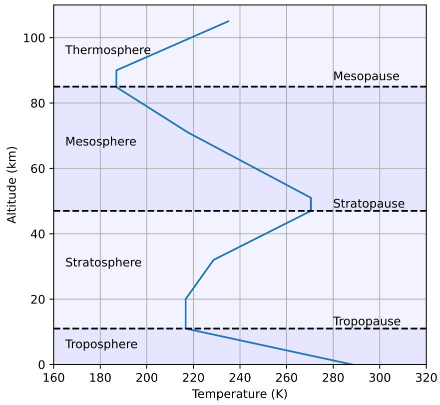

Fig 1. Example of temperature profile as a function of the altitude, International Standard Atmosphere model [8]

functional model and make it compatible with a surrogate model-based strategy, a Karhunen–Loève decomposition [7] is performed using a limited dataset of representative atmospheric conditions, defining a latent space. In a second step, a trajectory optimization is carried out for each atmospheric condition in the limited dataset to estimate the hypersonic vehicle performance. Subsequently, a surrogate model in the form of a Gaussian process [11] is trained to map the latent space representing the atmospheric conditions to the vehicle performance. This surrogate model enables uncertainty propagation via Monte-Carlo simulations, allowing the estimation of performance dispersion metrics (*e.g.*, quantiles) thereby quantifying the vehicle robustness to variations in the atmospheric conditions.

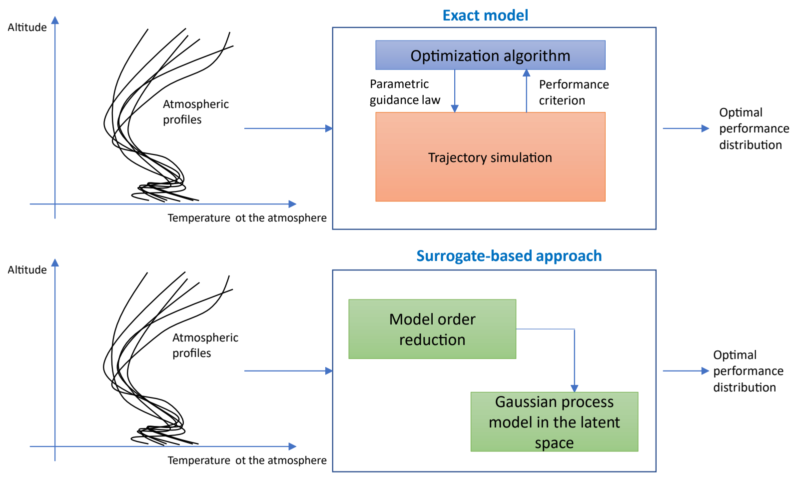

Fig 2. Uncertainty propagation using exact model (top), proposed surrogate-based approach (bottom)

The rest of the paper is organized as follows. In Section 2, the modeling of atmospheric dispersions as a stochastic process is described. Section 3 presents the surrogate-based approach for uncertainty quantification. This approach is then applied to the performance evaluation of a generic hypersonic vehicle in Section 4.

2. Analysis of atmospheric dispersions

Atmospheric conditions greatly vary depending on a large number of factors, such as the location on Earth, the seasons, the weather conditions, *etc.* Examples of such dispersions with the temperature as a function of the altitude are given in Figure 3. In this paper, only the temperature dispersion is considered, without loss of generality of the proposed approach about other quantities (*e.g.*, density, humidity).

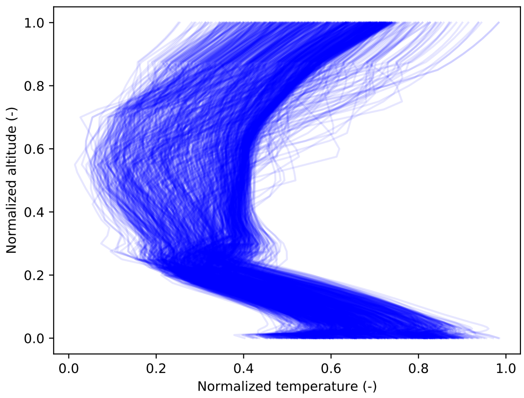

Fig 3. Example of dispersions of the temperature as a function for the altitude for different seasons (normalized data)

These atmospheric dispersions can be modeled as a collection of random variables indexed by the altitude. To analyze this joint probability distribution, marginal distributions of the normalized temperature at different altitudes are displayed in Figure 4.

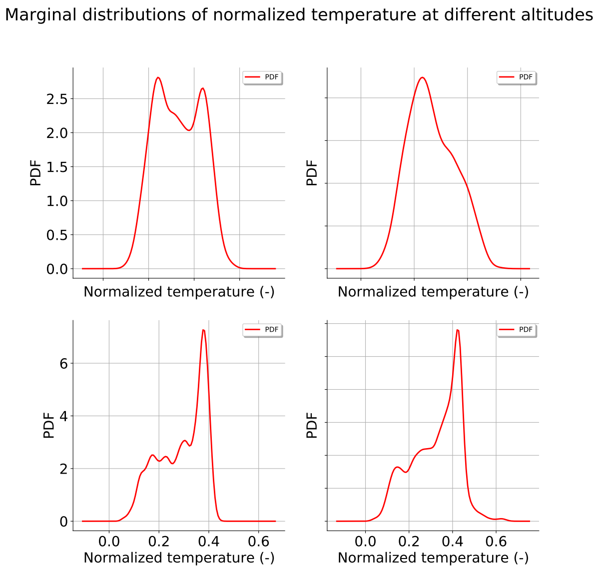

Fig 4. Marginal PDF of the normalized temperature at different altitudes

As depicted in this figure, the probability distribution of the temperature greatly varies from one altitude to another. Consequently, assuming a single probability distribution function (PDF) for all the altitudes is not a valid hypothesis. Furthermore, the PDFs cannot be modeled using classical univariate distributions (Normal, Weibull, LogNormal, *etc.*). Another important feature in terms of probabilistic modeling is the correlation (or statistical dependence) of the distributions at different altitudes. Indeed, the correlation of atmospheric conditions at two different altitudes h_1 and h_2 can greatly vary depending on the considered atmosphere layers. This is depicted in Figure 5 which illustrates the covariance matrix of the temperature. One can see in this figure that the temperatures at high altitudes are more correlated than temperatures at medium altitudes.

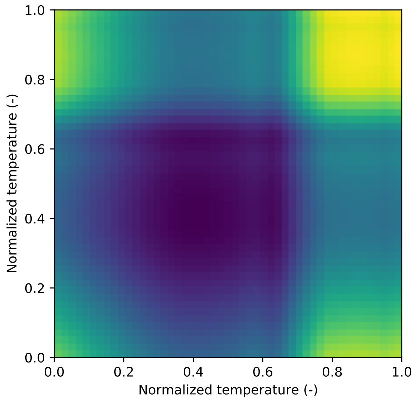

Fig 5. Covariance matrix of the normalized temperature

Consequently, the correlation of the uncertainty distribution for the different altitudes needs to be carefully defined. Using the Sklar theorem [5], a joint PDF of an aleatory vector can be decomposed between the influence of the marginal distributions (individual distributions of the variables composing the aleatory vector) and the influence of the statistical dependence between these variables. Figure 6 represents the statistical dependence of the atmospheric uncertainty for different layers of the atmosphere. Similarly to the marginal distributions, the shape of the statistical dependence suffers from large variations with the considered altitudes.

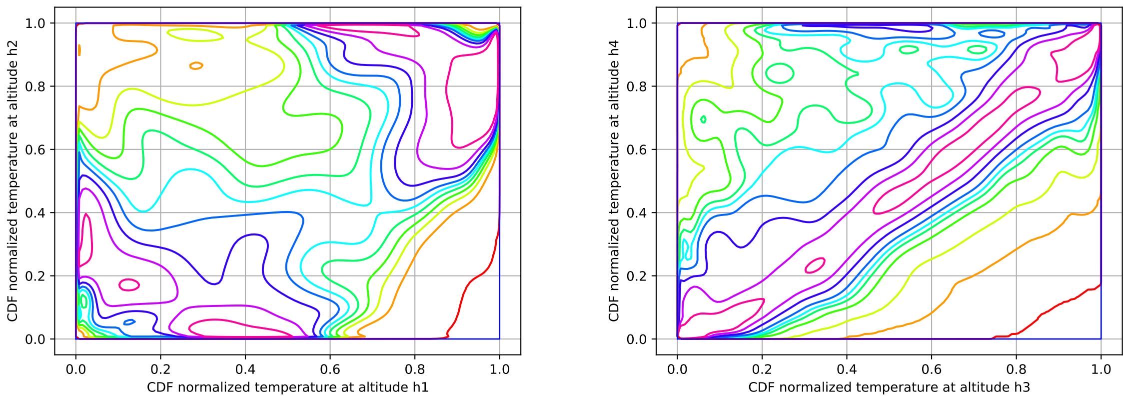

Fig 6. Statistical dependence analysis using copula for different layers of the atmosphere, on the left a statistical dependence in the tails of the distributions is visible

Let us suppose a probability space $(\Omega,\ \mathcal{A},\ \mathbb{P})$ with Ω the sample set, $\mathcal{A}$ an event space and $\mathbb{P}$ a probability measure. Using this probability formalism, the atmospheric conditions can be modeled as stochastic processes indexed by the altitude. For example, the temperature can be defined as $T(h,\omega)$ with h the altitude and $\omega \in \Omega$. In that way, for a given altitude, h_0, $T(h_0, \omega)$ represents a random variable defined using a marginal PDF. For a given value of $\omega = \omega_0$, $T(h, \omega_0)$ is a realization of the stochastic process, that represents a profile of temperature according to the altitude. The different profiles of temperature depicted in Figure 3 correspond to different realizations of the stochastic process $T(h, \omega)$. A stochastic process can be characterized by the definition of a collection of margins on a finite discretization of the altitude $\{h_1, \ldots, h_n\}$ with n the number of nodes; and a copula, as illustrated in Figure 7 for two altitudes. The number of nodes can be large in order to precisely describe the atmospheric evolution depending on the altitude.

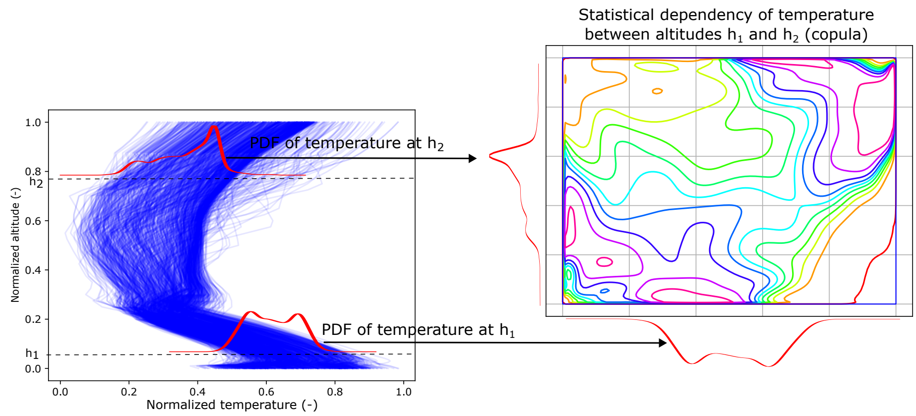

Fig 7. Illustration of probabilistic modeling of the stochastic process of the temperature as a function of the altitude

3. Goal-oriented surrogate-based approach for uncertainty quantification of atmospheric dispersions

In this section, an approach is proposed to build a surrogate model in order to carry out uncertainty quantification of the impact of atmospheric dispersions on a generic performance criterion. The estimation of hypersonic vehicles performance typically involves the optimization of the trajectory. This includes the determination of the best guidance law in order to minimize (or maximize) a given criterion (*e.g.*, the consumption). This optimization implies the integration of a system of ordinary differential equations according to the time. As a representative example, a simplified three-degrees-of-freedom system of equations of motion can be written as:

$$
\begin{aligned}
\dot{r} &= v\ sin(\gamma) \\
\dot{v} &= \frac{T\ cos(\alpha) - D}{m} - g\ sin(\gamma) \\
\dot{\gamma} &= \frac{L + T\ sin(\alpha)}{mV} - \frac{g\ cos(\gamma)}{v} + \frac{v\ cos(\gamma)}{r} \\
\dot{m} &= -q
\end{aligned}
$$

with r the radius, v the relative velocity, γ the flight path angle, T the thrust, D the drag, L the lift, α the angle of attack, m the mass and q the mass flow rate. This system of ODEs can be numerically integrated using for example a Runge Kutta algorithm [4] in order to provide the evolution of the state variables as functions of the time. For hypersonic air-breathing vehicles, both thrust, drag and lift coefficients depend on the ambient atmospheric conditions. Consequently, variations of such atmospheric conditions will induce variations of the computed terms present in the right hand side of the equations of motion. Thus, depending on the different atmospheric conditions, the resulting optimal control laws on angles such as pitch, angle of attack, steering as well as the mixture ratio will vary in the optimization process in order to reach the final trajectory conditions in terms of altitude, velocity, *etc.* and to ensure the stability of the vehicle.

To estimate the performance and robustness of the system, the naive approach that would consist in sampling by Monte-Carlo a large number of atmospheric conditions and optimizing the trajectory for these realizations of atmosphere is not affordable at a reasonable numerical cost. Indeed, this approach typically requires three nested loops with trajectory simulation (ODEs integration involving computational codes such as aeropropulsive model at each time step) embedded into the trajectory optimization, itself embedded into the Monte-Carlo sampling. In this paper, an approach relying on surrogate models is proposed. We suppose here that the goal is to estimate the dispersion of a generic quantity of interest (QoI, for example, the consumption of the vehicle) due to the atmospheric variability, without loss of generality. The proposed method is decomposed into several steps:

1. perform a model order reduction on the atmospheric dispersion through Karhunen–Loève (KL) decomposition in order to define a latent space corresponding to the coordinates of the atmospheric profiles in the Karhunen-Loève decomposition. This latent space is of lower dimension compared to the initial discretization of the atmosphere temperature as a function of the altitude;
2. select a limited number of atmospheric conditions and perform trajectory optimization to get the QoI values;
3. build a surrogate model of the relationship between the latent space (input) and the QoI (output), and carry out validation process on the surrogate model;
4. use this surrogate model to perform uncertainty quantification to estimate the uncertainty measure (here a quantile).

The main steps that are the model order reduction, the surrogate model construction as well as an example of uncertainty quantification study are described in the following.

3.1. Model order reduction of atmospheric variability

In practice, the functional model of the temperature as a function of the altitude is discretized on a mesh with a large number of nodes. Using directly a stochastic process on such a large dimensional space is often untractable with the available data. Instead, a functional Principal Component Analysis (also referred to as Karhunen-Loève decomposition in the stochastic context) can be used to reduce the dimension, by using a spectral decomposition of the stochastic process [7]. Using this formalism, any realization of the stochastic process **T** (here the temperature) may be expanded over an estimated basis (ω is dropped in the equation for the sake of conciseness):

$$\mathbf{T}(h) = \mu(h) + \sum_{k=1}^{\infty} \sqrt{\lambda_k}\xi_k \mathcal{L}_k(h) \tag{1}$$

with λ_k and $\mathcal{L}_k$ the eigenvalues and eigenvectors of the autocovariance function of the stochastic process. $\mu(\cdot)$ is the mean function, $(\xi_k)_{k\geq 1}$ are, in the latent space, the coordinates of the stochastic process realizations with respect to the deterministic basis functions $\mathcal{L}_k(\cdot)$. A set of orthonormal random variables $(\xi_k)_{k\geq 1}$ is defined by all the possible realizations of the stochastic process. Each random variable ξ_k involved in the KL decomposition is defined by a linear transform (using the orthonormality of the eigenfunctions):

$$\xi_k = \frac{1}{\sqrt{\lambda_k}} \int_{\mathcal{T}} \left(\mathbf{T}(h) - \mu(h)\right) \mathcal{L}_k(h) \mathrm{d}h \tag{2}$$

with $\mathcal{T}$ the integration domain. For numerical purposes, the KL expansion is truncated to the first N_{KL} most significant modes (with $N_{KL} \ll n$). These latter correspond to the most significant eigenvalues and in practice, the KL expansion corresponds to:

$$\mathbf{T}(h) \simeq \mu(h) + \sum_{k=1}^{N_{KL}} \sqrt{\lambda_k}\xi_k \mathcal{L}_k(h) \tag{3}$$

In that way, the stochastic process can be defined in the latent space through the random variables ξ_k whose dimension is significantly lower than the number of nodes. For the temperature, the normalized basis functions ($\mathcal{L}_k(h)$) are given in Figure 8. This model has been tested on a validation set of atmospheric conditions through embedding into the latent space, mapping back to the original space and analyzing the residuals (Figure 8). We can see a precise agreement between the estimated samples resulting from the KL decomposition and the reference ones.

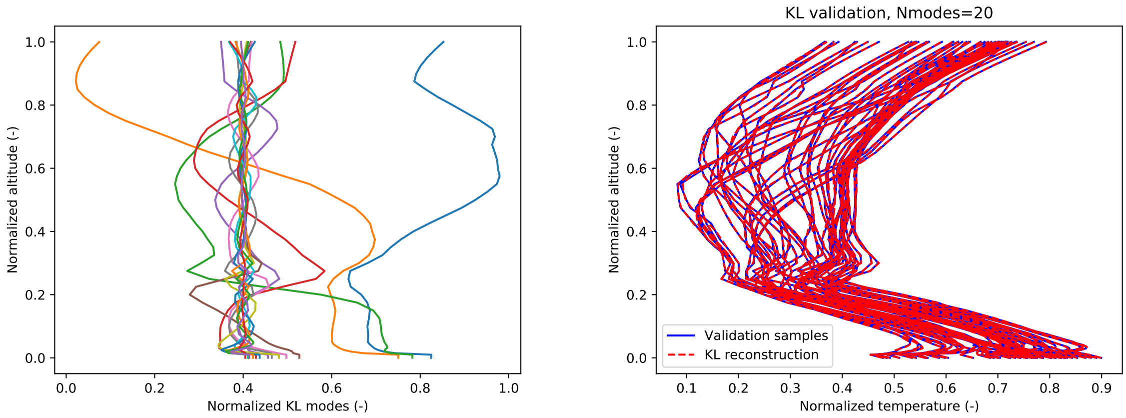

Fig 8. Normalized basis functions (modes) of the KL decomposition of the temperature (left). Validation of KL-decomposition for a decomposition of 20 basis functions over a validation set (right)

3.2. Surrogate model of the performance metrics as a function of the latent variables

In order to perform uncertainty quantification on the performance metrics, one needs to assess the mathematical relationship (mapping) between the atmospheric stochastic process **T** and the QoI y. A classical way is to define this mapping from the latent space on the ξ vector:

$$\begin{aligned} f : \mathbb{R}^{N_{KL}} &\rightarrow \mathbb{R} \\ \boldsymbol{\xi} = [\xi_1, \ldots, \xi_{N_{KL}}] &\rightarrow y \end{aligned}$$

The KL decomposition allows to reduce the input space for the mapping and therefore allows classical surrogate modeling techniques to be applied. In this work, conditioned Gaussian processes (GP) [10] are used to define this mapping. A Gaussian process is a stochastic process such as any finite collection of its random variables has a multivariate Gaussian distribution. A GP is fully determined by its mean and covariance functions. This covariance function is often defined through a parametric kernel function whose parameters are optimized by a supervised learning process (as well as the mean function parameters). The kernel function allows to model the covariance between two locations on the input space ξ and ξ' and is often a function of the distance between ξ and ξ' in the latent space (hypothesis of stationary model). After all the GP hyperparameters have been determined, the GP is conditioned on the data of the DoE $\{(\boldsymbol{\xi}_1, y_1), \ldots, (\boldsymbol{\xi}_N, y_N)\}$ with N the size of the DoE. Then, the GP allows to provide both the prediction at an unknown ξ and also the estimate a measure of its uncertainty through the predictive variance (Figure 9). Gaussian processes have been widely used in association with KL decomposition to define a mapping between the latent space and the quantity of interest [2, 3]. Here, the training data are composed of the projected atmospheric realizations on the latent space and the corresponding obtained QoI resulting from the trajectory optimizations.

3.3. Summary of the overall approach

In this section, the overall approach for uncertainty quantification is summarized.

- First, a DoE of N atmosphere realizations is performed. From each of these realizations, a trajectory optimization is performed in order to get the QoI value. At the end of this first step, a collection of N atmosphere realizations $\{\mathbf{T}_1, \ldots, \mathbf{T}_N\}$ and their corresponding QoI values $\{y_1, \ldots, y_N\}$ are available. This step requires the main part of the computational cost of the

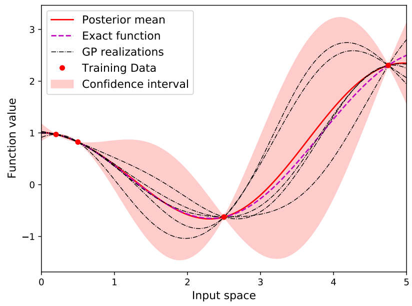

Fig 9. Generic Gaussian process prediction and associated measure of uncertainty, taken from [2].

approach.

- The second step consists in reducing the model order of the stochastic process **T**. For that purpose, a Karhunen-Loève decomposition is used in order to project the stochastic process realizations on a latent space of smaller dimension described by the basis formed by the eigenvectors $L_k(\cdot)$ and the latent variables coordinates $\boldsymbol{\xi} = \{\xi_1, \ldots, \xi_{N_{KL}}\}$. At this step, each realization of the stochastic process corresponds to a value of the $\boldsymbol{\xi}$ vector in the latent space. At the end of the second step, a correspondence between the realizations of stochastic process, their corresponding coordinates in the latent space and their QoI values is available: $\{(\mathbf{T}_1, \boldsymbol{\xi}_1, y_1), \ldots, (\mathbf{T}_N, \boldsymbol{\xi}_N, y_N)\}$.

- The third step consists in defining the mapping between the latent space and the QoI. To do so, a Gaussian process regressor is trained on the DoE composed of : $\{(\boldsymbol{\xi}_1, y_1), \ldots, (\boldsymbol{\xi}_N, y_N)\}$. At the end of the third step, the surrogate model is completely defined and can be used for uncertainty quantification.

- The last step consists in carrying out uncertainty quantification using the surrogate model. Example of uncertainty quantification studies (step 4) consists in sampling a large number of atmosphere realizations and projecting them in the latent space using the embedding mapping of the model order reduction. Once projected, the prediction of the performance criterion is carried out with the conditioned Gaussian process using the corresponding coordinates in the latent space. Finally, statistical metrics can be estimated on this large sample of performance criterion, such as mean, variance, quantile, PDF, *etc*. The overall approach is summarized in Figure 10. Confidence intervals on the estimated statistical metrics combining the Monte-Carlo simulation variability and the uncertainty of the Gaussian Process can eventually be assessed.

4. Application to hypersonic vehicle performance estimation

The proposed approach is applied in this section to the performance estimation of a generic air-breathing hypersonic vehicle. The goal is to estimate the effect of the atmosphere variability on the fuel consumption of the vehicle resulting from trajectory optimization. A first DoE of 400 atmosphere realizations has been randomly sampled based on an in-house ONERA model. From these latter, first the model order reduction approach is carried out, using the OpenTURNS library [1]. Figure 11 depicts the relative error of the model reconstruction with respect to the number of selected basis functions. In this work, selecting 20 basis functions corresponds of a relative error of 0.1% that is sufficiently accurate for the purpose of this study (Figure 11). It corresponds to a trade-off between the number of basis functions (and therefore the input dimension of the GP) and the relative error introduced by the model order reduction. This relative error is computed by projecting a validation set of atmosphere realizations in the latent space, then do the reverse-projection into the original space and comparing the reconstructed

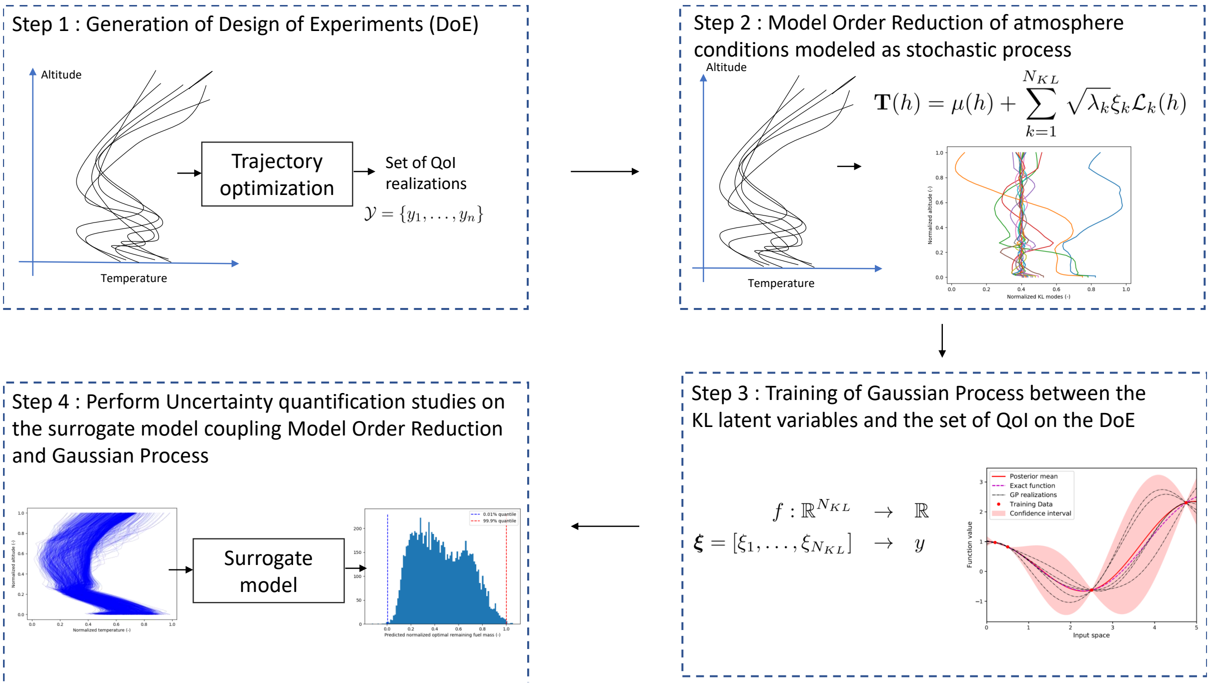

Fig 10. Overall approach for uncertainty quantification based on surrogate coupling Model Order Reduction and Gaussian Process

samples in the original space and the exact atmospheric realizations. Figure 12 illustrates the error between exact and estimate values using 1, 3 and 20 basis functions in the KL decomposition.

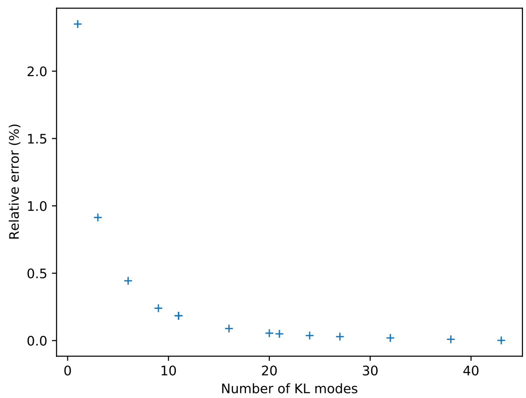

Fig 11. Relative error (in percentage) as a function of the number of selected KL basis functions (modes)

From the first DoE, trajectory optimizations have been carried out in order to get the values of the performance criterion. In this study, the remaining mass of fuel (the more, the better) has been set as the QoI. Figure 13 illustrates the resulting optimal normalized trajectories, with normalized relative velocity as a function of the normalized altitude for the different realizations of the atmospheric conditions.

The uncertainty quantification study consists in estimating the extreme quantiles (upper and lower) on the remaining fuel. To do so, a large number of atmosphere realizations have been sampled. Then, these realizations have been projected into the latent space to get the latent variables coordinates.

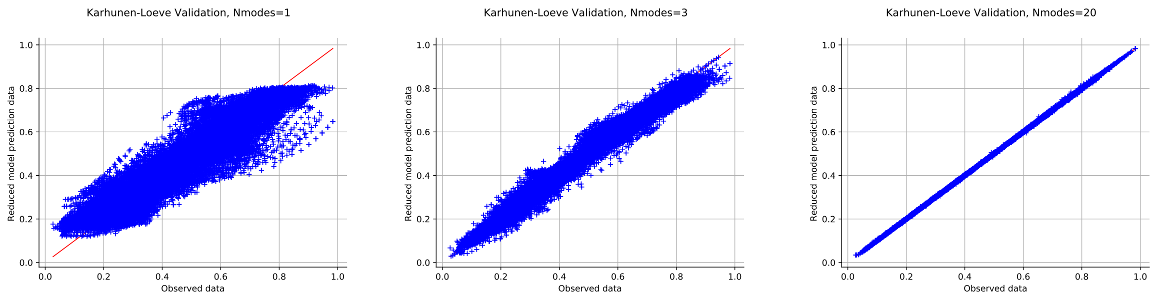

Fig 12. Errors of Model Order Reduction using 1 (left), 3 (center) and 20 (right) basis functions in the KL decomposition

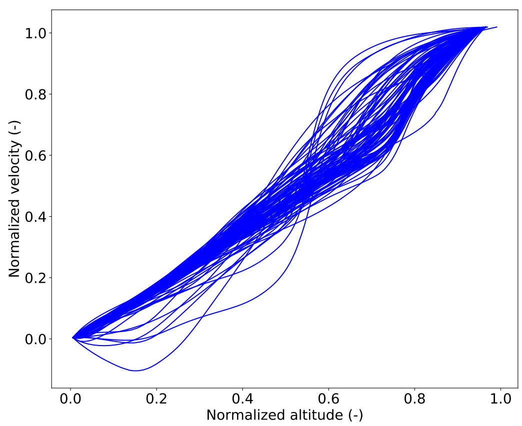

Fig 13. Optimal trajectories : velocity as a function of the altitude (normalized values for different realizations of the atmospheric conditions)

From these latter, the conditioned Gaussian process is used to predict the values of the remaining fuel mass. Histogram of this quantity is displayed in Figure 14. On this output sample, extreme quantiles can be eventually assessed. The PDF of the performance criterion can also be fitted in order to perform other uncertainty quantification studies. In addition, from these quantiles on the quantity of interest, it is possible to analyze which atmospheric profiles lead to these quantiles. One can mention that these profiles correspond to existing atmosphere realizations and are not determined based on some ”conservative margins”.

5. Concluding remarks

This paper describes a surrogate model-based approach for the uncertainty quantification of hypersonic vehicle performance due to atmospheric variability. Indeed, uncertainty quantification in the context of trajectory optimization involves a three nested loop approach with uncertainty propagation in the outer, uncertainty optimization in the middle and numerical ODE integration in the inner loop. Such a process is unaffordable when the equation of motions involves numerically costly calculations such as aero-propulsive forces estimation. By modeling atmospheric dispersions as stochastic processes, a method combining model order reduction and Gaussian process is defined using a limited number of trajectory optimizations. The overall approach is applied to a generic air-breathing vehicle in order to estimate extreme quantiles of the performance depending on the atmospheric variability. This method allows to improve classical engineering methods to adjust margins on vehicle design based on probabilistic performance evaluation. Examples of perspectives include the development of active learning methods

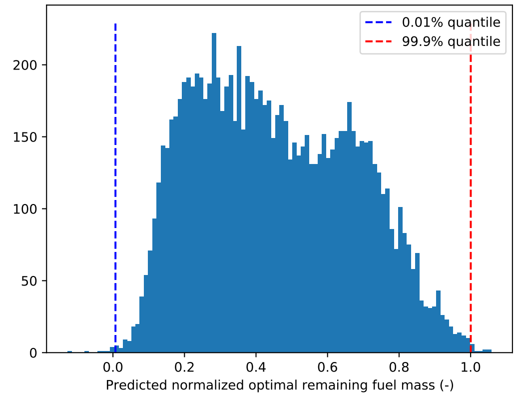

Fig 14. Histogram of the remaining fuel mass resulting from the uncertainty quantification and, the 0.01% and 99.9% quantiles

based on the uncertainty prediction of the Gaussian process in order to refine the surrogate-based model prediction.

6. Acknowledgments

The authors want to thanks M. Glen Sire for his help in the implementation of the trajectory simulation.

References

[1] M. Baudin, A. Dutfoy, B. Iooss, and A.-L. Popelin. Openturns: An industrial software for uncertainty quantification in simulation. In *Handbook of uncertainty quantification*, pages 2001–2038. Springer, 2017.

[2] L. Brevault, M. Balesdent, and J.-L. Valderrama-Zapata. Active learning strategy for surrogate-based quantile estimation of field function. *Applied Sciences*, 12(19):10027, 2022.

[3] L. Brunel, M. Balesdent, L. Brevault, R. Le Riche, and B. Sudret. A survey on multi-fidelity surrogates for simulators with functional outputs: Unified framework and benchmark. *Computer Methods in Applied Mechanics and Engineering*, 435:117577, 2025.

[4] J. R. Dormand and P. J. Prince. A family of embedded runge-kutta formulae. *Journal of computational and applied mathematics*, 6(1):19–26, 1980.

[5] P. Embrechts, F. Lindskog, and A. McNeil. Modelling dependence with copulas and applications to risk management. *Handbook of heavy tailed distributions in finance*, 8(1):329–384, 2003.

[6] B. Fidan, M. Mirmirani, and P. Ioannou. Flight dynamics and control of air-breathing hypersonic vehicles: review and new directions. *12th AIAA international space planes and hypersonic systems and technologies*, page 7081, 2003.

[7] R. G. Ghanem and P. D. Spanos. *Stochastic finite elements: a spectral approach*. Courier Corporation, 2003.

[8] International Organization for Standardization. Standard atmosphere, iso 2533:1975. 1975.

[9] A. J. Krueger and R. A. Minzner. A mid-latitude ozone model for the 1976 us standard atmosphere. *Journal of Geophysical Research*, 81(24):4477–4481, 1976.

[10] C. E. Rasmussen. Gaussian processes in machine learning. In *Summer school on machine learning*, pages 63–71. Springer, 2003.

[11] C. K. Williams and C. E. Rasmussen. *Gaussian processes for machine learning*, volume 2. MIT press Cambridge, MA, 2006.