Flutter of Periodically Stiffened Plates and Shells in Hypersonic Flow*

Gautam SenGupta†

University of Washington
Bothell, Washington,USA
matuag@hotmail.com

Keywords: Aeroelasticity, panel flutter, hypersonic flow, periodic structures

Abstract: This paper reviews application of the periodic structure theory for predicting flutter characteristics of periodically stiffened plates and shells in hypersonic flow. The method is particularly cost-effective when the number of spans becomes large.

1 Introduction

Recently, the topic of fluid-structure interaction in hypersonic flow has become of importance, as discussed in [1], where the flutter onset of a panel subjected to hypersonic flow has been investigated and the effects of various boundary conditions and structural nonlinearities have been presented. The purpose of the present paper is to review a method for calculating the flutter characteristics of periodic multi-bay panels commonly encountered in aerospace structures, on the basis of the periodic structure theory [3], [4], [5], [7], since the method is not widely known among the Aeroelastics community.

2 Flutter of Multi-bay Panels at High Supersonic Speeds

The multi-bay panel system originally considered by Dowell [2] is shown in Figure 1.

The equation of motion of a panel subjected to supersonic flow can be written as

$$D\frac{\partial^4 w}{\partial x^4} + \rho_s h \frac{\partial w^2}{\partial t^2} + c\frac{\partial w}{\partial t} = -\frac{2q}{\beta}\left[\frac{\partial w}{\partial x} + \frac{1}{U}\frac{\partial w}{\partial t}\right] \tag{1}$$

*Paper presented at the International Forum on Aeroelasticity and Structural Dynamics, IFASD 2019, 9-13 June 2019, Savannah, Georgia, USA

†Boeing Technical Fellow (Retired)

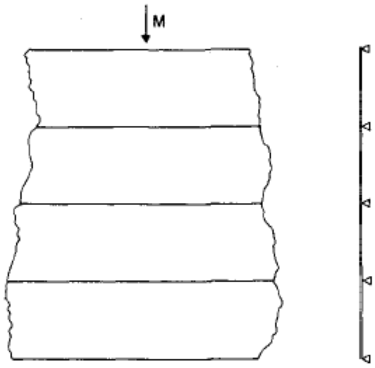

Figure 1: Multi-bay panel system considered by Dowell in [2]

where

$$
\begin{aligned}
w &= \text{Panel deflection}\\
D &= \text{Panel flexural rigidity}\\
h &= \text{Panel thickness}\\
\rho_s &= \text{Density of the panel material}\\
c &= \text{Structural damping}\\
U &= \text{Flow speed}\\
q &= \text{Dynamic pressure} = \frac{1}{2}\rho U^2\\
U &= \text{Flow velocity}\\
\beta &= \sqrt{M^2-1}\\
M &= \text{Mach number}
\end{aligned}
$$

The above equation is based on the assumption that the pressure difference between the top and bottom surfaces of the panel is given by the *piston theory* aerodynamics.

Satisfaction of the boundary conditions at the supports of an N-bay panel gives rise to $4N$ homogeneous, linear equations. Setting the determinant of coefficients equal to zero generates the characteristic equation for the eigenvalues. The functional relation can be expressed as

$$F_N(Z;\lambda) = 0 \tag{2}$$

where $Z = (\omega/\omega_o)^2 - i(\omega/\omega_o)g_t$, ω_o is the fundamental natural frequency of the panel, λ = Non dimensional dynamic pressure = $2qL^3/MD$, L is the distance between supports and $g_t = g_A + c$ = sum of aerodynamic and structural damping.

Solving for the eigenvalues Z we can compute

$$\omega/\omega_o = ig_t/2 \pm [(g_t/2)^2 + Z]^{1/2} \tag{3}$$

Unstable solution occurs if the imaginary part of the radical has an absolute value greater than $g_t/2$. After a few algebraic steps, this inequality reduces to

$$g_t < |Z_I|/Z_R^{1/2} \tag{4}$$

where $Z = Z_R + iZ_I$.

At the flutter boundary, $g_t = |Z_I|/Z_R^{1/2}$, and it can be shown that

$$|\omega_R/\omega_o| = Z_R^{1/2} \tag{5}$$

The above equation gives the *flutter frequency* in terms of the eigenvalues of the multi-bay panel and the inequality equation (4), gives the *flutter boundary* itself, in the λ, g_t plane. Based on the above formulation, the flutter boundaries for the one-bay and six-bay cases were derived and are shown in Figures 2, 3.

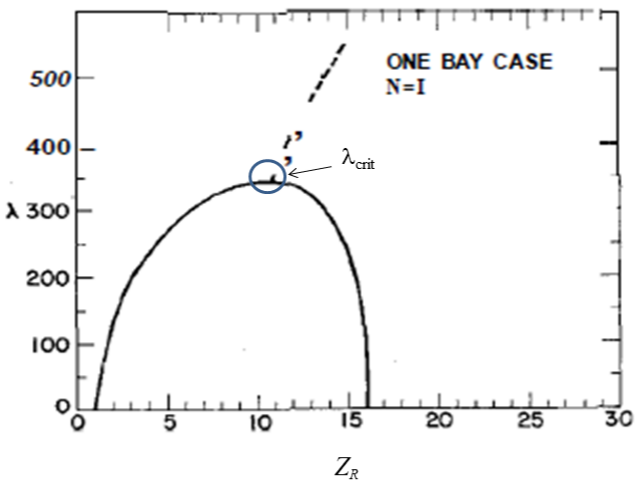

Figure 2: The flutter boundary for the one-bay case, from Ref. [2]

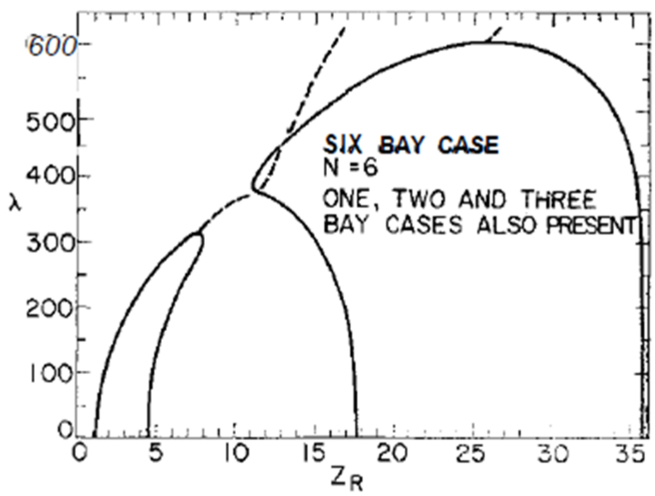

Figure 3: The flutter boundary for the six-bay case, from Ref. [2]

It was also observed by Dowell [2] that the natural frequencies of a six-bay structure fall into distinct groups as shown in Figure 4. It may be noticed that the frequencies of each group are bounded by the natural frequencies of the basic panel with simply-supported and clamped boundary conditions. He observed that the flutter boundaries are influenced by these distribution of the natural frequencies.

For a structure with a large number of identical bays, the above method becomes somewhat unwieldy. On the other hand, the periodic structure theory provides a cost-effective way to predict the flutter boundaries of a multi-bay structure with a large number of bays.

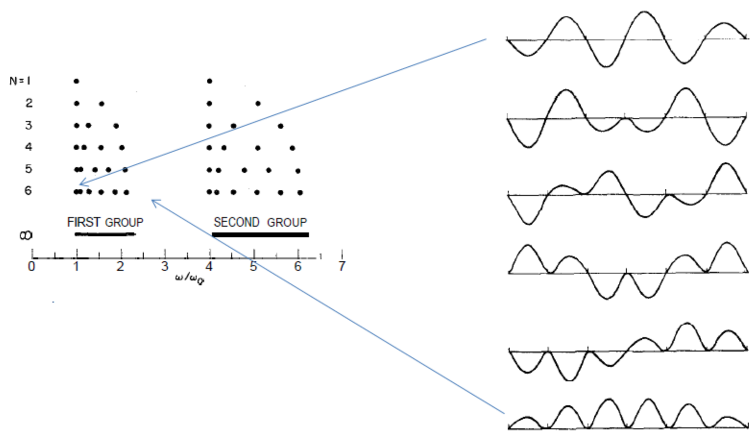

Figure 4: Distribution of the natural frequencies and mode shapes of a six-bay case, from Ref. [2]

3 Fundamentals of the Periodic Structure Theory

Distribution of the natural frequencies of multi-bay periodic structures in the absence of any fluid flow was studied by SenGupta [3] in terms of propagation of natural flexural waves along an infinitely long a periodically supported (or stiffened) panel.

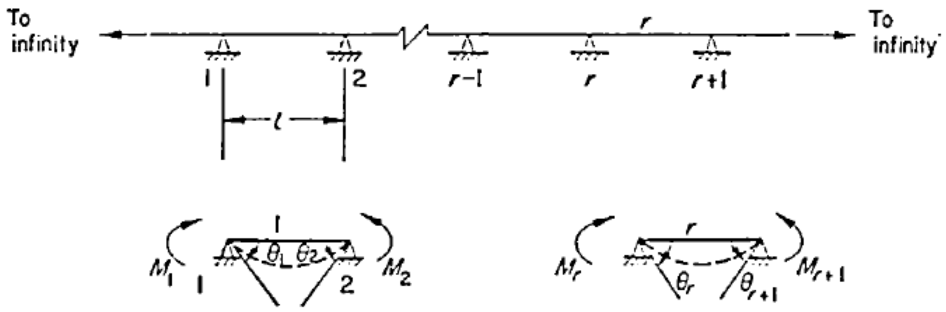

Figure 5: An infinite beam over periodic supports

In this approach, as a flexural wave travels from one bay to the next, the moments and slopes at the two ends of a periodic bay are related through a propagation constant as follows

$$
\begin{aligned}
M_2 &= M_1 e^{-\mu} \\
\theta_2 &= \theta_1 e^{-\mu}
\end{aligned}
$$

where μ is the propagation constant of the wave. In general, $\mu(= \delta + i\gamma)$ is complex, with the real part δ represents the decay of the flexural wave as it passes from one bay to the next, and the imaginary part γ represents the corresponding phase change over the peiodic length.

Figure 6 shows the variation of the decay parameter δ and the phase parameter γ with frequency.

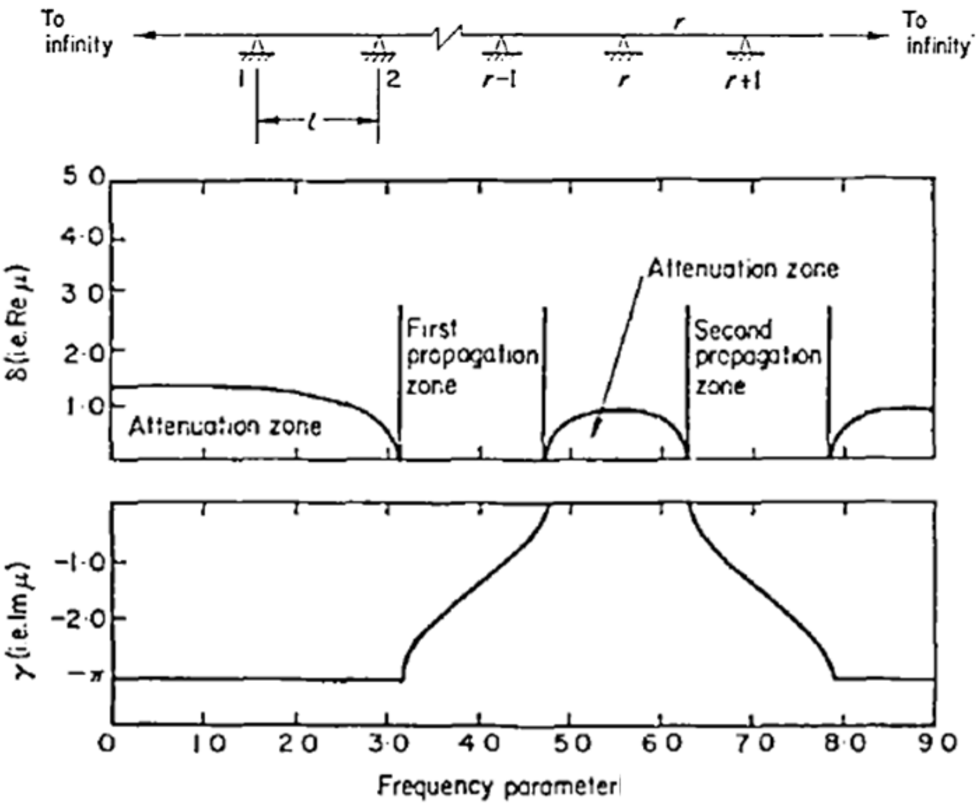

Figure 6: Stop and pass bands in a periodically supported beam

In [3] it was shown that the natural frequencies fall into free propagation zoes or pass bands, in which the flexural waves propagate freely without attenuation since $\delta = 0$. In these frequency bands, the imaginary part of the propagation constant is allowed to have certain discrete values and the eigenvalues or the natural frequencies are related to those discrete values.

In a finite N-bay periodic beam, any dynamic disturbance will give rise to flexural waves in both positive and negative directions which will be reflected back and forth between the boundaries. At any instant we should, therefore, expect both positive- and negative-going waves to be present simultaneously and the total bending moment at the rth support can be expressed as

$$\begin{aligned} M_r &= M_{r+} + M_{r-} \\ &= M_+ e^{i(\omega t - \gamma r)} + M_- e^{i(\omega t + \gamma r)} \\ &= (M_+ e^{-i\gamma r} + M_- e^{+i\gamma r}) e^{i\omega t} \end{aligned} \tag{6}$$

where γ is the imaginary part of the propagation constant μ in the pass band, where the real part δ of the propagation constant is zero.

From now on, we shall omit the term $e^{i\omega t}$, since it denotes only the time dependence, but its actual existence in appropriate places should be kept in mind. Thus,

$$M_r = M_+ e^{-i\gamma r} + M_- e^{+i\gamma r} \tag{7}$$

3.1 Natural frequencies of an N-bay beam simply supported at both ends

In this case, the boundary conditions are

$$\begin{aligned} M_0 &= 0 \\ M_N &= 0 \end{aligned} \tag{8}$$

Evaluating M_0 and M_N from equation (7) and substituting in equation (8), we have

$$M_+ = -M_- \tag{9}$$

and

$$M_+ \sin \gamma N = 0; \tag{10}$$

Since $M_+ \neq 0$, therefore

$$\sin \gamma N = 0 \tag{11}$$

and

$$\gamma = m\frac{\pi}{N} \tag{12}$$

where $m = 0, 1, 2....$ It can therefore be concluded that for the N-bay beam simply supported at both ends, γ can have only certain discrete values given by equation (above) and the natural frequencies of the N-bay beam will be the frequencies at which γ satisfies the above equation. Before proceeding any further, we need to take a closer look at the above equation. This equation is satisfied by any integral value of m and, therefore there seems to be no reason why the total number of natural frequencies in any propagation zone should be equal to the number of spans.

However, this paradox is entirely removed once we remember that in each propagation zone γ has the maximum and minimum numerical values of π and 0, respectively. If we re-write the above equation,

$$m = \frac{\gamma N}{\pi}, \tag{13}$$

the maximum integer values of m will be governed by the maximum value of γ, which is π. (As pointed out earlier, γ can be either positive or negative, therefore we shall from now on consider only the absolute value of γ.) Therefore,

$$(m)_{max} = \frac{|\gamma|_{max} N}{\pi}, \tag{14}$$

and, thus, the permissible solutions of equation (above) are given by

$$\gamma = m\frac{\pi}{N} \tag{15}$$

where $m = 0, 1, 2, ..., N$. The above equation still permits $N + 1$ solutions. However, $m = 0$ corresponds to $\gamma = 0$ which is associated with the clamped-clamped frequency of the individual bays in the first propagation zone. Since the extreme ends are simply supported, this cannot be a possible natural frequency of the N-bay beam. Thus, in the first propagation zone, m should have values from 1 to N giving N discrete values of γ and correspondingly N discrete natural frequencies in the first group. $m = N$ corresponds to $\gamma = \pi$, which predicts that the simply supported frequency of the individual bays should be a valid solution for the N-bay beam. This is acceptable, since extreme ends are simply supported.

In [3], it was shown that the natural frequencies of a periodic beam clamped at both ends or clamped at one end and simply supported at the other end can also be obtained in a similar manner. Application to the clamped case can be seen in figures 7 and 8.

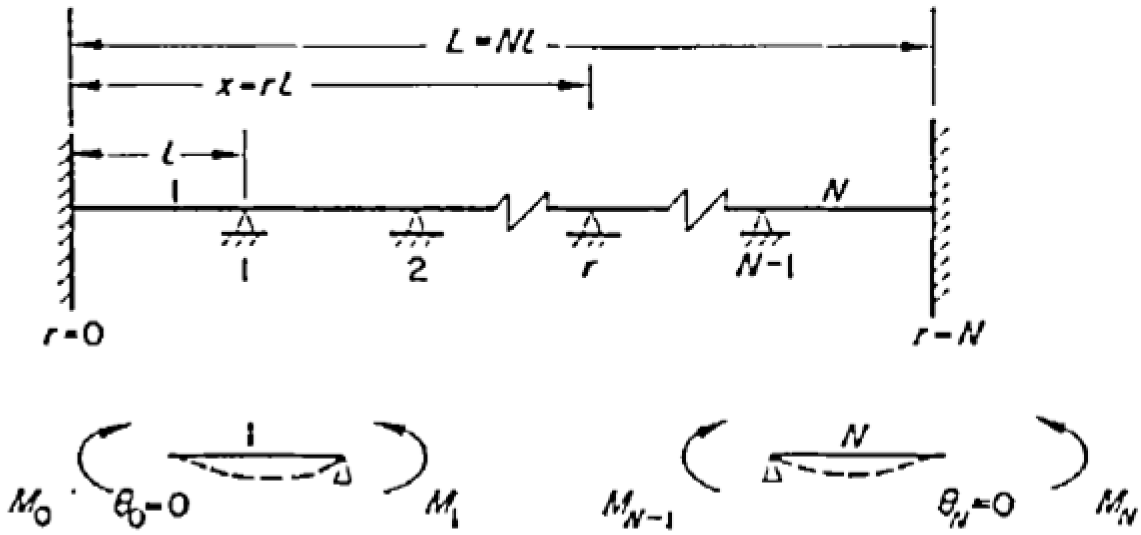

Figure 7: A periodically supported beam clamped at both ends

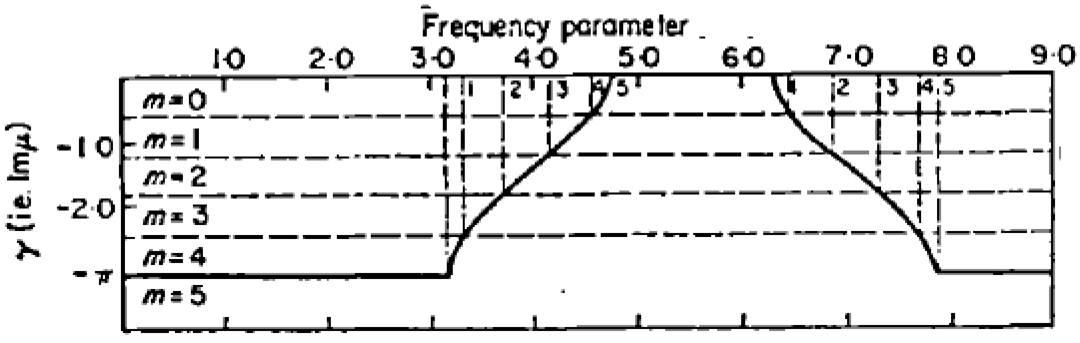

Figure 8: Determination of the natural frequencies of a 5-bay periodic beam clamped at both ends

3.2 Combination of the Periodic Strucre Theory with the Finite Element Method

The periodic structure theory has been combined with the Finite Element Method and applied to predict vibration characteristics of an aircraft fuselage with an enclosed acoustic medium with a large number of bays [4], as shown in Figure 9. It was shown that the computational costs can be substantially reduced by taking advantage of the periodicity of the structure, specially when the number of spans is large.

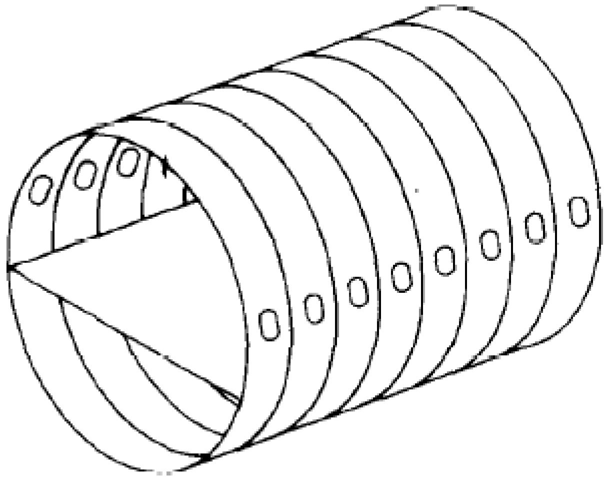

Figure 9: Modeling of an aircraft fuselage section, based on the Periodic Structure Theory

4 Application of the Periodic Structure Theory for Prediction of Panel Flutter at Hypersonic flow

The periodic structure theory was applied to predict the flutter boundaries of periodically supported panels exposed to high supersonic flow by Mukherjee and Parthan in [5]. Figure 10 shows the variation of the phase parameter with frequency in the absence of any flow ($\lambda = 0$) over the panel, as predicted in Ref. [5], and figure 11 shows the effect of the flow on the phase parameter for $\lambda = 100(A), 200(B)$ and $300(C)$.[1]

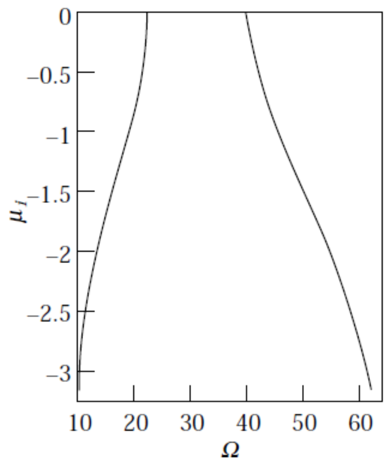

Figure 10: Variation of the phase parameter with frequency in the absence of any flow ($\lambda = 0$), from Ref. [5]

[1]Note that in Ref. [5] the phase parameter γ has been renamed as μ_i.

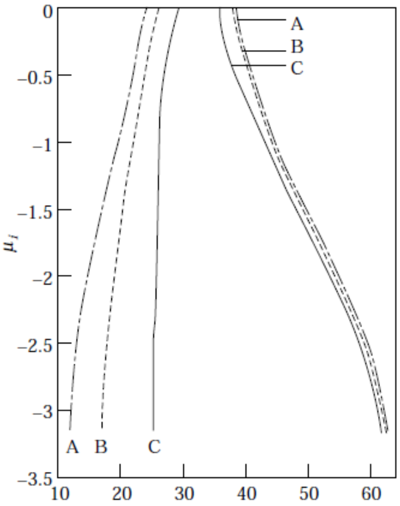

Figure 11: Variation of the phase parameter with frequency for three different values of λ, from Ref [5]

The variation of the phase parameter μ_i, and the dynamic pressure parameter λ with the frequency parameter Ω is shown in Figure 12.

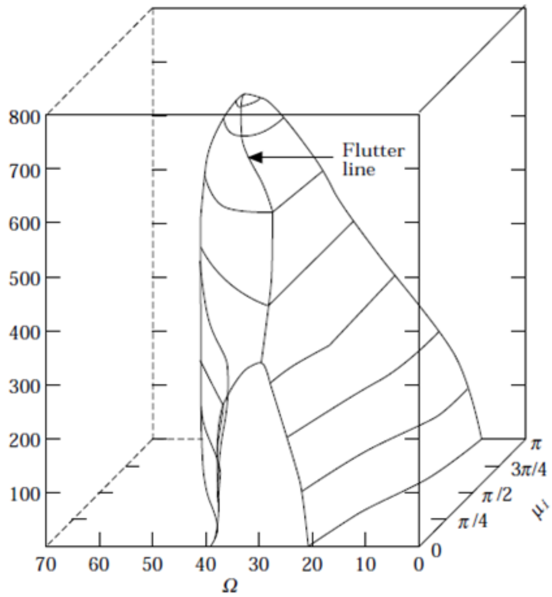

Figure 12: Variation of the phase parameter and the dynamic pressure with frequency, from Ref [5]

The supersonic flutter dynamic pressure of a 5-bay periodic panel can be obtained from the graph showing the variation of the phase parameter with the dynamic pressue by dividing the ordinate into five equal segments and projecting the points of intersection with the curve on to the abscissa, as shown in figure 13. Mukherjee and Parthan [5] matched their predictions with Dowell's results for various multi-bay panels, as shown in Figure 14, demonstrating the applicability of the periodic structure theory to predict hypersonic flutter characteristics of periodically supported panels. They also extended the method for predicting flutter characteristics of periodically supported panels with

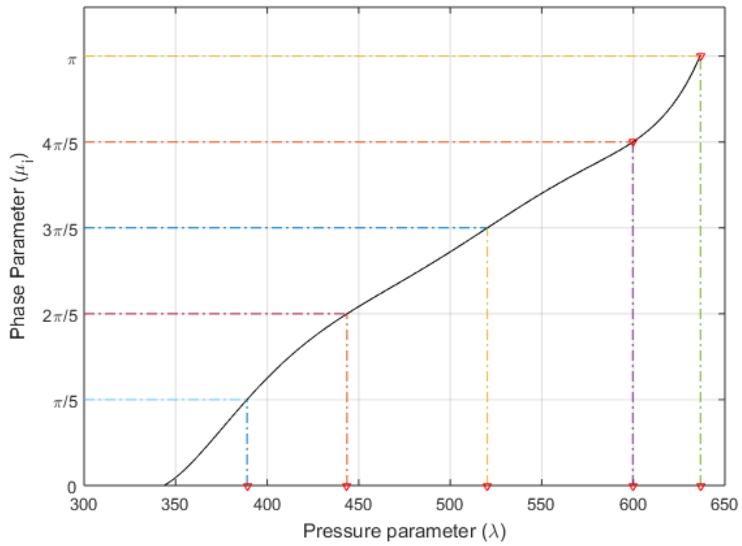

Figure 13: Prediction of supersonic flutter dynamic pressures of a five-bay periodically supported panels simply-supported at both ends.

both ends clamped in Ref. [6].

	λ_{crit}		
	Dowell	Present method	Ω_{crit}
One-span case	343·0	343·5	30·5
Two-span case (also includes one-span case)	485·0	481·0	42·0
Three-span case (also inclused one-span case)	423·0 545·0	420·0 546·0	34·80 45·95
Four-span case (also includes one- and two-span cases)	400·0 580·0	399·0 578·0	34·15 47·85

Figure 14: Comparison with Dowell's results, from Ref [5]

5 Flutter analysis of Periodically Supported Curved Panels

In addition, the periodic structure theory was also applied to predict the flutter boundaries of periodically supported curved panels (Figure 15) by Pany and Parthan in [7]. They were able to identify the flutter line (line of instability), using two methods - an exact method and the finite element method. They were also able to show that the critical flutter results for multi-supported curved panels can be obtained by discretizing the phase parameter based on the number of spans, as in the case of flat periodic panels.

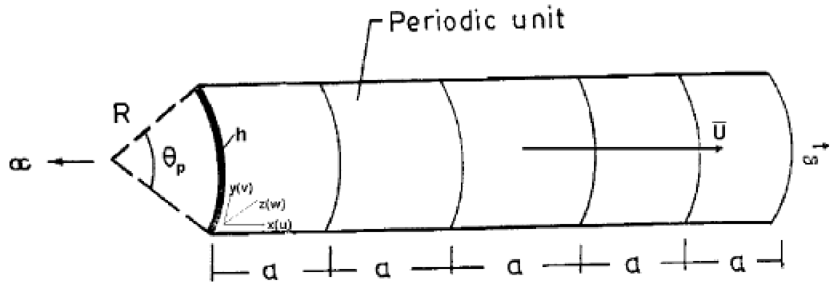

Figure 15: Flutter analysis of periodically supported curved panels, from Ref [7]

6 Concluding Remarks

In this paper, the method of predicting the flutter characteristics of multi-bay panels based on the periodic structure theory was reviewed. It was shown that application of the periodic structure theory provides a convenient and cost-effective method of finding the natural frequency distribution as well as flutter boundaries, specially when the number of spans is large.

Future research should take into account the presence of flexible periodic stiffeners [8], [9], [10] as well as structural nonlinearity associated with the in-plane stress terms, as discussed by Freydin & Dowell [1].

7 Acknowledgement

The author acknowledges his deep gratitude to Professor Earl Dowell, Dr. Somenath Mukherjee and Dr. Chittaranjan Pany for their permissions to show some of their equations, figures and results in this paper and in any future publications related to the topic.

References

[1] Freydin, Maxim and Dowell, Earl, "Fluid Structure Interaction in Hypersonic Flow", presented at the Aerospace Flutter and Dynamics Council meeting, St. Louis, Missouri, October 4-5, 2018.

[2] Dowell, Earl, "Flutter of Multi-bay Panels at High Supersonic Speeds", AIAA Journal, Vol. 2, No. 10, October 1964

[3] SenGupta, Gautam,"Natural Flexural Waves and Normal Modes of Periodically Supported Beams and Plate", J. Sound & Vibration, Vol. 13, No. 1, pp 89-101, 1970

[4] Jamshidiat, Hamid and SenGupta, Gautam,"Dynamics of Periodic Structures Interacting with an Enclosed Fluid Medium", J. Sound & Vibration, Vol. 148, No. 1, pp 103-115, 1991

[5] Mukherjee, S., and Parthan, S., "Wave Propagation in One-dimensional Multi-bay Periodically Supported Panels Under Supersonic Fluid Flow", J. Sound & Vibration, Vol. 186, No. 1, pp 71-86, 1995.

[6] Mukherjee, S., and Parthan, S., "Further Observations on Wave Propagation in One-dimensional Multi-bay Panels Under Supersonic Fluid Flow", J. Sound & Vibration, Vol. 186, No. 1, pp 71-86, 1995.

[7] Pany, Chittaranjan and Parthan, S., "Flutter Analysis of Periodically Supported Curved Panels", J. Sound & Vibration, Vol. 267, pp 267-278, 2003

[8] SenGupta, Gautam, "Natural Frequencis of Periodic Skin-stringer structures using a Wave Approach", J. Sound & Vibration, Vol. 16, Issue 4, pp 567-580, 1971

[9] SenGupta, Gautam, "Propagation of Flexural Waves in Doubly-periodic Structures", J. Sound & Vibration, Vol. 20, Issue 1, pp 39-49, 1972

[10] SenGupta, Gautam,"Vibration of Periodic Structures, Including Applications to Structural Dynamics, Structural Acoustics and Aeroelasticity", in preparation.

Copyright Statement