

STATISTICAL ANALYSIS OF DYNAMIC LOADS

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Abstract: Digitalization, big data, data analytics, machine learning... words very often listen but, is there anything real behind these buzzwords? Is there anything applicable to structural dynamic and aeroelasticity? This paper is aimed to show a real application of digitalization technology to dynamic loads.

Limit loads are the ones used in the structural design of the aircraft and many times related with a probability of occurrence around 10^{-5} per flight hour. Nevertheless, given a dynamic loads scenario reflected in the Airworthiness Regulations (taxi, dynamic landing, discrete tuned gust, continuous turbulence,...) there is only a reduce set of aircraft parameters (total weight, fuel contents, flight point, flaps or airbrakes configuration, etc) that provides the maximum loads that in turn would be used as limit loads.

This paper will show the methodology aimed to determine the probability of reaching limit loads in each one of the dots of the entire set of possible aircraft parameters (all weights, all fuel contents, all flight points, etc.) and not only “the critical” ones.

If the limit loads are reached in a given boundary of the “space of possible aircraft parameters” where the probability of reaching limit loads is 10^{-5} , then the probability of reaching limit loads inside that boundary will be lower than 10^{-5} (i.e. more remote than 10^{-5}) and the probability of reaching limit loads outside that boundary will be larger than 10^{-5} (i.e. more frequent).

The application of this methodology allows to determine accurately the associated risk in operation outside the guarantees of the Aircraft Flight Manual (AFM) and/or to fix the limits in the space of aircraft parameters that may allow operations outside that AFM guarantees.

An example of application of this methodology would be Statistical Analysis of Dynamic Taxi Loads due to military operations in unpaved runways. For those airplanes regularly operating in unpaved surfaces, their AFM exhibits the maximum allowable roughness of the runway profile. The methodology shown in this paper would help in assessing what would be the limits in the space of aircraft parameters and/or the accurate probability of reaching limit loads when operating in unpaved surfaces with more severe roughness than the ones in the AFM. This option would be a clear extension of the capabilities of the aircraft that will benefit the customers of it.

1 TAXI LOADS

1.1 Introduction

Military transport aircraft are required to operate in unpaved runways. Dynamic loads due to taxi in unpaved or uneven runways can be the sizing cases for some pieces or elements of the landing gear and the aircraft structure. They can also limit the operation of the aircraft on this type of runways.

The landing gear (L/G) and the aircraft (A/C) shall withstand the taxi loads that appear in such type of severe operations. The non-linearities present in the landing gear constitute the main difficulty in the simulation of these loads. These non-linearities are related with its kinematics and with the elastic characteristics of some of its components.

Taxi loads computation requires the coupled analysis of a non-linear model of the L/G with the linear dynamic model of the A/C. Although the L/G model is validated with a set of L/G drop tests and the A/C model is validated with the Ground Vibration Test, to obtain meaningful taxi loads numerical simulations, the coupled model results should be matched to a comprehensive set of aircraft taxi tests in which the aircraft is taxied many times over a variety of different obstacles (short and long) and unpaved runways.

The process to derive a suitable numerical model to simulate aircraft taxi loads requires a significant effort from the analyst and can be split in the following steps:

- Development of a taxi loads software tool.
- Complete aircraft taxi test specifications, test performance and test results analysis.
- Matching the numerical model to the taxi test results.
- Taxi loads analysis loops for design, check-stress and certification processes.
- Industrial application of the tool and the matched numerical model to determine the runway roughness in which the aircraft is able to operate with guaranteed safety.

1.2 Taxi loads software tool

The coupled system of equations that defines the movement of the system aircraft-landing gear taxiing at constant speed over a surface of prescribed roughness is a non-linear system that should be solved numerically in an iterative process in the time domain.

Airbus DS Military Aircraft Aeroelasticity and Structural Dynamics domain has developed a software tool (DYNTAXI) to solve this problem. The first operational version was already available by the end of the 80's [1] following the most general way to state it [2, 3, 4]. Many improvements have been added to the code since then, based on component tests and full-scale tests. Essentially, the code uses the normal modes obtained from any Finite Element Method (FEM) commercial software (like MSC/NASTRAN). The A/C structure is considered a linear system governed by a set of equations in modal coordinates. The L/G equations keep all the kinematic and elastic non-linearities of tires and shock absorber (S/A), considering as rigid all the structural components of the L/G. The coupling of these two systems leads to a system of L/G equations that takes into account its non-linear behavior and its interaction with the flexible A/C structure.

As a result of the system integration, in addition to the loads in the L/G elements and the parameters defining its behavior (tire deformation, S/A displacement...), the interaction forces between the aircraft and the landing gear can be obtained at the pintle points. These loads are then applied to the aircraft structure to obtain the aircraft response on the considered runway.

1.3 Models validation from tests

The analytical method needs to be validated. The validation is achieved by comparison of the numerical simulation with test results. Figure 1 shows the dynamic model validation flow chart.

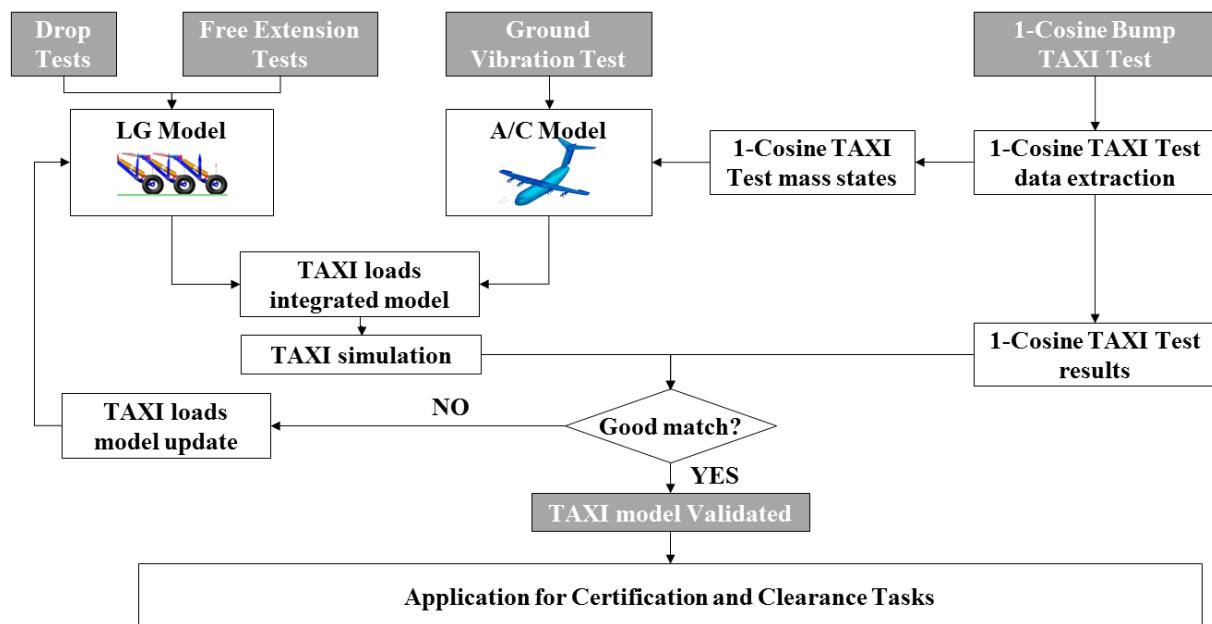


Figure 1: Taxi Loads Numerical Simulation Model Validation Flowchart.

The validation tests include:

- Ground Vibration Test (GVT) for validation of the aircraft dynamic model.
- Drop test for validation of the landing gear model and to demonstrate the L/G dynamic characteristics. The most relevant parameters to tune are S/A loads vs. S/A strokes.
- An extensive tests campaign of the aircraft taxiing/running over known (1-cos) bump [5-7], varying A/C speeds, center of gravity, brakes, thrust and flap settings, is performed to validate the coupled L/G and A/C model, studying the system response to an obstacle representative of a real unpaved runway but under controlled conditions. The key factor in the matching process is to tune, as close as possible, the S/A speed [8]. Figure 2 shows two time-histories corresponding to the NLG vertical force (left) and Wing root bending moment (right). Continuous blue line represents the numerical simulation while the red line is the measured test results [9]; correlation is excellent.

Finally, trials with landing and takeoff configurations on representative unpaved runways are performed to check the behavior of the validated L/G-A/C system in a real scenario.

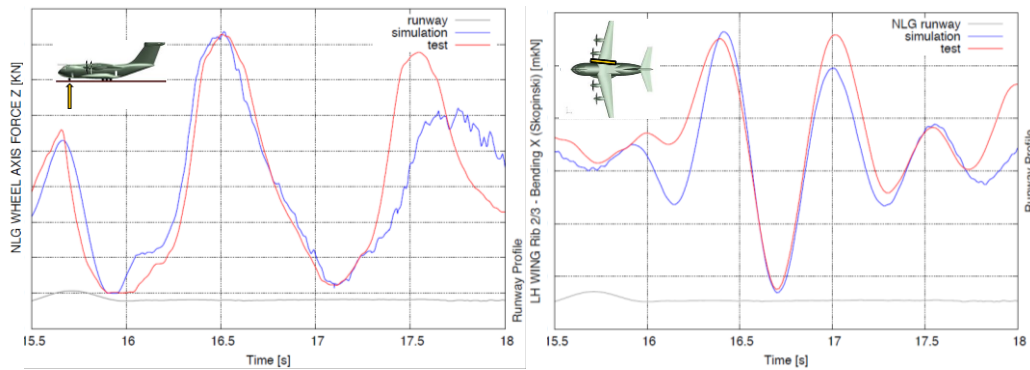


Figure 2: Example of Test-Simulation NLG (left) and Wing (right) loads comparison. (1-cos) bump.

1.4 Taxi loads analyses

Once the numerical model has been validated and the software tool reproduces with enough accuracy the taxi behavior of the aircraft, the final stages of the aircraft design process can be addressed. This is the direct engineering problem, which consists on determining the taxi loads that the aircraft must withstand to fulfill with the specifications required for the certification.

For military transport aircraft, required to operate in unpaved runways, those taxi loads size some components of the landing gear and the aircraft structure. Figure 3 shows an example of aircraft components sized by taxi loads [10].

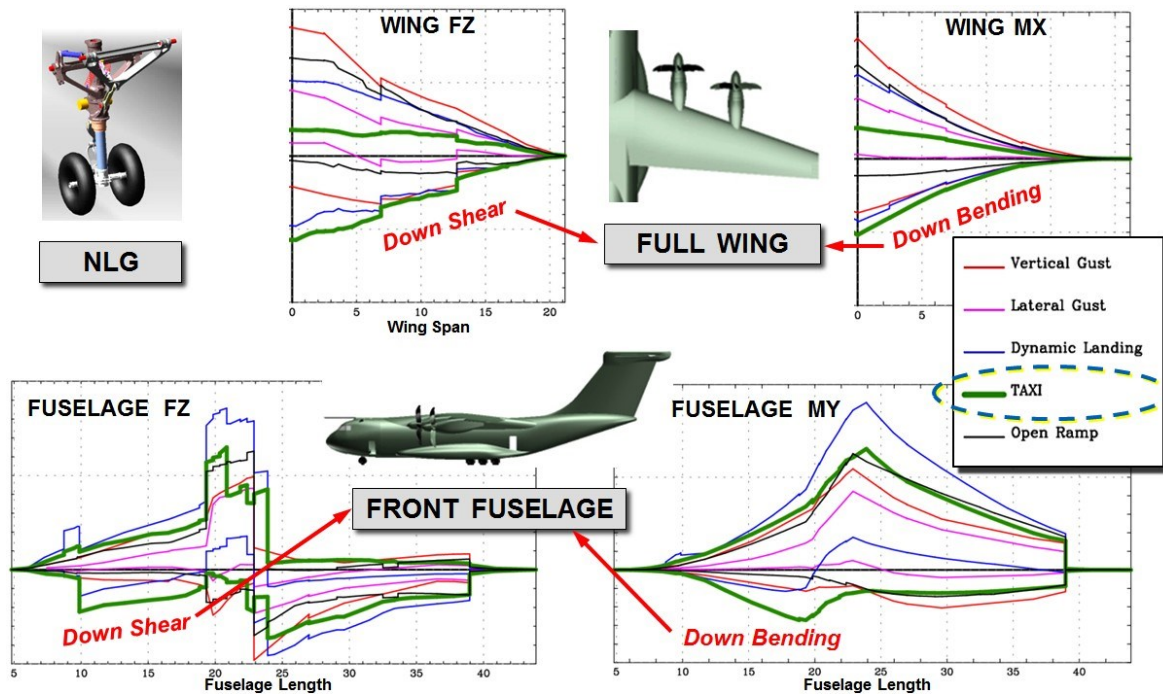


Figure 3: Example of aircraft components sized by taxi loads.

When the design phase is completed, after having achieved certification, the structural capability is already known and frozen; then, the inverse engineering problem can be addressed to explore the full capabilities of the aircraft. This inverse problem comprises the EBH (Equivalent Bump Height) [11] Curves Calculation process by performing a numerical analysis to each runway that determines which is the most severe 1-cos shape bump or trough for each wavelength. This way, for each runway profile, a unique EBH curve is obtained that can be

compared easily and directly with other runways EBH curves, giving an accurate estimation of the expected loads levels for each runway.

The EBH method [11] can also provide to the customer the most severe runway roughness profile on which the aircraft is able to operate with guaranteed safety. The EBH curve is obtained in an iterative process. For a certain obstacle wavelength the software tool computes what is the height that the A/C is able to pass without achieving any limit load or displacement in the L/G or A/C structure. This in turn means that each point of the EBH curve is characterized because a certain "limit load or displacement" has been reached in that point (otherwise the A/C could pass a taller obstacle).

These EBH curves will be calculated as a function of aircraft operational weights. The larger the weight of the aircraft, the less severe the roughness in which it is able to operate. That information is provided to the customer in the Aircraft Flight Manual (AFM). Figure 4 shows a typical definition of allowable runway roughness in the AFM.

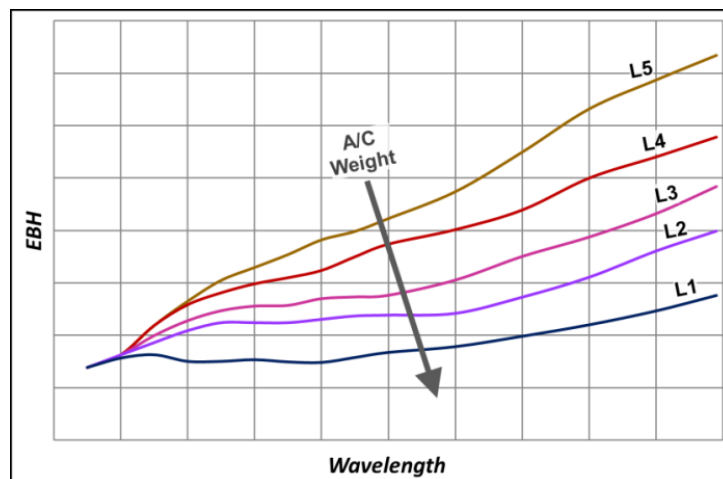


Figure 4: Typical definition of allowable runway roughness in the AFM.

Prior to a typical operation on an unpaved airfield, the customer should measure the runway profile and compute its EBH curve, then, this airfield EBH will be compared with the curves provided in the AFM. This will constitute a pass-fail criterion for the operation.

2 STATISTICAL ANALYSIS OF TAXI LOADS

2.1 Introduction

The definition of EBH curves has constituted the usual procedure to define the taxi operational limitations of an aircraft in order to preserve its airframe integrity. However, more demanding requirements in the current state of the art of military air transport have awakened the need among the customers to explore the aircraft taxi capabilities beyond its traditional operations and to request the quantification of the risk that this kind of operations could involve.

Customers demand has motivated Airbus to explore a new statistical approach [12] with the objective to complement the definition of the AFM EBH curves and decrease the conservatism intrinsic in their computation method by introducing the concept of probability in the assessment of the taxi loads problem.

The purpose of this section is to present the methodology developed to make it possible the risk quantification by means of the probability calculation of limit load exceedance at any point of the aircraft structure, given a specific runway roughness profile and an aircraft configuration.

The methodology of statistical analysis of taxi loads includes the following sequence of steps:

1. Selection of the AFM EBH curve to be studied and identification of the critical taxi cases associated to the bumps defining such roughness level.
2. Determine the loads monitor magnitudes to be analyzed, based on the critical taxi cases previously identified and on the experience from previous loads loops.
3. Define the relevant aircraft operational parameters involved in a taxi loads scenario. Then, select the statistical parameters by analyzing the criteria for the association of these parameters to define the 2D probability maps included in the statistical analysis, based on its influence on the selected monitor magnitudes.
4. Selection of a relevant number of critical “reference” taxi cases associated to the selected monitor magnitudes. For those reference cases, several taxi cases are defined in order to account for the effect of all the parameters and to cover the entire “mesh” of the 2D probability maps.
5. Computation of the taxi cases that define the “mesh” of each map.
6. Calculation of the iso-load curves for each probability map by interpolation of the computed taxi loads.
7. Determine the probability of limit load occurrence, in the AFM EBH curve, for selected monitor magnitudes from the corresponding probability maps, taking into account all the selected critical parameters.
8. Quantify the operational risk in a more severe runway than the one specified in the AFM.

In next sections, these steps are described in more depth.

2.2 Selection of AFM EBH curve, loads monitor magnitudes and reference taxi cases

Due to the high complexity of the probability calculation task, the problem is reduced to the analysis of taxi cases at a representative roughness level (i.e. L4 curve from Figure 5) and a unique bump wavelength as a matter of illustrative example for the application of this approach to any other roughness scenario. Among the set of bumps that define the roughness level L4, the bump selected as reference obstacle to perform the statistical analysis is the one closest to the point of tangency between both curves, L4 and DEF-STAN B (see Figure 5).

The two parameters defining this reference 1-cos bump are gathered in next table.

Wavelength [m]	Height [cm]
50	18

Table 1: Reference obstacle 1-cos bump.

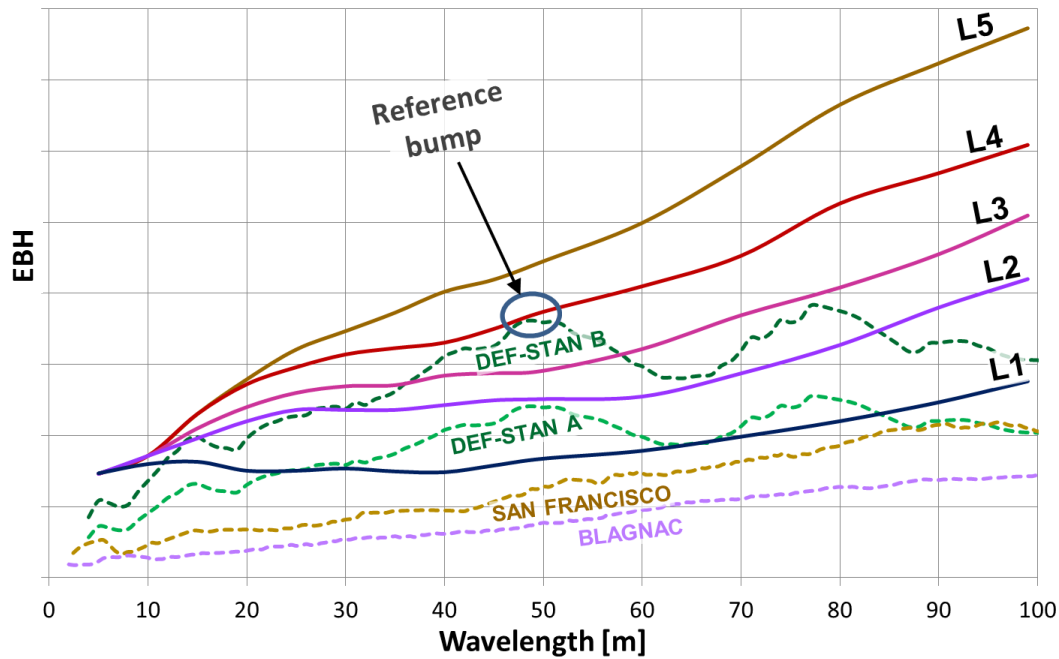


Figure 5: Example of EBH curves and reference bump selected for analysis.

With the aim to simplify the analysis, two monitor magnitudes are selected for the statistical analysis:

- Wing Root down bending moment (WR-MX), representative of the Wing component.
- NLG vertical force (NLG-FZ), representative of the NLG and Nose Fuselage.

The objective of the selection of the “Reference” taxi cases is to find the A/C configuration, operational parameters and runway ambient conditions in taxi operations that lead to the most critical loads for the two magnitudes of interest proposed for this statistical analysis. Among the all taxi case run to define the roughness level L4 and bump wavelength of 50 meters [10], the most critical cases found for the two magnitudes studied are gathered in next table.

Magnitude	WR-MX	NLG-FZ
Maneuver	Landing	Landing
Total Mass [T]	115	115
Fuel Mass [T]	25	8
Payload [T]	8	25
Xcg	Aft	Fore
Speed [Ktas]	100	75
Thrust	Ground Idle	Max Reverse
Brakes	ON (ABrake)	ON (ABrake)
Flaps [°]	47	40
Altitude [ft]	14600	9000
ISA offset [°C]	+24	+40

Table 2: “Reference” taxi cases associated to the reference bump selected for analysis.

2.3 Identification of relevant statistical parameters

The statistical parameters selected should fulfill these conditions:

- They should influence the results of the selected magnitudes (wing root down bending, NLG vertical force).
- They should be uncorrelated among them.

The statistical parameters selected for analysis in the present statistical approach are the following:

- *FUEL MASS* is a relevant parameter in the WR-MX monitor magnitude since their inertia has a direct effect on the dynamic response of the wing root.
- *TOTAL MASS*, *XCG* and *BRAKES* are relevant in the NLG-FZ magnitude due to the relevant initial pitch down attitude induced by all of them.
- *FLAPS SETTING*, *ALTITUDE* and *ISA OFFSET* are important parameters due to their direct implication in the aerodynamics of the aircraft affecting the apparent weight.
- *SPEED* is considered crucial in the analysis as it is the main factor involved in the excitation of a particular mode frequency.

2.4 2D probability maps and casuistry of cases

The parameters selected as relevant are grouped in pairs to define the X and Y axis of the 2D probability maps. Hence, each map will reflect how the variation of each parameter affects the probability of limit load occurrence. Four types of 2D probability maps are defined:

- *TOTAL MASS – FUEL MASS*. With this map, the overall mass configuration of the aircraft is completely defined.
- *BRAKING COEFFICIENT* (μ_{brk}) – *Xcg*. This map shows the variation of two factors that are closely related to the pitch attitude of the aircraft.
- *ALTITUDE – SPEED*. Both parameters have a direct impact on the lifting capabilities of the aircraft, which affect its apparent weight during taxi maneuvers.
- *ISA OFFSET – FLAPS DEFLECTION*. This map is also related with the aircraft lift alleviation.

Figure 6 shows the four defined 2D probability maps. Each one of the cross (X) inside the maps represents one typical combination of parameters values that defines one taxi case to perform the probability calculation.

For a typical statistical analysis of taxi loads, the total amount of numerical simulations to be performed would be the combination of the number of taxi cases defined within the selected probability maps by the different bump heights considered. This is in the order of several hundred cases.

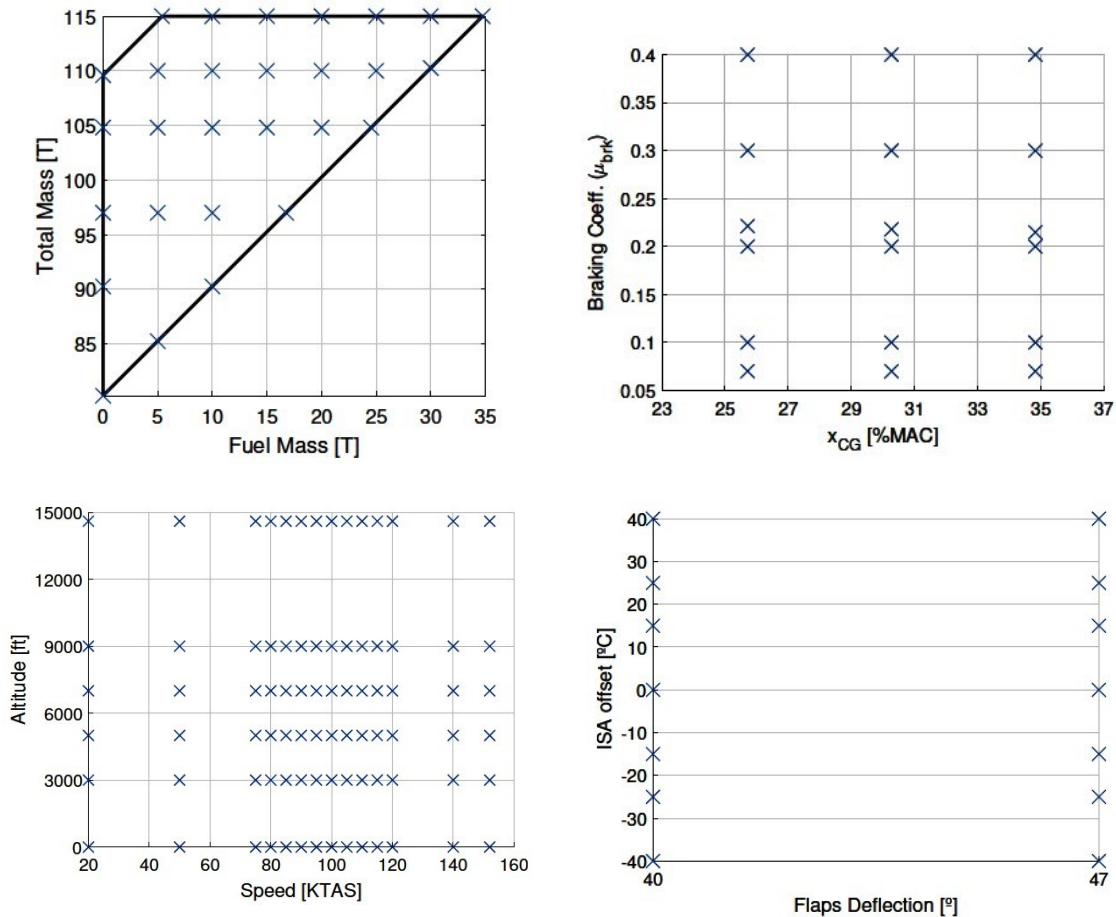


Figure 6: 2D probability maps.

2.5 Definition of iso-load curves

The iso-loads curves represent different load magnitude levels expressed as a fraction of the structural limit.

Because the calculated taxi loads correspond to discrete points within of each probability map, an interpolation procedure of the resulting loads is needed to obtain the iso-load curves.

Figure 8 and Figure 9 show examples of 2D probability maps with the corresponding iso-load curves.

2.6 Probability calculation methodology

The probability is calculated in two sequential steps:

1. Probability Map (PM)

The probability of exceeding a specific limit load percentage as a consequence of the variation of two parameters defining a taxi case can be computed from its probability map. Therefore, applying Laplace's rule [13] in a 2D probability map, under the assumption of equiprobability, the probability would be given by the ratio of the area of the map envelope that is equal or above a given limit load percentage value divided by the total area considered in its corresponding envelope:

$$PM_{(Parameter1) \cap (Parameter2)} = \frac{AREA \geq LIMIT \ LOAD \ PERCENTGE}{TOTAL \ AREA} \quad (1)$$

Figure 7 shows a graphical example of the probability of exceeding the 90% limit load, from the corresponding iso-load map: $AREA \geq 0.9$ LIMIT LOAD (blue color), $TOTAL AREA$ (blue+green).

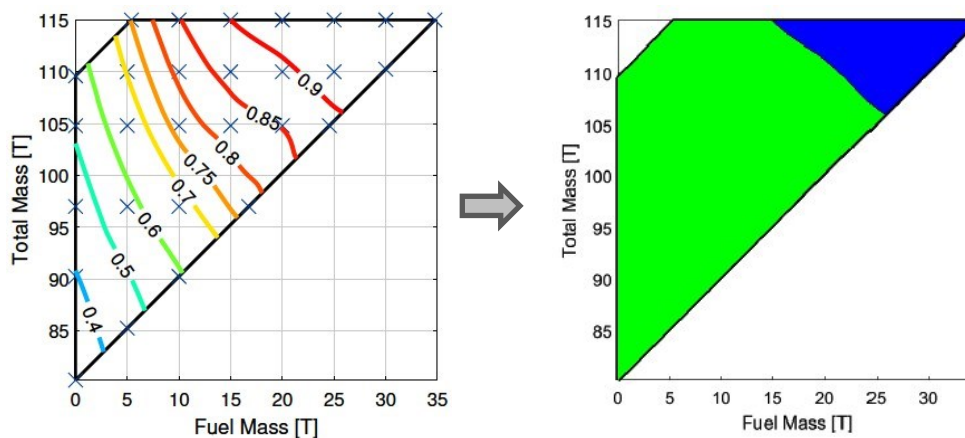


Figure 7: Graphical example of probability map calculation

2. Reference Case (RC)

In order to calculate the probability for a specific reference case, the contribution of the corresponding 4 probability maps must be considered. These 4 maps are independent events due to the fact that the relevant parameters selected for the statistical analysis are uncorrelated.

The formula describing the intersection of independent events is [14]:

$$P(A \cap B \cap C \cap D) = P(A) \cdot P(B) \cdot P(C) \cdot P(D) \quad (2)$$

Applying this equation (2) to the particular case of the combination of the four probability maps for a given reference case, the probability is computed according to the following expression:

$$P(RC) = PM_{(P1) \cap (P2)} \cdot PM_{(P3) \cap (P4)} \cdot PM_{(P5) \cap (P6)} \cdot PM_{(P7) \cap (P8)} \quad (3)$$

2.7 Probability curves at design bump height

The statistical analysis is performed first at the design EBH curve selected for study (EBH L4 curve [10], reference bump of wavelength 50m and 18cm height) in order to validate the methodology.

The set of results, concerning the definition of the iso-load curves for the probability maps considered to study the two magnitudes of interest (wing down bending and NLG vertical force), are presented. Then, the probability curve to assess the risk of limit load exceedance.

Figure 8 shows the 4 iso-load curves for the monitor magnitude wing down bending (WR-MX). The results show:

- The map of *total mass* vs *fuel mass* presents a behavior as expected. For a constant total mass of the aircraft, the loads increase when the amount of fuel is increased.

- From the map of *braking coefficient* vs X_{CG} , loads at the wing root are critical for backward CG and medium braking, which corresponds to an intermediate aircraft pitch attitude.
- The effect of the *altitude* and *speed* is small except for those speeds close to excite the frequencies of the wing bending moment that leads to critical loads at high altitude (low air density leads to lift decrease).
- The results of the map *temperature* vs *flaps deflection* conclude that the effect of this two parameters on the wing load variation is also small.

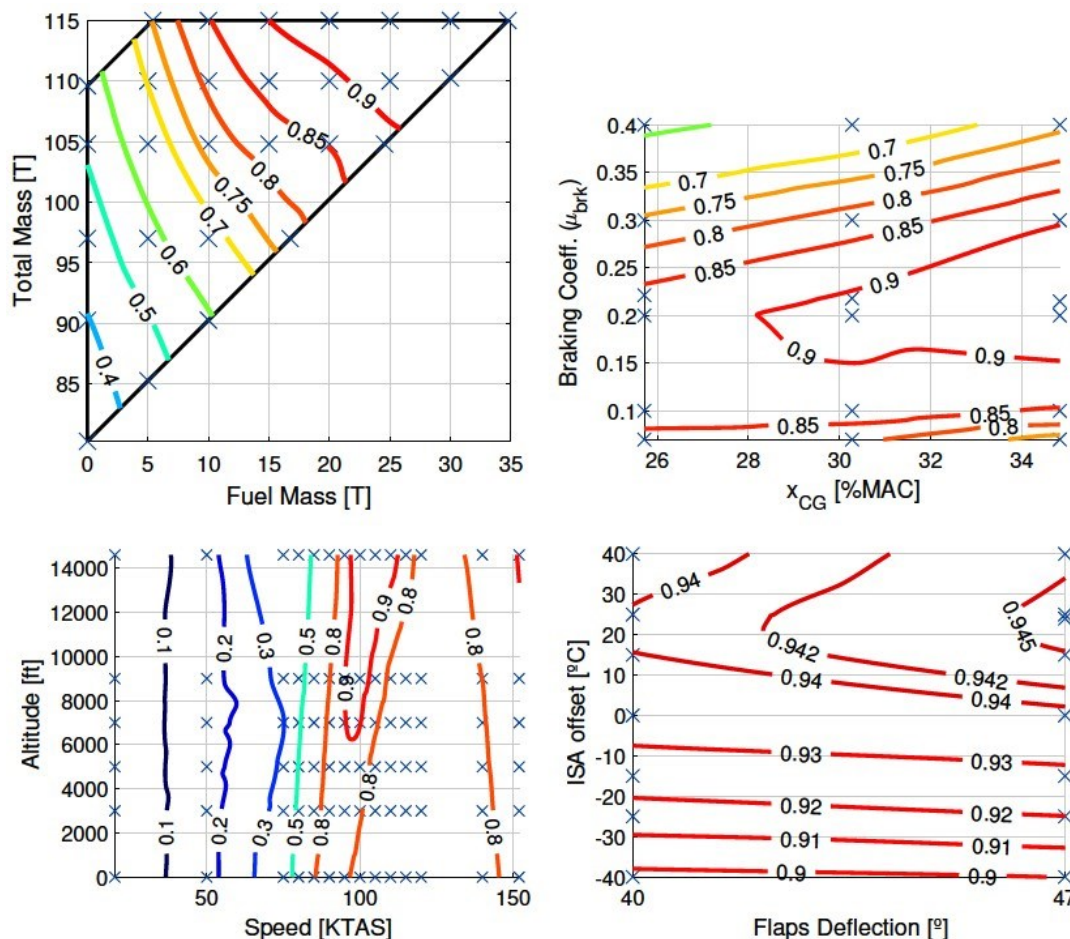


Figure 8: Iso-load curves for WR-MX. Reference bump from the selected EBH L4 curve

In the same manner as for the wing down bending, a similar sensitivity analysis is performed for the monitor magnitude NLG-FZ (see Figure 9), finding following conclusions:

- Now the map of *total mass* vs *fuel mass* shows that, on the contrary that the wing root load, for a constant total mass, the NLG load magnitude decays when the fuel is increased because the increase of fuel is compensated by an decrease of payload whose load path directly affects the NLG loads.
- From the map of *braking coefficient* vs X_{CG} , loads at the NLG are also critical for medium braking, but now for forward CG, since this configuration leads to a pitch down attitude of the aircraft that generates a higher compression on the NLG.
- The other two maps of iso-load curves for the magnitude NLG-FZ show similar behavior that for the WR-MX magnitude.

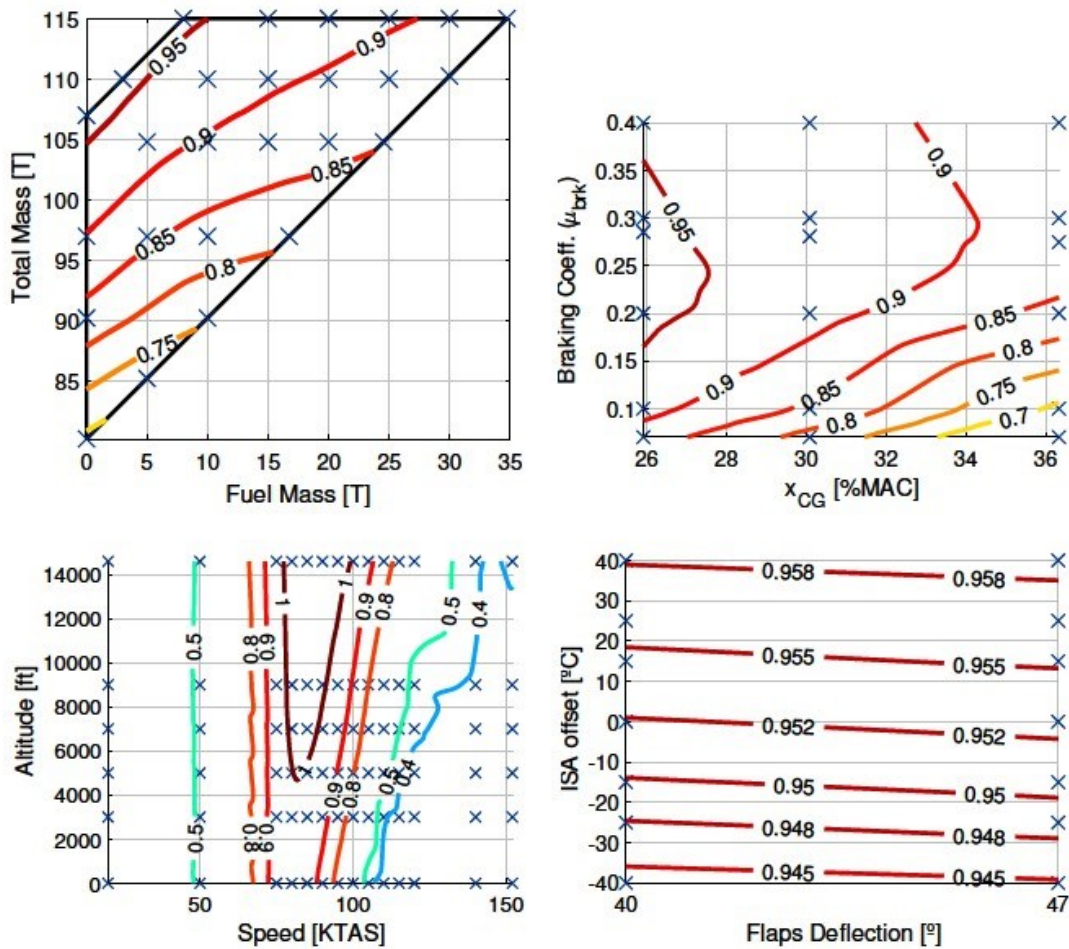


Figure 9: Iso-load curves for NLG-FZ. Reference bump from the selected EBH L4 curve

Following the probability calculation philosophy developed in Section 2.6, the probability curves for both magnitudes of interest, WR-MX and NLG-FZ, were computed at the design bump height (18 cm) of the roughness level L4 considered for study in the present statistical analysis. Results are shown in Figure 10.

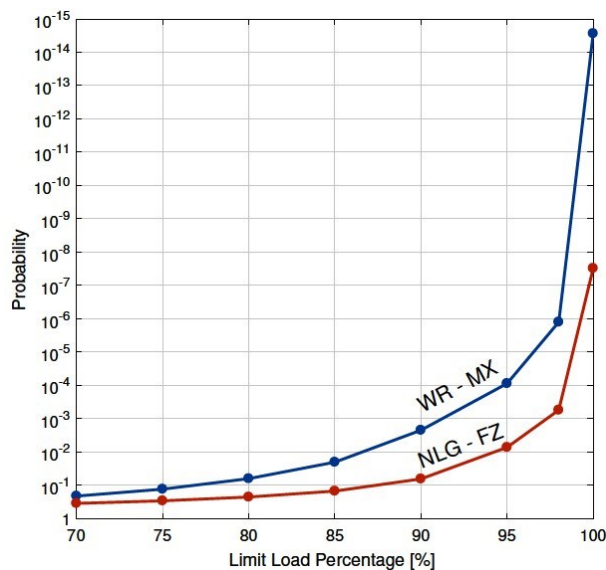


Figure 10: Probability curves at design bump height (18 cm). WR-MX (blue) and NLG-FZ (red)

The most critical magnitude found in these results is the NLG-FZ, whose probability of limit load occurrence is smaller than 10^{-5} (maximum threshold value typically considered in the Airworthiness Regulations by which limit loads are expected to not be exceeded if taxi operations are carried out within the limitations imposed in any AFM EBH curves). As a consequence, this result validates the methodology developed for the present statistical analysis.

2.8 Probability curves for operation in a more severe runway

The methodology used for the operation in the design EBH curve can be applied to explore the taxi capabilities beyond the traditional operations in terms of runway severity. The consideration of a more severe runway is translated into an increase in the maximum allowable bump height defined at a particular wavelength.

Therefore, to study the effect of operating in more severe roughness profile, the statistical analysis procedure was accomplished for four additional bump heights considering height increments of 1 cm from the maximum allowable one (EBH L4 curve, wavelength 50 m, bump height 18 cm).

The results of this analysis shows that the overall effect of operating in a more severe runway comprises that, as long as the bump height is increased, the areas enclosed by the iso-load curves of a particular limit load ratio progressively increase. This behavior occurs since an increase in the bump height is translated into a more severe excitation of the undercarriage.

Since the length of the bump remained constant, increasing the bump height does not modify the frequency of the excitation experienced by the aircraft. Therefore, the most critical cases in the *altitude-speed* probability maps are always encountered within the same speed range, near the resonant velocity, which remains unaffected. Moreover, not only in the *altitude-speed* maps, but the trend of the iso-load in the rest of maps also remains unchanged because the way each parameter affects the load magnitudes is independent of the bump height.

Figure 11 shows the probability curves obtained at bump heights of 19, 20, 21 and 22 cm for both magnitudes of interest studied in the present statistical analysis. Also, the reference probability curve (height 18 cm) for the operation in the EBH design curve is presented.

- For the monitoring station WR-MX, the probability of occurrence of 100% limit load on the runway roughness level allowed in AFM (bump height of $h=18$ cm) is in the order of 10^{-14} . If the aircraft is operated in a more severe runway, for instance bump of $h=19$ cm, then the probability of reaching 100% limit load drops to 10^{-4} . If the bump height is $h=20$ cm, then the probability drops again to 10^{-3} .
- Similarly, if the monitoring magnitude is the NLG-FZ, the probability of occurrence of 100% limit load (bump height of $h=18$ cm) is in the order of 10^{-7} . If the aircraft is operated in a more severe runway, for instance bump of $h=19$ cm, then the probability of reaching 100% limit load drops between 10^{-3} and 10^{-2} . If the bump height is $h=20$ cm, then the probability drops again between 10^{-2} and 10^{-1} . This would mean that statistically one out of 50 taxi operations in that specific runway would lead to limit load occurrence.

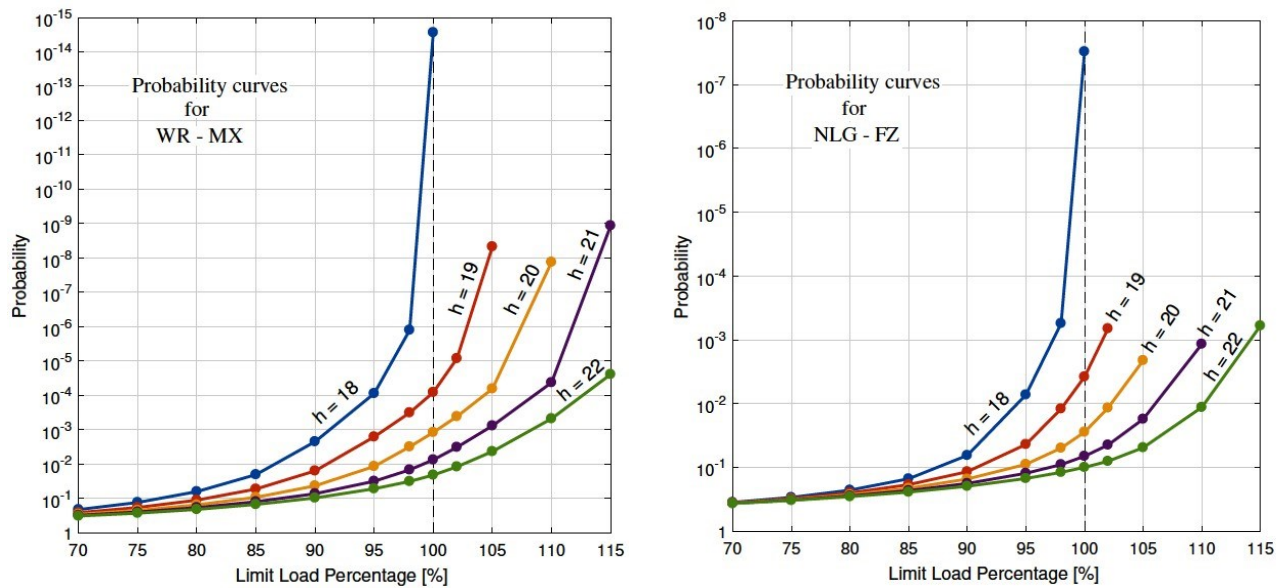


Figure 11: Probability curves at several bump heights. WR-MX (left) and NLG-FZ (right)

3 SUMMARY AND FUTURE WORK

3.1 Summary

This paper has presented a new methodology developed to quantify the risk of performing taxi operations on unpaved runways beyond the typical operations reflected in any AFM EBH curves.

The quantification of such risk implies the determination of the probability of limit load exceedance at any point of the aircraft structure, with the main objective of providing clearance for the operation in more severe runways than that for which the aircraft is designed. With this approach, the customer knows the probability of exceeding limit loads in a very rough runway and can take the corresponding decision.

The methodology involves simple statistical concepts and simplifying assumptions to get feasible computation time with accurate results.

The concept has been demonstrated by applying it to a reference bump from one of the design EBH curves included into a typical AFM. Positive conclusions can be drawn from this validation since the limit load occurrence probability for both magnitudes of interest is above the 10^{-5} threshold value at the design EBH bump analyzed.

Finally, as an example of application, the statistical analysis procedure has been accomplished for four additional bump heights considering height increments of 1 cm from the maximum allowable one, and whose results provided probability of occurrence below 10^{-5} , as expected.

This methodology is not only applicable to dynamic taxi loads, but also to statistical analysis of any type of dynamic loads like Landing and Gust.

3.2 Future work

The statistical approach has been particularized for a single point of one of the roughness levels defined in the AFM. However, to provide assessment of the risk to the customer, the statistical analysis should be performed for all the different wavelengths at which an EBH curve is defined and for every roughness level considered in the AFM.

Among the future considerations the assumption of equiprobability should be revised, since not each possible configuration of the aircraft studied is operated in the same proportion of time. The objective would be to develop a more realistic probability distribution that better represent the probability of the different configurations in which the aircraft is able to operate. Having this data available, collected from the customers operations, the probability maps could be divided in several regions, each being multiplied by a different influence coefficient. Also, the pondering weights associated to each thrust setting can be further refined depending on the percentage of time in which each event occurs.

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