COMPRESSOR BLADE AND DISC OPTIMAL STRUCTURAL DESIGN

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Abstract: The paper concerns optimal structural design of blade and disk of compressor for aviation engine. Structural optimization takes into account the shape of blade and disk design. Methodology for optimal design was applied to optimize the shape of the blade by changing the offsets and sizes of the blade section profile. The problem of mismatching minimization of the blade shape in operating position and predefined aerodynamic blade shape and natural frequency retuning is discussed. The estimation of the flutter boundaries is considered for design process. The problem of disk weight minimization with constraints on permissible radial displacement is investigated. Optimization criteria and constraints, parameterization model, design model, design parameters are discussed. All results were obtained using finite element analysis program connected with the sequential quadratic programming (SQP) optimization procedure.

1 INTRODUCTION

One of the important task in modern engine design process is to providing high engine efficiency, economical efficiency, minimum weight, reliability and stability of engine parameters during the flight cycle. It is possible with using multidisciplinary optimization methods for static, dynamic and aerodynamic analysis.

In the design process is used mathematical model of compressor stage that determine the characteristics of the work process and the mathematical models of the disk, blade and joint. The diagram in Fig. 1 shows the design process of the project. The design process is divided into three phases: divergence, transformation and convergence [1]. In divergence phase is used the engineering one-dimensional models for defining the main sizes and estimating the mass of the structure. Then these basic parameters is transferred as initial data for transformation phase models which used for optimal design.

The design process starts from the set of conceptual designs in the divergence phase and goes to the detailed design of parts in the transformation phase. The main contradiction are solved in the transformation phase in design process of construction units and parts. Quality assessment of the project is performed in convergence phase. Models and methods for optimal design is used in the transformation phase to ensure satisfaction of requirements of the minimum weight, strength and manufacturability.

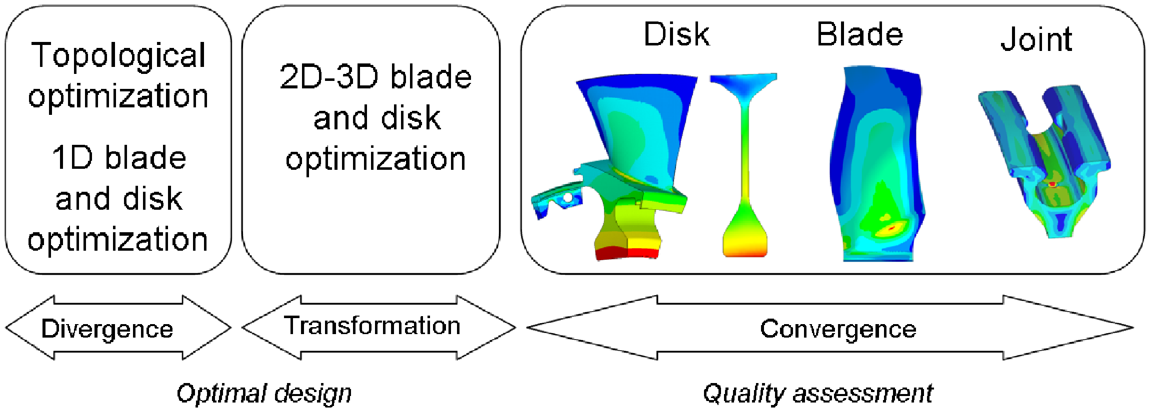


Figure 1: The design process

Optimal design consists of several sequential stages [2-4]: defining of objective function, constraints and criteria; parameterization of a shape, defining design variables; creating calculation model; choice of an optimization algorithm; solving optimization problem. The structural design problem of the compressor stage is most often used criteria for the minimum weight, the minimum clearance between rotor and stator while satisfaction the constraints on strength and manufacturability. In computer aided design the blades and discs are presented as the parameterized geometric objects. The complexity and dimensionality of the calculation model is determined based on the acceptable calculation accuracy and available time to solving the problem. The choice of optimization method is associated with the design variables, the dimensionality of the parametric space, the smoothness of the objective functions and constraints, varying set of constraints on the iterations.

The optimal structural design of compressor stage is performed with optimal design software of structural elements of gas turbine engine (GTE). In Fig. 2 the optimization software consist of several modules which solve various tasks. The analysis module is based on the finite element method (FEM). The optimization module is based on the sequential quadratic programming (SQP) algorithm [5].

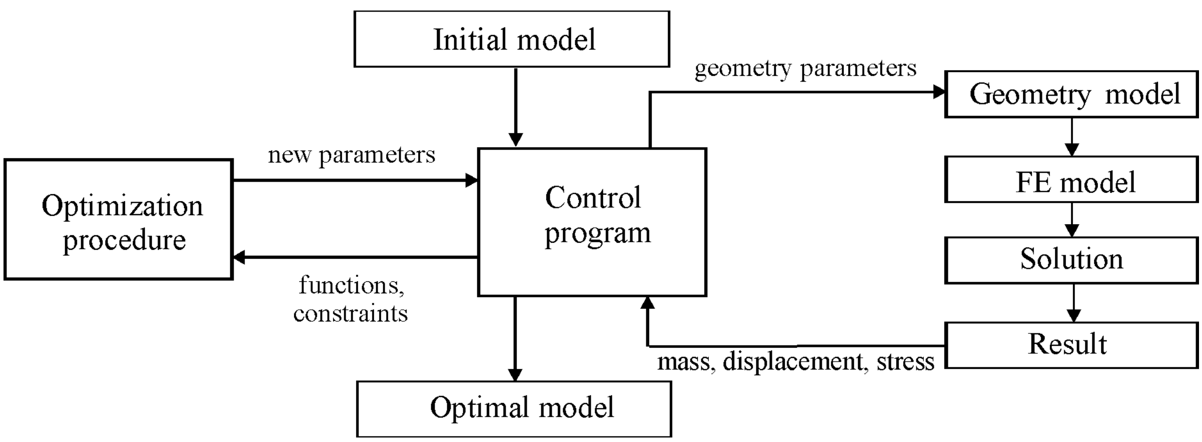


Figure 2: The diagram of the interaction between the modules in the optimization software

One of modules is a control program which performs interaction between modules. The control program runs each module in determined order, processes a data flows between geometry, finite element analysis and result modules. This program includes an interactive graphical environment which allows defining initial data, design variables, objective function, constraints, to control and operate the optimization process. Optimization module modifies design variables using SQP algorithm as search procedure. After changing parameters the program makes updating a geometric model and runs finite element analysis of the modified model. The values of objective function and constraints are calculated in finite element analysis procedure and are sent to the optimization procedure. Calculation of function gradients for SQP algorithm uses forward difference. The control program also includes a termination criterion of iteration process. The versatility of the software is that the analysis module can use a different FEM program and optimize various structural elements. The developed software module reduces computational time and increases the efficiency of the optimal design of GTE parts.

This paper provides examples of the application of optimal design software for compressor stage design. Various design features are considered: formulation of the optimization problem, parameterized models of blade and disk, the results of optimal weight design subjected to various constraints.

2 BLADE OPTIMIZATION

In the design of compressor blades is considered several objective functions and criteria [6] that minimize weight and stress, minimize the mismatching of the blade shape in the operating position and the calculated aerodynamic shape, natural frequency retuning. The shape of the blade is defined parameterized geometric model. Constrains are based on the maximum permissible stress and displacement, the frequency retuning from any resonance.

Applying centrifugal and gas forces can make significant mismatching of blade shape in the operating position from a predetermined aerodynamic shape which is result of gas-dynamic design of the compressor stage. Usually "cold" and "hot" shape of the blade is considered. "Cold" shape of the blade is called a shape corresponding to the unloaded blade. "Hot" shape of the blade is the shape after application to the blade centrifugal forces, gas loads and operating temperatures. Thus the main objective optimal design is the creation such optimal design of a "cold" shape blades which "hot" shape after applying loads has minimally different from the predetermined aerodynamic shape. It is also important to ensure minimum weight and permissible strength constrains.

Blade sectional profile is formed by the intersection of the aerodynamic surfaces with revolution surfaces formed by three-dimensional curved lines of the gas flow. The offset of these sectional profiles within a small range along and around the axis of rotation while keeping the angles of leading and trailing edges has small influence on gas-dynamic characteristics of the flow but significantly affect the stress-strain state of the blades and forces and moments at the joint.

In some cases such sectional profile can be replaced by a set of cylindrical or plane sections of the aerodynamic surfaces of the blade. In Fig. 3 shows design variables of blade sectional profile: 1 – i-th section; 2 – the surface of the sectional profile offset. The sectional profile offset is determined by the offset of the geometric points of the sectional profile along the axis of rotation through the parameter x_i and in the circumferential direction around the axis of rotation through the parameter s_i. Geometric points such as the center of mass of the sectional profile and the points of leading and trailing edges define a spatial curves offsets by interpolation that can be free or follow some specified behavior.

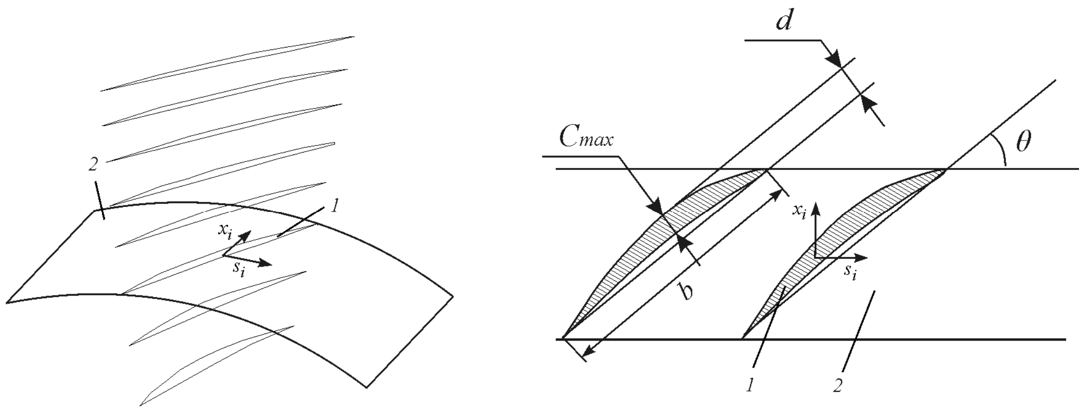


Figure 3: Design variables of blade sectional profile

The goal for minimum weight in the first approximation can be achieved by specifying the distribution area for the full-strength blades. In subsequent approximations depending on the design goals and constraints need to define all design variables of the sectional profile: chord length of profile b, the maximum thickness of the profile C_{max}, curved profile d, offsets sectional profile x_i, s_i and the angle of the installation section to the front of the compressor θ (Fig. 3).

The modification of the chord and the parameters of the sectional profile can change the aerodynamic performance of the blade. Therefore, the range of parameters variation is small and must be consistent with aerodynamic calculation. Mostly the size of the chord and the parameters of the sectional profile vary with the natural frequency retuning when set limitations on the natural frequency of the blade $f_j(\omega) \notin [k(\omega-\Delta\omega), k(\omega+\Delta\omega)]$, where k is the number of harmonic, ω - rotation speed, $\Delta\omega$ - margin rotation speed.

When the goal to reduce the stresses in the blade as a objective function is chosen norm corresponding maximum stress in the blade. Minimum norm can be achieved by changing the section offsets with the requirement of constant angles of the sectional profiles. Optimization of weight and stress is possible also when changing the chord and profile parameters.

An important task is the design of the initial "cold" shape of blades for a predefined aerodynamic shape with constrains as mass, natural frequencies and static stresses. In this case the design variables can be offsets, angles, chords and sectional profile parameters. The objective function is

$$F = q_\Phi F_\Phi + q_M F_M + q_\sigma F_\sigma + q_f F_f, \tag{1}$$

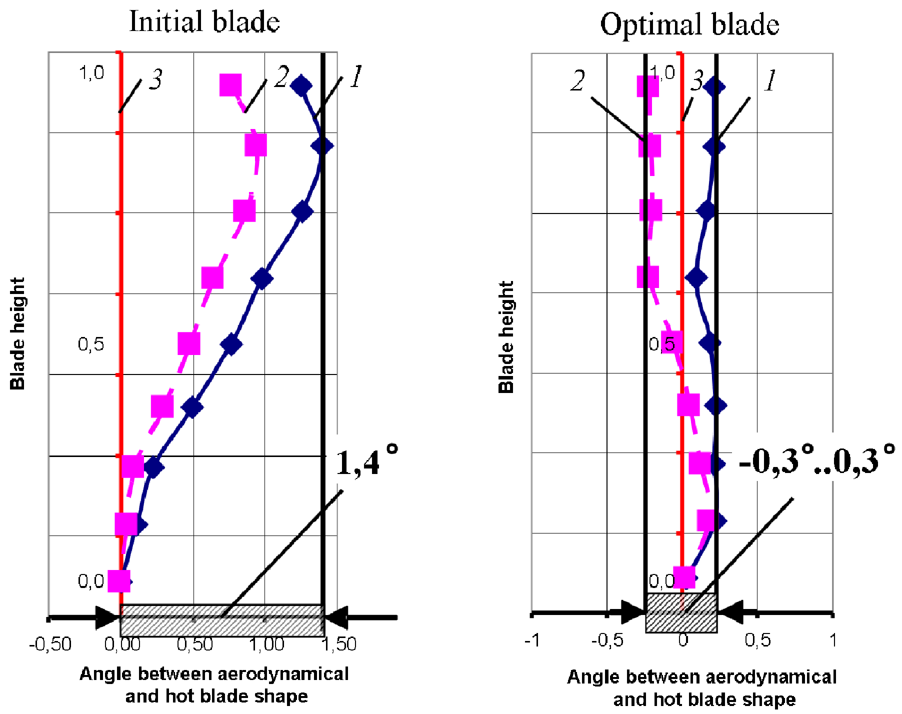
where are the components of the objective function provides: F_Φ – a minimum of mismatch predefined aerodynamic position of the blade and the position that takes the cold blade after applying the workloads; F_M – minimum mass; F_σ – minimum stress; F_f – frequency retuning; and $q_\Phi, q_M, q_\sigma, q_f$ – specially penalty coefficients. For solving multiobjective optimization problems with the objective function is used methods of constrained and unconstrained optimization. For specific optimization task the coefficient of the objective function is equal one. The remaining coefficients are equal to zero, and the functions with zero coefficients are translated into constrains and are used to solve constrained optimization methods. In another case, selecting the coefficients from the formula (1) based on the experience of the designer. The multiobjective problem can be converted to one criterion problem and used effective methods for nonlinear programming for unconstrained optimization.

Optimization are used for the structural design of blades with the objective of increasing overall efficiency. The design could be to perform aerodynamic and structural optimization separately or simultaneously. This paper presents an shape optimization of blades with the main objective of placing blade in predefined aerodynamic position.

The problem of mismatching minimization of the blade shape in operating position and predefined aerodynamic blade position is discussed. Structural optimization of a single blade is considered without direct aerodynamic constraints. The aerodynamic characteristics of blade are sustained by applying limits on the allowable changes of design variables. Finite element method is used to calculate structural characteristics of a blade. Static pressure along blade surfaces is one of the inputs for strength calculations.

The goal of optimization is to find the initial "cold" shape of the blade providing minimum mismatching of the blade from the predefined aerodynamic shape for takeoff and cruise modes. The objective function is the minimum of the absolute difference between the angles of the chord blade section at take-off and cruise mode and sections aerodynamic shape. As parameters are used cubic dependence of the offset sectional profile in circumferential and axial directions and the changing the installation angle of the sectional profile. The total number of design variables of the blade profile is 9. The maximum value of the offset sectional profile restricted to not more than 30 mm. Stress calculations were carried out with the centrifugal load and the gas forces at takeoff and cruise modes. In the calculation were taken into account geometric nonlinearity and the dependence of centrifugal forces on the displacements.

The calculation results of the initial blade is shown in Fig. 4a where is shown the mismatching of the blade in the operating position from predefined aerodynamic blade position. The angles between the aerodynamic shape chords of sections 3 and hot blade shape at takeoff 1 and cruise 2 mode by the blade height is shown. The y-axis is the aerodynamic shape 3. The coordinates of the blade height set on the leading edge of the blade.



(a) (b)

Figure 4: The mismatching of the blade in the operating position at takeoff *1* and cruise *2* from predefined aerodynamic blade position *3*

Fig. 4b shows the angles between the chords of the sectional profiles of the optimal blades at takeoff and cruise mode and the aerodynamic shape chords. The optimum blade at cruise mode has the mismatching angles of the "hot" shape sections from aerodynamic shape less than 0.28º compared with the angle of mismatching of 0.94º for the initial blade. On takeoff mode blade has mismatching angles of not more than 0.29º compared with the angle of mismatching of 1.39º for the initial blade.

3 BLADE FREQUENCY RETUNING

In the design of the blades is often necessary to change the natural frequency of the blades to avoid the resonance at the operating range. Unsteady loads on the blades such as the pressure pulsation can be generated by the guide vanes, air bleed holes, waves in the flow. Using the optimization procedure to search for the desired frequency allocation greatly simplifies the design task.

When tuning of the natural frequencies set limits on the natural frequency of the blade $f_j(\omega) \notin [k(\omega-\Delta\omega), k(\omega+\Delta\omega)]$, where k - the number of harmonic, ω - rotation speed, $\Delta\omega$ - margin rotation speed. The objective of optimization is to increase or decrease the natural frequency of the blades so that their intersection with the harmonics of the rotor was not within the operating range. As a variable parameters used quadratic dependence offsets blade sections in the circumferential and axial directions and the ability to change the angle of the installation sections. The maximum values of the offsets is limited to not more than 30 mm. Also it is used the quadratic dependence of section characteristics: maximum thickness variation profile C_{max}, curved profile variation d, the variation of the chord length b. The values of the design parameters section is limited to not more than 5%. The total number of blade profile design variables is 12. The limitation of the maximum values of equivalent stress is not more than 600 MPa.

Fig. 5 shows the Campbell diagram with the first three frequencies of the initial and optimal blade and five harmonics. On the diagram with the circle marked a possible resonance when 3rd frequency f_3 intersects with the 4 harmonica within the operating range 90…100% rotor speed, indicated by the shaded area. Also the circle marked a possible resonance, when the 2nd frequency f_2 intersects with 3 harmonica on the boundary of the operating range. Such a resonance should be considered in the values spread of natural frequencies due to the manufacturing tolerance and flexibility of a disk with blades. As a result of optimization 2nd and 3rd frequencies (f_{2opt} and f_{3opt}) decrease, and the point of intersection with the harmonics is out of the operating range as shown by the crosshairs.

It should be noted that to ensure a global minimum in the problem of optimal design of the blade is difficult. Therefore, the designer needs to choose an initial “cold” the shape of the blade from some set of results that matching to the calculated aerodynamic shape and that do not increase the weight of the blade.

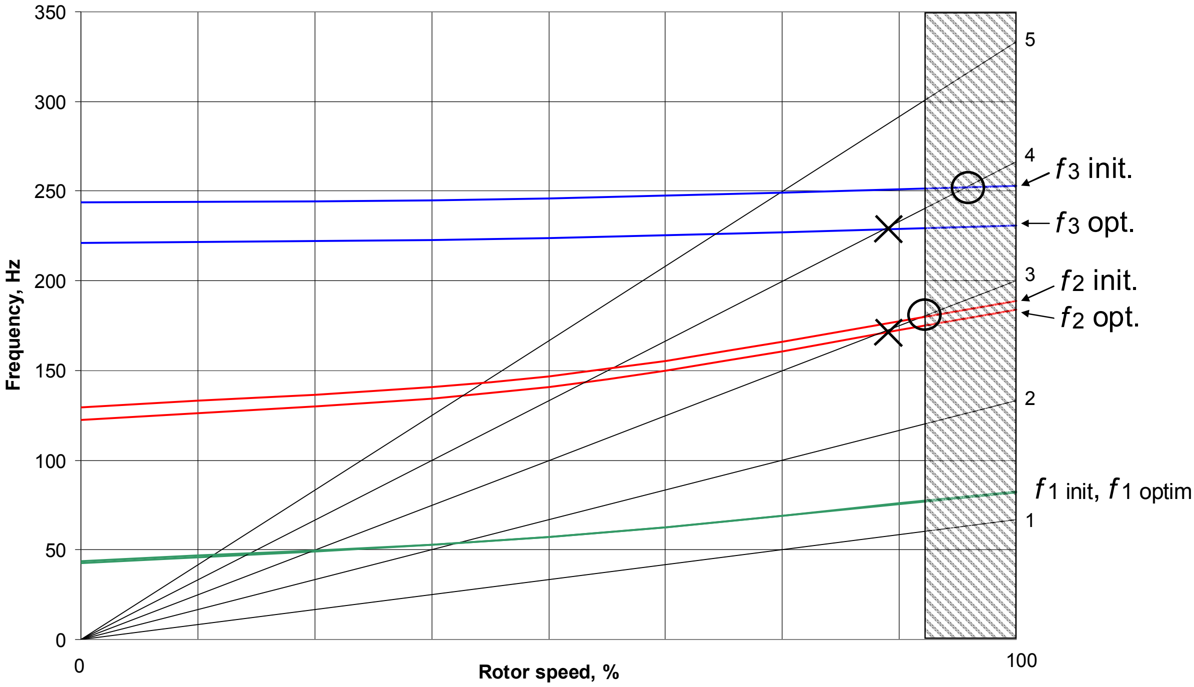


Figure 5: Campbell diagram

Methodology for optimal design was applied to optimize the shape of the blade by changing the offsets and sizes of the blade section profile. In the optimization problem was considered various criteria and constraints: minimize stress, minimization of mismatching angle between the chord of the aerodynamic shape and "hot" blade shape, the frequency tuning out from resonance.

4 BLADE FLUTTER PREDICTION

Calculation of flutter can be used for frequency tuning in order to eliminate flutter zone of the operating range of the blade [7].

The problem of blade flutter prediction is relevant on the engine design stage. Modern technology of multidisciplinary simulation that combines the vibrating blade model and the response model of gas flow in gas-dynamic tract is used to determine the influence of flow parameters on natural modes and frequencies. Geometrically nonlinear aeroelastic blade models (pretwisted beam, shell and 3D solid body) that consider aerodynamic stiffness in the blade assembly can be implemented.

Blade flutter analysis and prediction problem is significant for modern high-loaded gas turbine engines and stationary power units. Different compressor blade flutter aspects are discussed in [8-12]. Turbine blade flutter phenomenon is less investigated and experimentally confirmed. However, the increase of the last turbine stages loading has led to the necessity of flutter investigation in modern stationary GTU [13-16].

There are two mechanisms of flutter initiation exist - the mechanism of the blade-disk system interaction with flow and blade torsion and bending vibrations generation in the flow. The first is mostly determined by speed and non-uniformity of the gas flow in engine flow path, cascades interaction features and the gas flow through them, since in practice blade vibrations generation with the features, peculiar to flutter, occurred for different engine operational regimes. Common classification of different flutter types and its schema are shown in fig. 6. Taking the different character of the blade flow into account (seal jump and break-up zones existence and location, time and spatial phenomenon scales) flutter type investigation should be carried out with independent application of different models and methods. Schematics, shown in fig. 6 is general and refers to the cascades flutter (all the blades are involved). Besides the blade interaction by means of the flow, the interaction by means of disk, structural damping in blade root, etc, is also of the considerable meaning.

However as it was shown in [17-20] classic bending-torsion flutter may occur due to the high dynamic non-homogeneity of the blade assembly. This type of the flutter occurs as a result of the bending and torsion blade vibration types interaction.

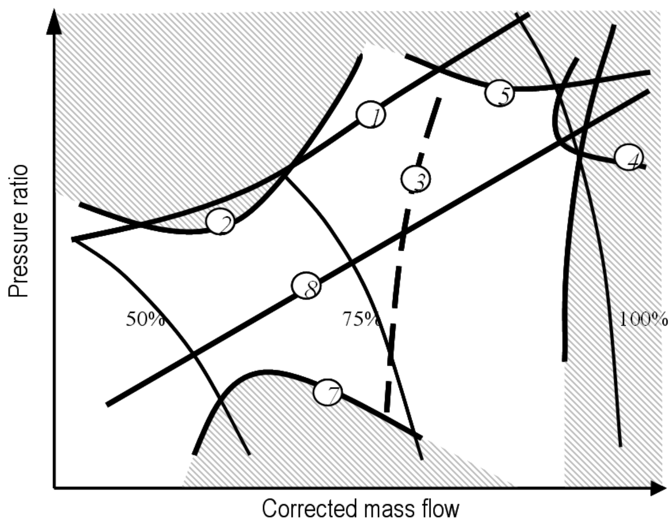


Figure 6: Compressor map; 1 – surge line; 2 – subsonic stalled flutter; 3 – bending-torsion flutter; 4 – supersonic unstalled shock wave flutter; 5 – supersonic stalled flutter; 6 – supersonic unstalled flutter; 7 – choke flutter; 8 – operating line

In case of classic plane wing flutter such interaction is determined by the dependence of pressure distribution over the wing profile on orientation angle. Profile mass center, stiffness and pressure center relative location are also significant [21-23]. Blade profile twist, cross-section asymmetry and convolution lead to the blade displacements and rotation angles coherence. Blade tension under the action of centrifugal loads field causes blade cross-section spin-up and its bending, which has an influence on the blade vibration frequencies spectrum. So, geometric nonlinearity makes an additional contribution into the blade and blade assembly flutter phenomenon. Therefore flutter prediction problem and its diagnostics should be guided by multidisciplinary investigation of gas flow process in the blade assembles of turbo machines and aeroelastic vibrations analysis of the separate blades as well as the blade assembly.

Method of blade bending-torsion flutter boundaries estimation is considered. By means of the multidisciplinary simulation blade stiffness matrix, which considers tangential and aerodynamic stiffness matrices, is found and used for vibrations analysis. Flutter boundaries are defined by the moment of bending and torsion blade vibrations frequencies coincidence.

5 ESTIMATION OF BLADE FLUTTER BOUNDARIES

To estimate the flutter boundaries and use it for design process, a certain characteristic, which is continuous with respect to blade geometrics, is necessary. As for wing flutter problem, where flow aerodynamic stiffness is proportional to the square of flow velocity, the natural frequencies behavior can be investigated:

$$M \cdot \ddot{q} + \left(K_t + \beta^2 \cdot K_A\right) \cdot q = 0\,,$$

where β – independent parameter. The value of parameter β that gives the frequency with positive real part has a meaning of flutter boundary.

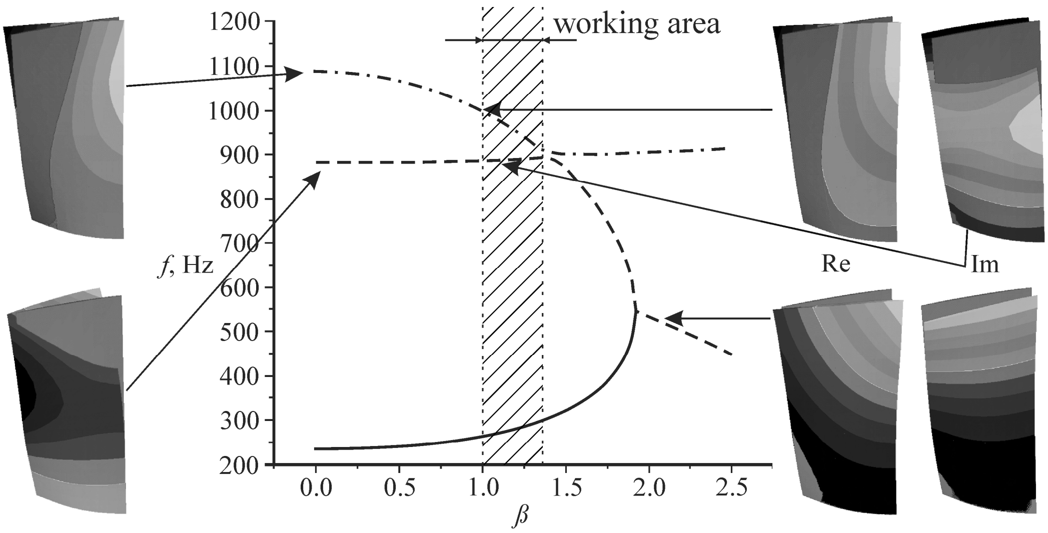


Figure 7: Aerodynamic stiffness influence, $\bar{w} = 0$

In fig. 7 the aerodynamic stiffness influence on blade natural modes and frequencies is shown for non-rotating system. Five sections have been used (0, 0.25, 0.5, 0.75 and 1 of the blade height) for tangent matrix calculation, aerodynamics stiffness matrices for 3 sections (0, 0.5 and 1) have been calculated, for other sections these matrices have been obtained using interpolation considering section rotation. Hub section has been fixed. Blade stiffness matrix has been obtained without considering rotation.

In case of two imaginary frequencies combination a couple of roots $\pm\lambda + i \cdot \omega$ appears, one mode is stable, another is unstable. For this roots the natural modes are imaginary too, what corresponds to motion

$$q(t) = e^{\lambda t} \cdot \left[q^1 \cdot \sin(\omega \cdot t) + q^2 \cdot \cos(\omega \cdot t)\right],$$

i.e. vibrations in two bending-torsion modes with phase shift $\pm\pi/2$. Small instability zone doesn't necessary occur for second and third natural modes combination. The inversion of these modes may happen without frequencies combining. Increasing parameter β over the working area leads to frequencies combining and flutter.

In fig. 8 natural frequencies and modes change is shown. The blade tangent stiffness matrix K_t considers rotation. Second bending and torsion modes inversion occurred, because, unlike fig. 7, second and third frequencies are not combined.

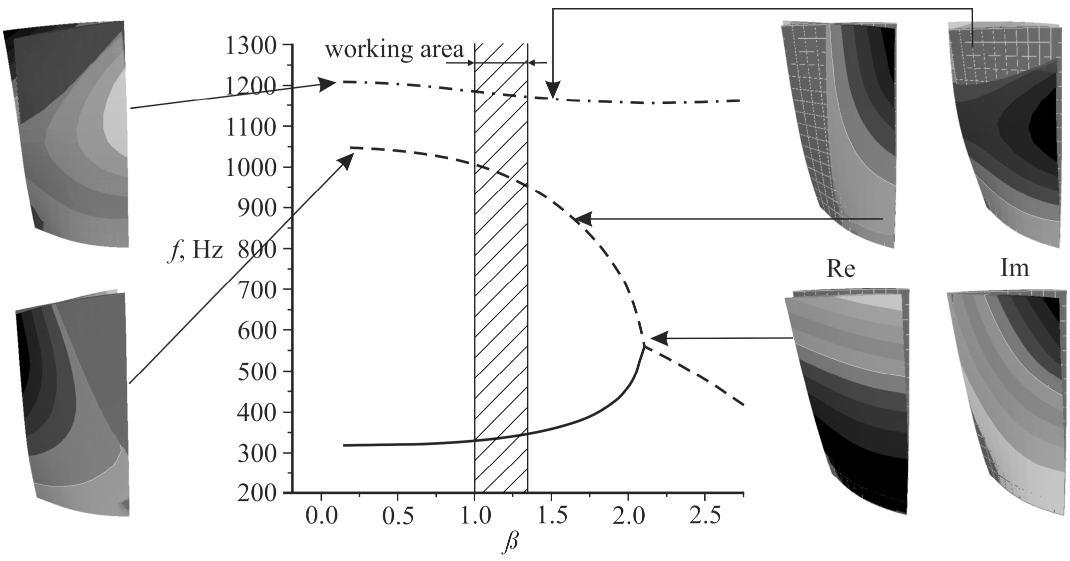


Figure 8: Aerodynamic stiffness influence, $\bar{w} = 1$

6 DISK OPTIMAL DESIGN

The disks of the rotors are the critical engine parts which operate at high temperatures and centrifugal loads and determine the weight of the rotor. The common optimization goal is to minimize disk weight while performing strength and stiffness restrictions. We consider the problem of finding the optimal disk with different temperature gradient between the rim and hub and various constraints on the permissible radial displacement of the rim of the disc with the strength criterion. The analysis is performed on the example of the middle compressor stages from titanium alloy.

During operating the compressor disks are heated so between the rim and the hub of disk the temperature difference reaches 600°C or more at takeoff mode. This temperature difference provides thermal stress in the disks that can be reallocated by changing the shape of the disk and thus changing the mass, or by using an additional heating of the hubs in the disks to reduce the temperature difference.

For efficient operating of the HPC is should be maintained a required level of radial clearances. The biggest change of the clearances occurs at take-off mode with maximum speed and the temperature difference between the rim and hub for a unheated disk. The limitation of radial displacement of the disks will allow to obtain an efficient design of the compressor. The optimum shape of the parts and the mass of the structures depend essentially on the constraints that apply to the structure.

For the optimization of rotor HPC was created a universal disk template suitable for a wide class of optimization problems as shown in Fig. 9. This template uses parametric model of axisymmetric disk shape which includes 19 design parameters in Fig. 9a. Among them six design variables (Fig. 9b) determining the thickness of the web and the size of the hub are changed in the optimization process. The remaining 13 design parameters are used to define the dimensions of the rim by the designer in the interactive mode. Such a simplified parameterization of the disks shape allows to quickly obtain the optimal variant of the structure, to explore more disk design variant and faster to evaluate the behavior of the optimal solution with changing of design variables.

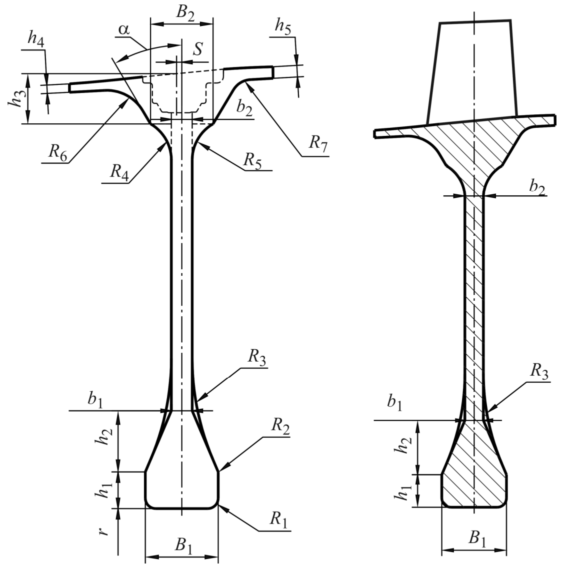


Figure 9: A parametric model of a disk

The goal of disk optimization problem is to minimize the disk weight with stress and displacement constraints by disk shape changing. Technological and constructive constraints which determine parameters limits are also considered. Constraints on radial and axial displacements of rim disk provide the desired stiffness. The disk minimization problem is solved with take-off loads and thermal load which calculated in the thermal analysis procedure.

The problem of optimizing the disk shape is considered in the axisymmetric formulation finite elements in order to speed-up the optimization process. Four-node element with the first-order approximation is used. For the simulation of centrifugal load coming from the blade is used plane stress elements with a thickness. The thickness distribution on the blade is set to take into account the centrifugal load from the real three-dimensional blade. Loads include the rotor speed and the temperature distribution on the disk from operating mode.

Fig. 10 shows a generalized dependence of the relative mass M / M_{min} of optimal disks on the permissible radial displacement U_r of the disk for different temperature gradient ΔT between the rim and hub: 1 – gradient 200°C, 2 – gradient 300°C, 3 – gradient 400°C. The value of the relative disk mass M / M_{min} is calculated on the minimum mass among all optimum disks. Also Fig. 10 shows the optimal shape of disks for different loading conditions and constraints. With weakening of the constraints on the radial displacement the disc becomes less rigid and has less weight. Reducing the permissible radial displacement by 20% leads to an increasing weight by 50% for the disk with a temperature gradient of 400°C. The disk weight increases by 70% with more heated the hub and the temperature gradient of 200°C. The various temperature difference between the rim and the hub allows to estimate the gain in weight for various HPC disks at equal radial displacement. For hard limit on the radial displacement the optimal disk weight varies nonlinearly.

The disk becomes more lightweight when permissible radial displacement constraint is increased. But by increasing the permissible radial displacement and receiving less stiffness disk restriction on the stress becomes significant that set a limit to further reduce weight.

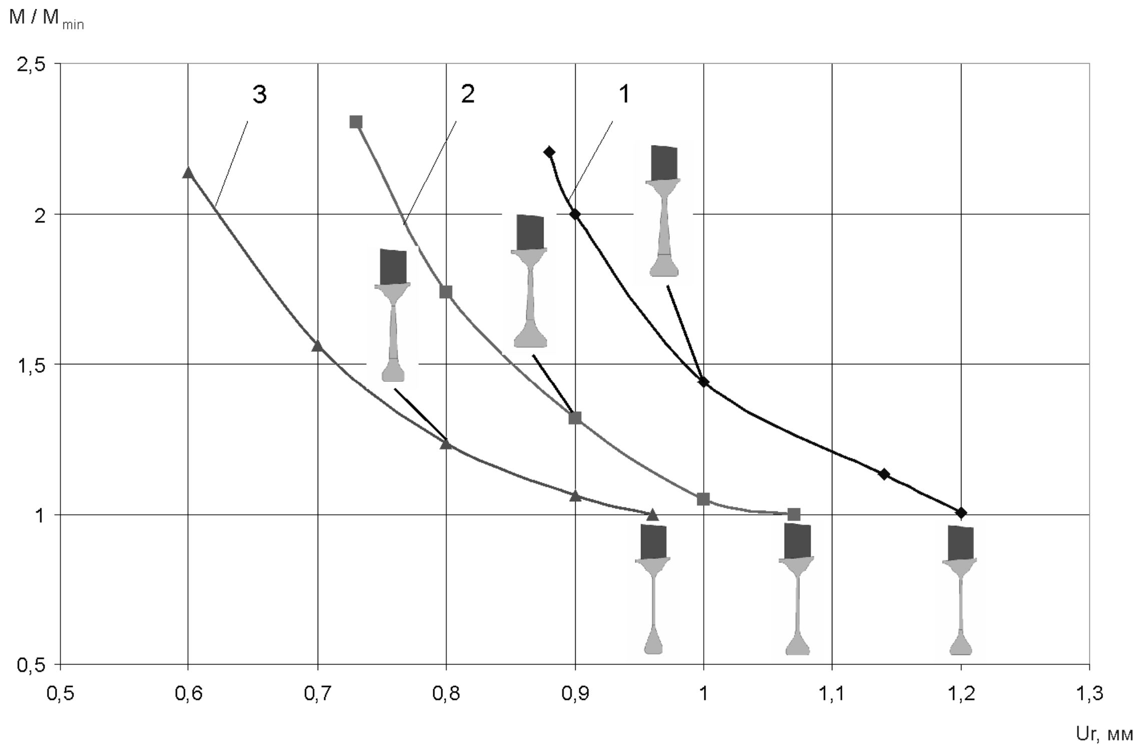


Figure 10: Dependence of the optimal disk mass on the permissible radial displacement for some temperature gradient between rim and hub of disk

7 CONCLUSIONS

The common optimization goal is to reduce the weight, but combining with proposed effective techniques, we can also optimize to achieve predefined cycling strength, blade tip clearance values and other goals. The problem of weight minimization is solved with both blade and disk stiffness optimization. Optimization approach was applied to optimize the shape of the blade by changing the offsets and sizes of the blade section profile. The blade design uses several objective functions and criteria that minimize weight and stress, minimize the mismatching of the blade shape in the operating position and the predefined aerodynamic shape, natural frequency retuning. The estimation of the flutter boundaries is considered for design process.

The disk optimization allows obtaining necessary construction stiffness with the stress restriction. The search for optimal disk with different temperature gradient between the rim and hub and various restrictions on the permissible radial displacement of the disk rim with respect to the strength criterion showed that the results of solving nonlinearly dependent on restrictions and the disk thermal loads. This confirms the need to manage the thermal condition not only disks, but the whole design of GTE, changing temperature gradients, stiffness and durability while minimizing weight of the whole structure.

The problem of blade and disk optimization with certain constraints on weight, stress and displacement is investigated by using an in-house finite element analysis program connected with the sequential quadratic programming algorithm in optimization procedure.

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