

## A STATIC AEROTHERMOELASTIC RESPONSE ANALYSIS METHOD CONSIDERING HEAT FLUX UNCERTAINTY

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**Abstract:** This paper proposes an analysis using the interval method to consider the uncertainty of the heat flux in hypersonic aerodynamic heating. This method is needed to address the significant errors of hypersonic aerodynamic heat computations and provide results that can be more easily verified experimentally. The proposed method is used to analyze the aeroelastic response of hypersonic vehicles based on aerodynamic-thermal-structure coupling. The heat flux input for the heat conduction analysis is modeled using uncertain interval parameters based on the interval method. Next, the analysis equations are obtained using a finite element method and subsequently solved using a matrix perturbation method. The evolution of the temperature field of a wing during flight is determined. Based on the intervals of the temperature field, a genetic algorithm is used to obtain the temperature field that imposes the largest load on the structure. Finally, based on this temperature field, the relevant static aeroelastic analysis is performed. The results demonstrate that the research methods presented in this paper can accurately consider the impact of heat flux uncertainties on the static aeroelastic response of hypersonic vehicles based on aerodynamic-thermal-

structure coupling. The uncertainty of the aerodynamic heat flux has a significant impact on the analysis results for the static aeroelastic response.

## **1 INTRODUCTION**

Hypersonic vehicles encounter severe thermal environments in flight. The structure is subjected to a non-uniform temperature field due to aerodynamic heating, resulting in thermal stresses. These thermal stresses change the structural stiffness and cause thermal deformations. Therefore, aeroelastic analysis for hypersonic vehicles that considers the impact of this thermal effect is significant.

Analysis of structural thermal conduction is an important part of an aerothermoelastic analysis of hypersonic vehicles. The aerodynamic heat flux is an important parameter in the analysis of structural thermal conduction. However, the results of heat flux analyses and the truth always differ to some extent; in other words, uncertainty exists. In addition to the heat flux, uncertainty also exists in the calculation results, such as the parameters of the structural materials, the distribution of aerodynamic pressure, and other parameters. The uncertainty of these parameters complicates already complex aeroelastic problems. The uncertainty problem in the field of aeroelasticity has gained considerable attention in recent years. In a workshop organized by the U.S. Air Force Office of Scientific Research and the U.S. Air Force Research Laboratory, the significance of the uncertainty problem in aircraft aeroelastic design was clearly identified [1-4]. Research on this uncertainty problem may become a popular direction in the field of aeroelastic design in the future. However, at present, research in this field is in its infancy.

Thermal parameters, such as the heat flux input, heat conduction coefficient, heat exchange coefficient, and thermal capacitance, are treated as certain quantities in the conventional analysis of heat conduction. However, the uncertainty problem of aerodynamic heat flux is

more prominent in aerodynamic heat computations for hypersonic flow because of the complexity of flow phenomena, the complexity of the relevant experiments, and the current lack of experimental data. Thus, it is necessary to first consider the uncertainty of the aerodynamic heat flux in the analysis of hypersonic aerodynamic-thermal-structure coupling [5].

Researchers worldwide have developed multiple analysis and processing methods to consider the impact of uncertain factors in engineering practice. There are three main types of methods: the stochastic method, fuzzy method, and non-probabilistic set method. The non-probabilistic set method is a new analysis method used to process uncertain information. This method is divided into the convex model method and interval analysis method. The non-probabilistic set method has two main advantages. First, it is not necessary to know the characteristic functions of uncertain variables, such as probability density functions or fuzzy membership functions. Instead, only the variation range of the uncertain variables must be known. Second, this method provides the response range of the structural system or the robust stability margin .

The interval analysis method is suitable for such a situation. The statistical information is not sufficient to describe the probability distribution or membership functions of uncertain parameters; only the interval of the uncertain parameters is known, and only the interval of the structural response is expected to be determined. This situation often occurs in practice. Therefore, using interval theory and the analysis method to study the uncertainty factors in engineering is critical in many situations. However, few researchers use the interval analysis method in the field of aerothermoelastic analysis when considering the uncertainty of parameters.

Therefore, this paper proposes an analysis method for the static aeroelastic response of hypersonic vehicles based on aerodynamic-thermal-structure coupling using the interval

method to consider the uncertainty of the heat flux. The heat flux input for the heat conduction analysis is considered using interval parameters. The element matrices and load vectors, which are both represented as interval parameters, are obtained using a finite element method. The interval finite element equations of the entire structure are obtained by the finite element assembly and subsequently solved using a matrix perturbation method. The temperature field intervals of a wing at every moment in the flight trajectory are obtained. Based on the intervals of the temperature field, a genetic algorithm is used to obtain the temperature field that imposes the severest load on the structure. Finally, a static aeroelastic analysis is performed using this temperature field, and a number of valuable conclusions are obtained.

## **2 INTERVAL ANALYSIS METHOD FOR THE TRANSIENT TEMPERATURE FIELD CONSIDERING THE HEAT FLUX UNCERTAINTY**

For the problem of transient heat conduction, the most general case is that structural parameters and heat flux input are uncertain. In this case, physical parameters, such as the thermal conductivity, material density, material specific heat, and heat flux input, as well as the amplitude of the initial conditions, such as the initial temperature, are all regarded as interval variables. Thus, the problem becomes one of solving an interval mathematics problem that has interval parameters, given interval initial values and boundary value conditions [6].

Then, the governing equation of the interval finite element for the structural transient heat conduction is<sup>[9]</sup>

$$\mathbf{M}(\alpha') \left( \frac{\partial \mathbf{T}}{\partial t} \right)_t + \mathbf{K}(\alpha') \mathbf{T}_t = \mathbf{Q}(\alpha'). \quad (1)$$

Here, the subscript  $t$  indicates that the parameter is calculated at time  $t$ .

This paper uses a numerical integration method to solve the above equation.

### 3 STATIC AEROELASTIC ANALYSIS OF THE WING OF A HYPERSONIC VEHICLE BASED ON AERODYNAMIC-THERMAL-STRUCTURE COUPLING

#### A. Structural statics/dynamics equation in a thermal environment

The kinematic equations of the statics/dynamics for the structure of a hypersonic vehicle's wing based on the finite element method in physical coordinates is expressed as

$$\mathbf{M}_s \ddot{\mathbf{q}} + \mathbf{K}_s^*(\mathbf{T}) \mathbf{q} = \mathbf{F}_s(t), \quad (2)$$

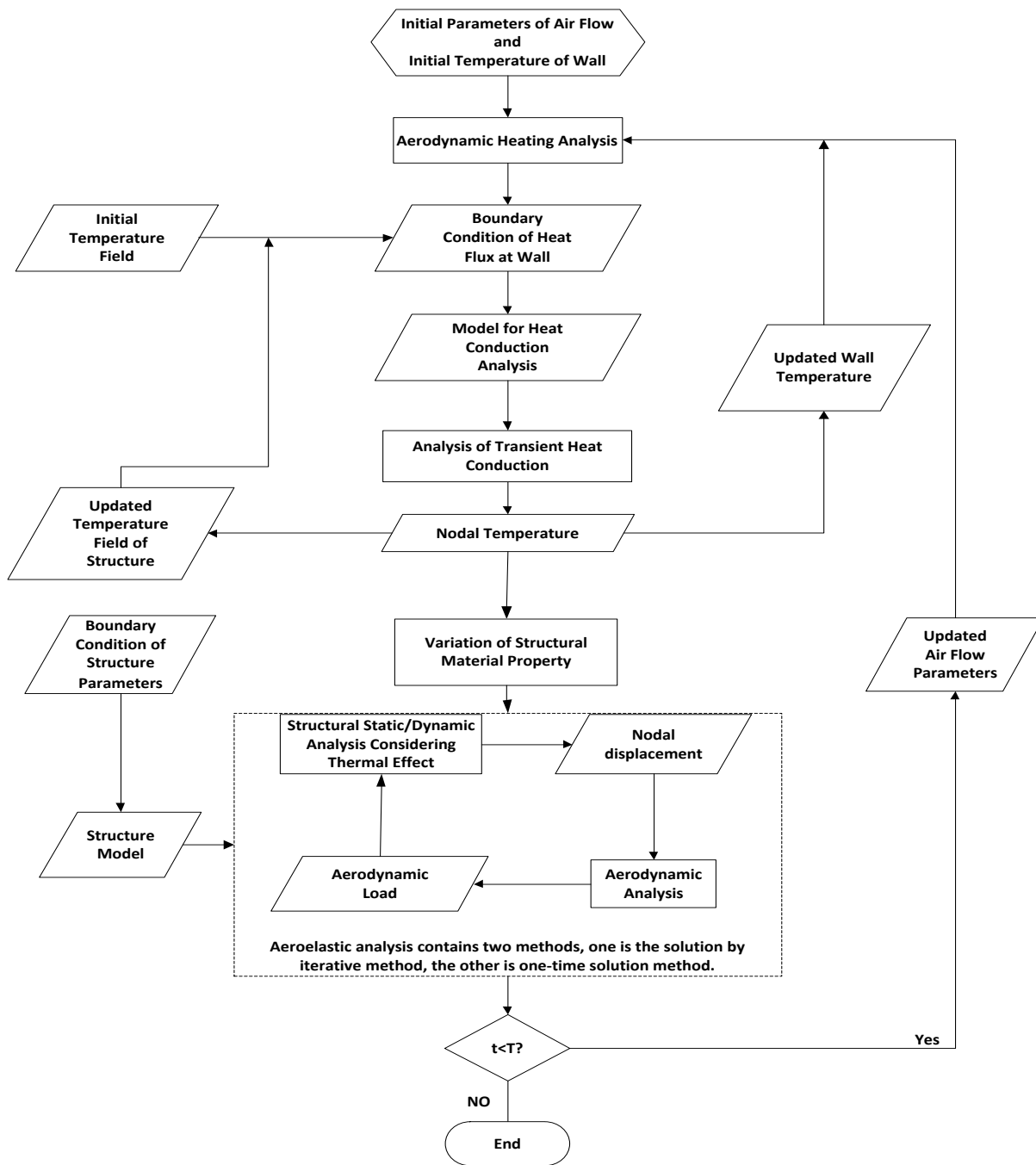
where  $\mathbf{M}_s$  is the mass matrix,  $\mathbf{F}_s$  is the load vector,  $\mathbf{q}$  is the displacement vector, and  $\mathbf{K}_s^*(\mathbf{T})$  is the modified stiffness matrix, expressed as

$$\mathbf{K}_s^*(\mathbf{T}) = \mathbf{K}_s(\mathbf{T}) + \mathbf{K}_G(\mathbf{T}). \quad (3)$$

Here,  $\mathbf{K}_s(\mathbf{T})$  is the classical stiffness matrix of structure. Because the material properties of the structure are temperature dependent,  $\mathbf{K}_s(\mathbf{T})$  is a function of the temperature.  $\mathbf{K}_G(\mathbf{T})$  is the additional geometrical stiffness matrix caused by thermal stress and is related to the temperature field.  $\mathbf{K}_s(\mathbf{T})$  and  $\mathbf{K}_G(\mathbf{T})$  are assembled from  $\mathbf{K}_{el}$  and  $\mathbf{K}_{\sigma e}$ , respectively.

- **Analysis framework**

Fig. 1 presents the framework of the hypersonic aerodynamic-thermal-structure coupling analysis<sup>[10-12]</sup>.



#### 4 AERODYNAMIC-THERMAL-STRUCTURE COUPLING ANALYSIS OF A WING CONSIDERING THE HEAT FLUX UNCERTAINTY

The interval analysis method can be used to obtain the temperature field interval of the wing structure of a hypersonic vehicle in the given heat flux input interval with relative ease.

To ensure the aircraft's safety in flight, the corresponding aeroelastic response analysis must be based on the most severe thermal environment in this temperature field interval.

The wing structure of a hypersonic vehicle consists of two parts: the insulating layer and structure layer. The insulating layer influences the heat conduction analysis of the structure and, to a lesser extent, the mechanical properties of the structure. Thus, in this paper, the temperature field intervals in the insulating layer and structural layer of the model are processed separately to determine the most severe thermal environment for the aircraft.

For the insulating layer of the model, because the temperature of its wall will significantly influence the heat flux input obtained by the aerodynamic heating calculation, the temperature field of the insulating layer is assumed to be the one that maximizes the heat flux input in a given interval. The analysis of the aerodynamic heating calculation process demonstrates that the aerodynamic heat flux input is maximized when the temperature of the wall is minimized.

Because the temperature field significantly influences the properties of the structure layer, one must use an optimization method to determine the most severe temperature field of the structure layer. Here, a genetic algorithm is used to perform this task<sup>[13]</sup>. The optimization formula is

$$\begin{cases} \text{find } \mathbf{T} \\ \text{min } \varphi(\mathbf{T}) \\ \text{s.t. } \underline{\mathbf{T}} \leq \mathbf{T} \leq \bar{\mathbf{T}} \end{cases}, \quad (4)$$

where  $\mathbf{T}$  is the temperature field of the structure,  $\underline{\mathbf{T}}$  and  $\bar{\mathbf{T}}$  are the upper and lower bounds of the temperature field interval, respectively, and  $\varphi(\mathbf{T})$  is objective function that maximizes the thermal deformation of the structure at the wingtip.

## 5 CALCULATION AND ANALYSIS

### 5.1 Model description

The model used in this paper uses the low-aspect-ratio wing of a hypersonic aircraft, and its wing span, chord length and thickness are shown in Fig.2[6].

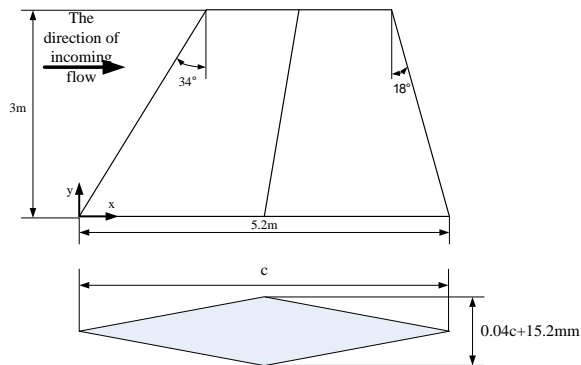


Fig. 2 Configuration of the hypersonic aircraft wing.

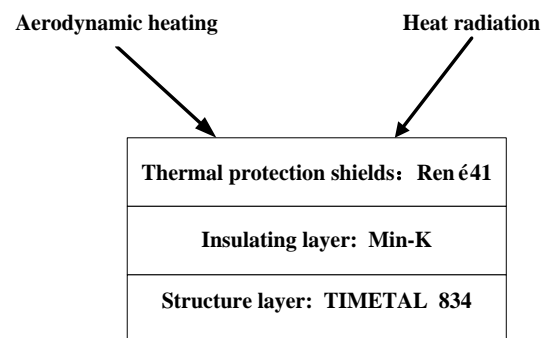


Fig. 3 Scheme of thermal protection shields and structure layer on the aircraft surface.

Because a hypersonic aircraft in flight encounters a severe thermal environment, a number of thermal protection measures must be taken to protect the wing structure, as shown in Fig. 3. The proposed model uses 7.6 mm-thick thermal protection shields added at the upper and lower surfaces of the wing to achieve this goal.

### 5.2 Analysis results without considering uncertainty

In this section, the static aeroelastic analysis of the wing under the aerodynamic-thermal-structure coupling framework that does not consider uncertainty is performed with the altitude, angle of attack, Mach number, and load factor given. The specific parameter values used in the analysis are as follows:  $Ma = 8$ ,  $H = 15\text{km}$ ,  $\alpha = 6^\circ$ , longitudinal overload = -1 g, initial temperature of the structure  $T_{ref} = 293\text{K}$ , Prandtl number  $Pr = 0.86$ , and specific heat ratio of the atmosphere  $\gamma = 1.4$ .



The radiant emissivity of the surface layer of the structure, i.e., the thermal protection shield, is 0.85, the time interval for the aerodynamic heating and heat conduction analysis is  $\Delta t = 4s$ , and the total calculation time is  $t_{total} = 600s$ .

Table 1 presents the results of the analysis. After being in flight for 600 s, the pressure on the lower surface of the wing is clearly higher than that on the upper surface due to the positive angle of attack. Thus, the heat flux and temperature on the lower surface are significantly higher than those on the upper surface. The temperature on the leading and trailing edge of the structure layer is relatively high, reaching 1,100 K. The temperature of the middle region is lower, reaching approximately 350-500 K. Thus, the thermal protection layer is very effective. The structural bending deformation is not significant, and the maximum displacement at the wingtip is 106.1 mm. However, because the temperature on the lower surface is higher than that on the upper surface, there is a clear upward wrap deformation, as shown in Fig. 4.

Table. 1 Comparison of the results of the temperature field between the states considering and not considering uncertainties at 600 s.

|  | The maximum deformation at wingtip (mm) | The maximum temperature on structure layer (K) | The average temperature on structure layer (K) | The maximum temperature on upper surface (K) | The average temperature on upper surface (K) | The maximum temperature on lower surface (K) | The average temperature on lower surface (K) |
|--|---|--|--|--|--|--|--|
| Not considering the uncertainty of heat flux input | 106.1                                   | 1,119  | 551  | 1,347  | 1,124  | 2,040  | 1,663  |
| Considering the uncertainty of heat flux input     | 122.7                                   | 1,288  | 584  | 1,338  | 1,121  | 2,057  | 1,664  |

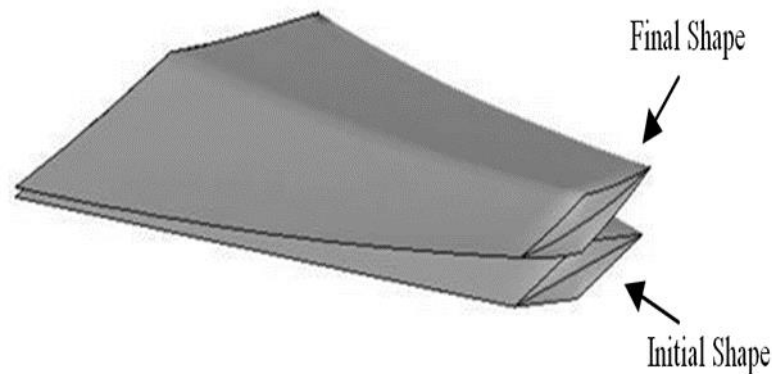


Fig. 4 Deformation of the wing at 600 s.

### 5.3 Analysis results considering uncertainty

Based on section 5.2, the analysis considering the uncertainty of the heat flux input for the example of a hypersonic wing under the aerodynamic-thermal-structure coupling framework is completed. The uncertainty of the heat flux input is set to 10%. The severest temperature field of the thermal protection shield is calculated quickly. The field is calculated every 4 s. For the structure layer, the temperature field is calculated every 100 s to reduce computational consumption because its temperature changes slowly and it is computationally expensive to calculate the most critical temperature field using a genetic algorithm.

Table 1 compares the results between the states considering and not considering the uncertainty. Fig. 5 illustrates how the radius of the temperature field interval (the maximum range of the temperature perturbation) changes over time at the leading edge of the wingtip on the structure layer, the middle point of the wing root on the upper surface, the middle point of the leading edge on the structure layer, and the central point on the upper surface. Fig. 5 also presents the ratio (in percent) of the radii of the temperature field interval over the average temperatures ( $\Delta T / T^c$ ) at these positions after 600 s.

The maximum deformation of the wingtip increases by 16.6 mm once the uncertainty of the heat flux input is considered. Moreover, the average temperature of the structure layer

increases by 5.99%, i.e., by 33 K. The temperature on the surface is subject to heat radiation and that the temperature fields on the surface in the two cases are highly similar. Over time, the radius of the temperature field interval gradually increases, indicating that the temperature field of the wing becomes increasingly uncertain.

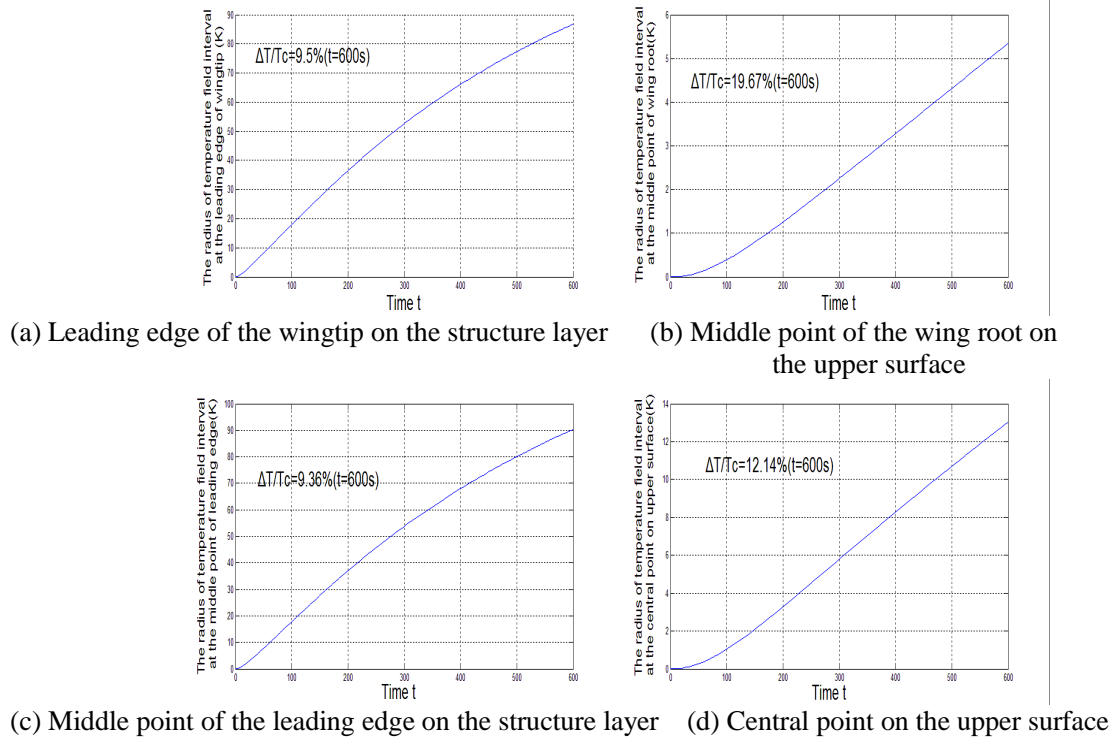


Fig. 5 Temperature at different positions of the model structure layer over time.

## 6 CONCLUSIONS

In this paper, the interval method was introduced for the analysis of the transient heat conduction of a hypersonic vehicle's wing structure. The temperature field intervals at every moment in the flight trajectory were obtained considering the uncertainty of the heat flux input. Next, based on the intervals of the temperature field, a genetic algorithm was used to obtain the temperature field that results in the most severe load on the structure. The analysis method for the static aeroelastic response of a hypersonic vehicle based on aerodynamic-thermal-structure coupling considering the uncertainty of the heat flux was proposed and developed. The following conclusions can be drawn:

1) The uncertainty of the hypersonic aerodynamic heat flux has a considerable impact on the temperature field obtained by the heat conduction analysis of the structure as well as on the distribution of the structure stiffness; thus, the uncertainty impacts the analysis result for the static aeroelastic response.

2) The interval method can effectively predict the transient temperature field of the structure when the impact of the heat flux uncertainty on the heat conduction is considered. The temperature field is calculated quickly. The interval method is easily integrated into the aerothermoelastic analysis framework of the aerodynamic-thermal-structure coupling for hypersonic vehicles considering parameter uncertainties.

3) The use of an optimization algorithm, such as a genetic algorithm, to calculate the most severe temperature field of the structure is computationally expensive. Therefore, the insulating and structure layer must be processed separately in the analysis.

This paper focused on the study of the applicability of the interval method to the analysis of the static aeroelastic response of hypersonic vehicles based on aerodynamic-thermal-structure coupling considering the parameter uncertainty. This paper only considered the uncertainty of the heat flux input of the vehicle. The impact of uncertainties in other parameters (e.g., the pressure distribution of the aerodynamic force and the material properties of structure) on the aeroelastic response are not considered. However, the impact of the uncertainty of other parameters on the result may be important. The impact of various uncertain factors will be considered together in further studies to further develop the research method proposed in this paper.

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