NONLINEAR GROUND VIBRATION IDENTIFICATION OF AN F-16 AIRCRAFT
PART I: FAST NONPARAMETRIC ANALYSIS OF DISTORTIONS IN FRF MEASUREMENTS

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Abstract: Although they are generally modeled as linear systems, aircraft structures are known to be prone to nonlinear phenomena. A specific challenge encountered with fighter aircraft, besides aero-elastic nonlinearity, is the modeling of the wing-to-payload mounting interfaces. For large amplitudes of vibration, friction and gaps may be triggered in these connections and markedly impact the dynamic behavior of the complete structure. In this series of two papers, the nonlinear dynamics of an F-16 aircraft is investigated using rigorous methods applied to real data collected during a ground vibration test campaign.

The present work focuses on the detection, qualification and quantification of nonlinear distortions affecting frequency response function (FRF) measurements. The key idea of the approach is to excite the structure using a random signal with a user-defined amplitude spectrum, where only a set of well-selected frequencies is different from zero in the band of interest. It is demonstrated that this careful choice of the input frequencies allows, without any further user interaction, to quantify the importance of odd and even nonlinear distortions in the output spectra with respect to the noise level. At high excitation amplitudes, the F-16 dynamics is found to exhibit substantial odd nonlinearities and less significant, yet not negligible, even nonlinearities.

1 INTRODUCTION

Ground Vibration Testing (GVT) is typically done near the end of the development process of an airplane. Its purpose is to obtain an experimental validation and/or improvement of the structural dynamical model of the plane to allow better understanding of its behavior. One of the goals of this campaign is to predict the flutter behavior and to plan the safety-critical in-flight tests of the aircraft [1]. The methods developed for GVT used in most current industrial tests use linear time invariant (LTI) identification methods. These methods are well behaved as long as the excitation level of the input signal remains low enough so that the system is excited in its linear domain. When the aircraft excitation passes a certain level, the nonlinear (NL) distortions become prominent. This causes the LTI methodology to break down. Still using it leads to an incorrect interpretation of the measurement results and leads to bad decisions in the following steps of the development.

Different approaches exists how to deal with these nonlinear distortions. In this paper, two methods are explained and combined [2, 3]. The first method, the Fast Method (FM), answers the question if nonlinear distortions are present. Next, it provides the classification of the distortions, answering the question whether or not the nonlinear are distortions odd (i.e. x^3) or even (i.e. x^2). This method also provides knowledge about the dynamics of the system allowing to track the shift of a resonance frequency. The second method provides the Best Linear Approximation (BLA) of the system combined with the level of the nonlinear distortions (no split in even and odd nonlinearities).

In this paper, the two methods are introduced and applied to an F-16 Fighter Falcon of the Belgian army (Figure 1). The measurements are shown and discussed. The two methods use a special case of the random phase multisine excitation. This signal is introduced in the next section.

Figure 1: F-16 Fighter Falcon of the Belgian Army. (FA-03)

2 INPUT SIGNAL SELECTION

The goal is of the experiment is to get more information about the F-16 Fighter Falcon without changing the usual setups for vibration testing. This means that the hardware can not be changed. The only way to gather more knowledge is to change the signal applied at the input. Exciting the F-16 using a broadband signal allows the practitioner to quickly recover more resonance frequencies and responses at the frequencies of interest. A modified version of the random phase multisine is used to this end. The random phase multisine is defined below:

$$u(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N/2-1} A_k \sin(k\omega_0 t + \varphi_k) \quad (1)$$

with $A_k \in \mathbb{R}^+$ the user defined real amplitude of the sine wave with a frequency $k\omega_0$ and a random phase value φ_k distributed in $[0,2\pi[$. This signal excites the system at the frequencies $k\omega_0$ within the band of interestF_{band}. It looks like a periodic random signal in the time domain. This is shown in Figure 2 for a random phase multisine with $f_s = 200Hz$, $N = 8192$, $F_{band} = 1\ to\ 60\ Hz$. The advantage when compared to a random noise excitation, is that the user has full control over the amplitude spectrum A_k.

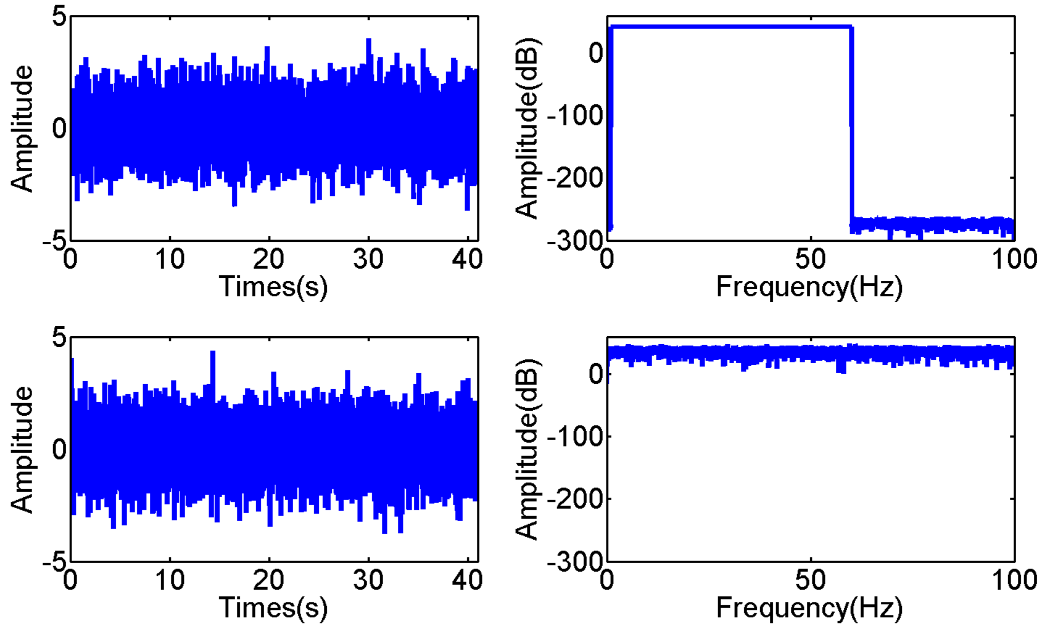

Figure 2: Difference between a multisine and random noise. Upper: time domain and frequency domain plot of the random phase multisine. Lower: time domain and frequency domain plot of a Gaussian signal. $f_s = 200Hz$, $N = 8192$ and $F_{band} = 1\ to\ 60\ Hz$.

A single realization of the multisine and a noise signal covering the same spectral band is shown on the right in Figure 2. While this signal allows much more control and insight into the system, it still needs to be altered be able to detect, quantify and qualify the nonlinear system behavior.

2.1 Detection, quantification and qualification of the system

Qualification of the nonlinear distortion means separating the odd from the even nonlinear distortions. This provides more knowledge about the system and allows the user to get insight in the dynamics of the system. Shifting of the resonance frequencies with variations of the input signal power can lead to (bad) phenomena, including instability during flight tests.

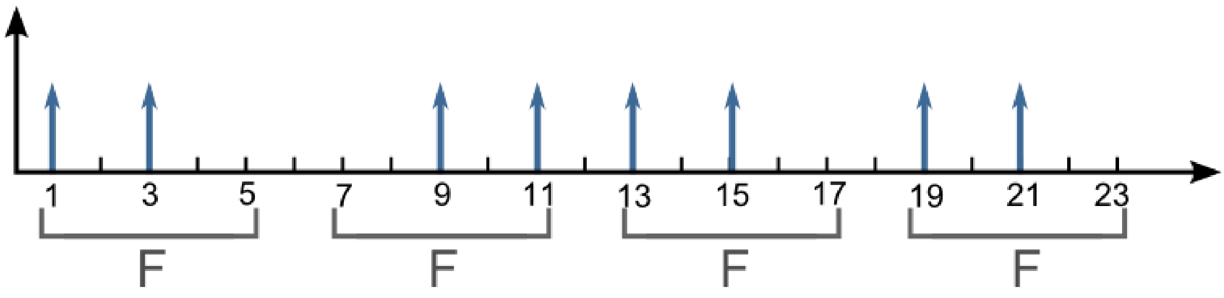

Figure 3: Frequency domain plot of a random phase multisine with detection lines for the odd nonlinear distortions. one odd harmonic in each set of F consecutive harmonics is eliminated. In this example, F= 3

To obtain this additional information, the random phase multisine is altered. Instead of using a full random phase multisine signal where all the spectral components present in the multisine are excited, only the odd lines (2k+1) are excited.

This signal does not excite the system at the even frequency components. Also, exactly 1 randomly selected odd spectral line in each group of F consecutive harmonics is eliminated. This leaves some of the odd lines unexcited. The resulting spectral grid of the signal is shown in Figure 3. The idea behind this type of signal is simple and intuitive. Depending on the type of the nonlinear distortion present in the system, the resulting output spectrum will be different. An example is shown in Figure 4. If the system is linear, the frequencies excited at the output exactly match the frequencies excited at the input, but have a different amplitude and/or phase. No other frequencies will be energized besides the ones present at the input. However, if the system creates nonlinear distortions, the energy inserted into one frequency at the input can be spread over multiple frequencies at the output. Figure 4 shows the output of an even and odd nonlinear system excited by an odd random phase multisine with detection lines. When the signal is applied to an even nonlinear system, only the even output frequencies will be excited. This happens because the sum of an even combination of odd frequencies is even. If the excited system is an odd nonlinear system, the frequencies at the output are odd. This allows to qualify the system. Note that if the system contains a combination of a linear and an odd nonlinear part, the qualification proves impossible if the odd random phase does not contain the odd detection lines. Removing one randomly selected odd harmonic in each group of F consecutive harmonics allows the qualification between the linear and the odd nonlinear contributions of the system.

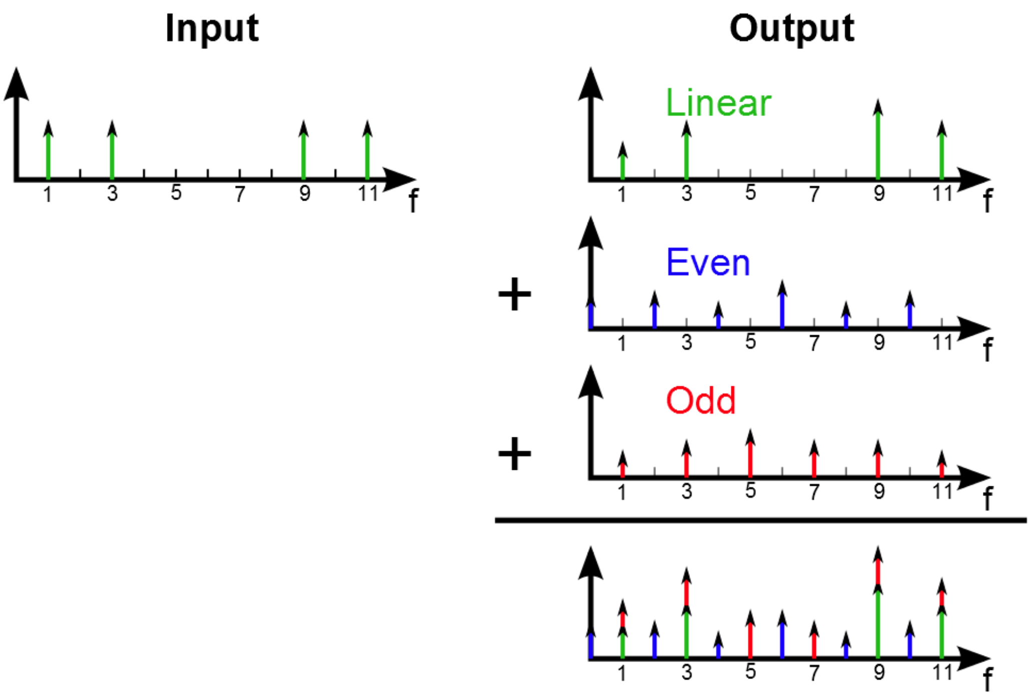

Figure 4: Output of a linear, an even (second order), an odd (third order) and a combination of all three systems using an odd random phase multisine with detection lines.

Most systems contain linear, odd nonlinear and even nonlinear contributions. The output of such a system can be seen at the bottom of Figure 4. Note that this type of signal has less than half the frequency resolution of a full multisine. If the same frequency resolution is required for the random phase odd multisine as the full multisine, the sampling frequency f_s should be lowered or the amount of points N should be increased by a factor of 2 or more.

The proposed method is a very fast way to obtain information about the system that is being identified. Only one signal realization and some signal periods are to be measured to get information about the FRF, the even and the odd nonlinearities, and the power spectrum of the disturbing noise.

A lot of systems in the industry are being used at a certain point of operation. To obtain a reliable and accurate linearized model, it is an advantage to measure the Best Linear Approximation (BLA) at this point of operation. This means that the LTI-identification methods can be used to identify the system while treating the nonlinear distortions as a kind of noise perturbation. This is shown in the next section.

2.2 Finding the Best Linear Approximation

When applying an odd random grid random phase multisine to a nonlinear system, it is possible to see the influence of the odd and even nonlinear distortions. However, the output frequencies that contain energy at the input contain linear and odd nonlinear contributions at the system's output. When the odd nonlinear contributions become more prominent, the calculated Frequency Response Function (FRF) becomes noisy as seen in an in Figure 5. Averaging over several periods does not help, the result still remains noisy. However, applying a different phase-realization of the same signal several times as an input to the system, and averaging over the realizations of the excitation will reduce the noise on the FRF.

This means that the nonlinear distortions can be treated as stochastic noise. The Frequency Response Function (FRF) contains the following contributions:

$$G(j\omega) = G_{BLA}(j\omega_k) + G_s(j\omega_k) + N_G(k) \quad (2)$$

with $G_{BLA}(j\omega_k)$ the best linear approximation (BLA), $G_s(j\omega_k)$ the stochastic nonlinear contributions, and $N_G(k)$ the errors due to the measurement output noise [2]. The second method, the Robust Method (RM), takes advantage of this equation. The principle of this method is shown in Figure 6. The measurement noise level can be found by averaging over successive periods. The level of the nonlinear distortions is found by averaging the FRF over different realizations of the input signal. This can be multisine signals with the same power and bandwidth excitation, but with a different set of random phases. By averaging over different realizations, the nonlinear distortions that behaves as noise, will be averaged to zero.

This method allows a better Signal to Noise Ratio (SNR) due to the long measuring time. However, this is also a downside of the method as measuring multiple periods of multiple realizations takes a lot of time. Therefore, the Fast Method and the Robust methods have been combined to be able to measure on the F-16 Fighter Falcon. The results of these measurements are shown in the next paragraph.

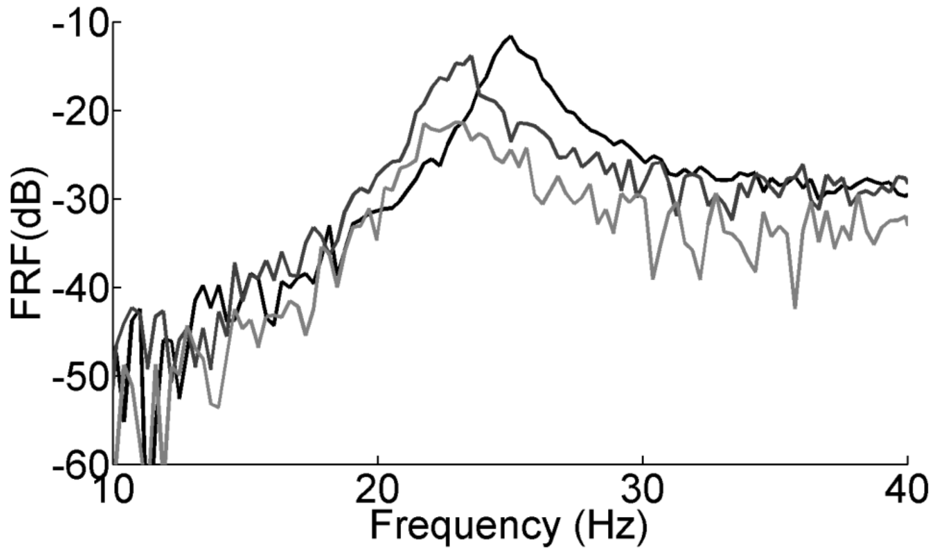

Figure 5: Example of a the increasing noise due to the nonlinear distortion when increasing the power of the input signal (black to gray)

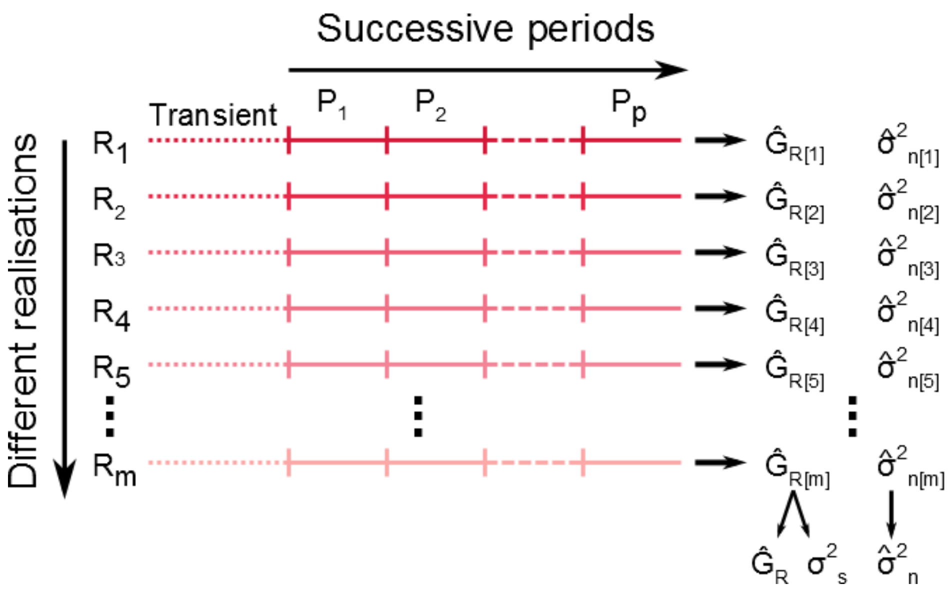

Figure 6: Principle of the Robust Method to find the Best Linear Approximation

3 MEASUREMENT RESULTS

3.1 Measurement setup

A Single Input Multiple output (SISO) measurement setup is used. The right wing is excited with a shaker that is fed with the random phase multisine as shown in Figure 7. As much as 160 accelerometers are placed on the aircraft to measure the response. The focus of the two day measurement campaign is to analyze the behavior of the connection between the right wing tip and the missile. The connection can be seen as a bolted connection which behaves as a softening spring. Three power levels were used to excite the wing with a set of signals with the following specifications: sampling frequency $f_s = 200Hz$, $N = 8192$ number of data points per period, band of excitation $F_{band} = 1\ to\ 60\ Hz$, $M = 10$ realizations and $P = 3$ periods. The total measurement time for one power level took just over 20 minutes. Both methods described above will be used to analyze the measurement results. The band of interest analyzed in the results ranges from 3Hz to 15Hz due to the modes of interest being in that range.

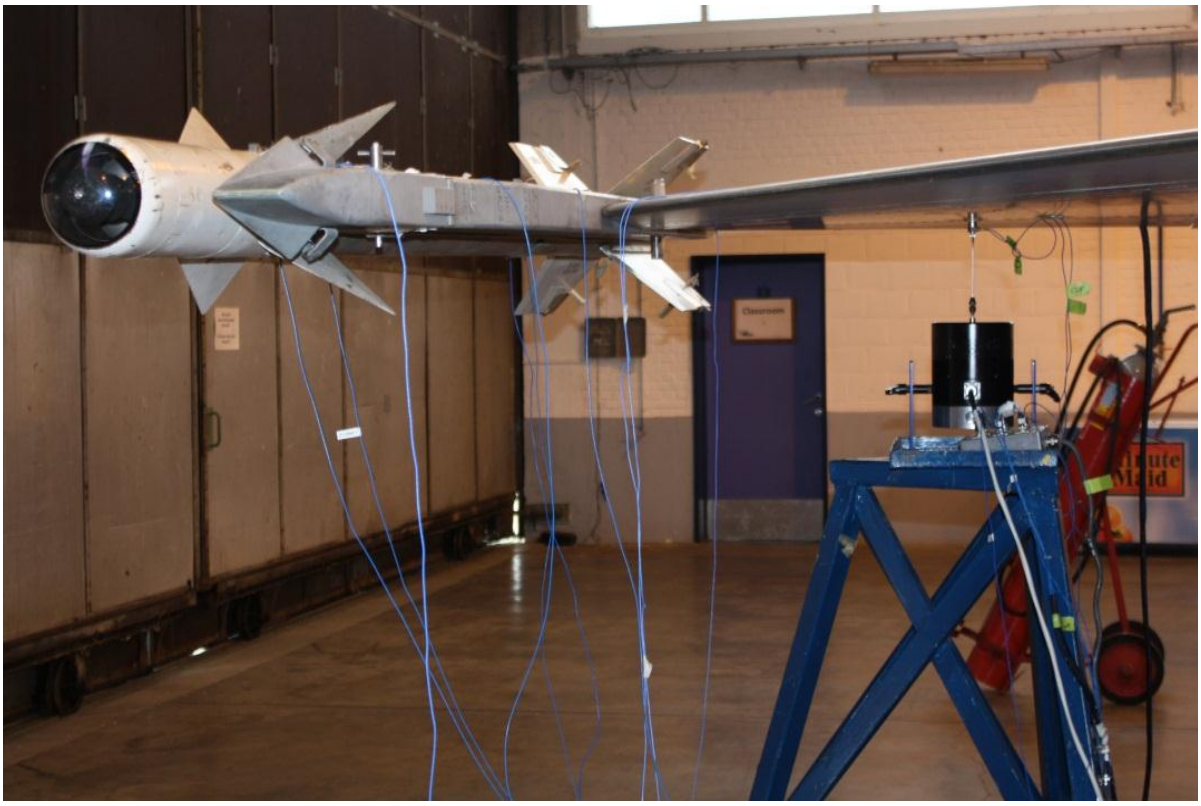

Figure 7: The setup of the shaker and the accelerometers

3.2 Experimental Results

3.2.1 Applying the Fast Method

The first result shown in Figure 8 shows the acceleration output at the missile side resulting from an excitation with an odd random phase random grid multisine with a low power applied at the input. The level off odd and even nonlinear distortions is close to the noise level at almost all frequencies, except for the anti-resonance - resonance pair closest to 9Hz where the even detection lines appear to be significant. This means that globally speaking the system is behaving linear. Increasing the input power results in an output where the nonlinear distortions become more prominently present over the complete frequency span, as shown in Figure 9. All the nonlinear distortions are much higher than the noise over the complete band now. Here, in the neighborhood of the the resonance frequencies, the odd nonlinear distortions are dominant unlike what happened at the low excitation level. Using the signal levels at the resonance frequencies allows one to estimate the degree of the nonlinear distortions, and to check the even/odd behavior that is expected. From Figure 8 to 9, the excitation is raised by 15dB. Hence, the even distortions should increase by 30dB or more, while the odd third order ones should increase with 45dB. Comparing the even/odd distortion near the resonance frequencies at 7Hz and 9Hz shows that this is indeed the case.

An odd nonlinear distortion can cause changes of the dynamics of a system, which can be dangerous for stability/damping [2]. Odd nonlinear distortions tend to shift the resonance frequencies, and is shown in 3.4

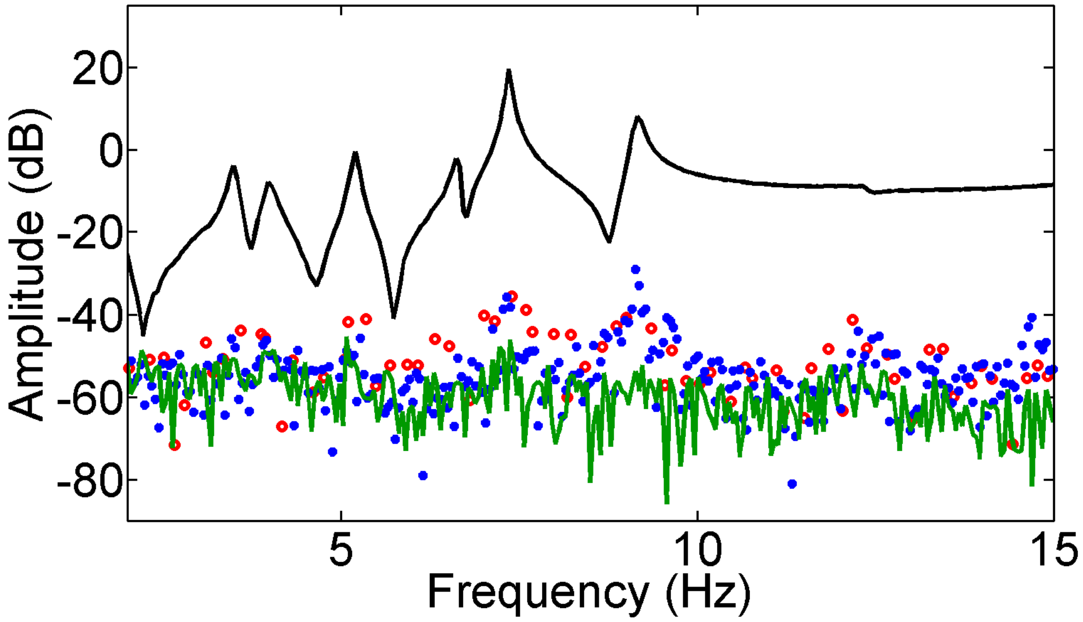

Figure 8: Output spectrum measured for a low power multisine excitation at the input. Black = Output spectrum, red o = Odd detection lines, blue . = Even detection lines, green = Measurement noise level

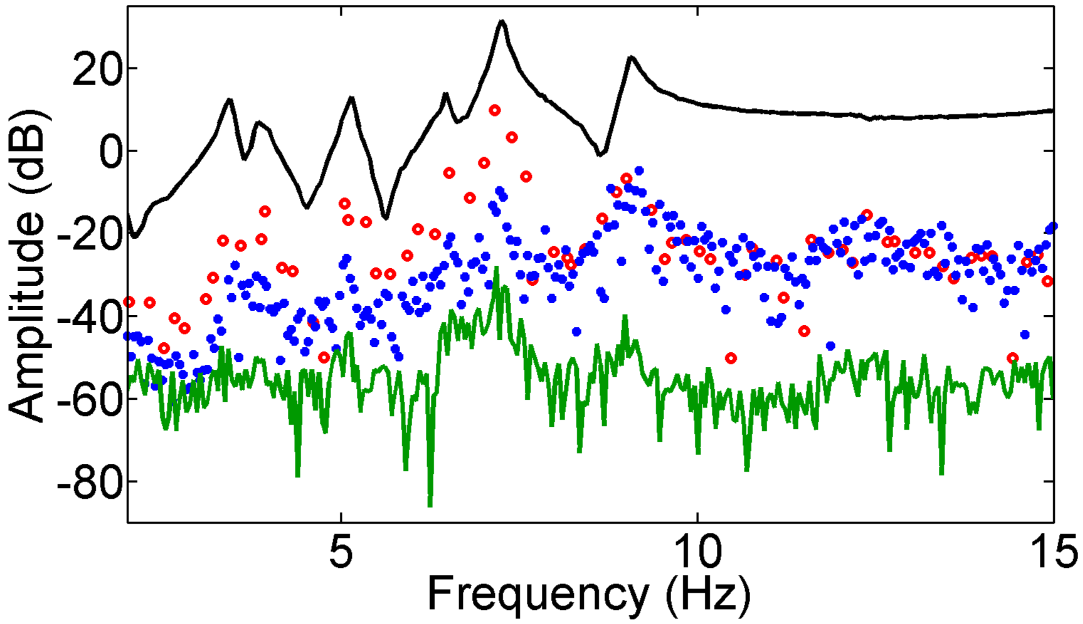

Figure 9: Output spectrum measured for a high power multisine excitation at the input. Black = Output spectrum, red o = Odd detection lines, blue . = Even detection lines, green = Measurement noise level

3.2.2 Applying the Robust Method

Section 2.2 shows that, averaging over the realizations allows to obtain a high SNR version of the BLA. Figure 10 shows the measured BLA for increasing excitation power (blue = lowest, red = highest). If the system would be LTI, the result would always be the identical. However, the presence of the nonlinear distortions results in an increased damping for increasing the excitation power. Also, a shift in the resonance frequencies is visible in Figure 10, all the resonance frequencies shift to lower frequencies.

When focusing on one level, it is possible to show both the level of the nonlinear distortions and the noise level. This is shown in Figure 11. Notice that this method can also be applied using a full random phase multisine to attain a higher resolution if the same signal specifications are used.

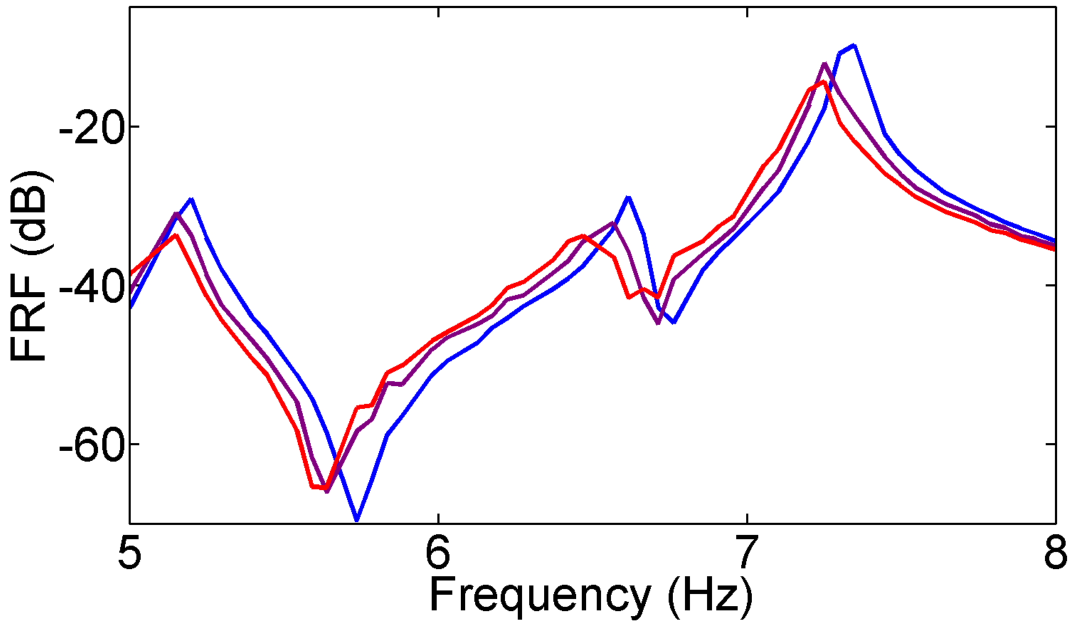

Figure 10: Result of the $G_{BLA}(j\omega_k)\}$ of the F-16 Fighter falcon with increasing power (blue to red)

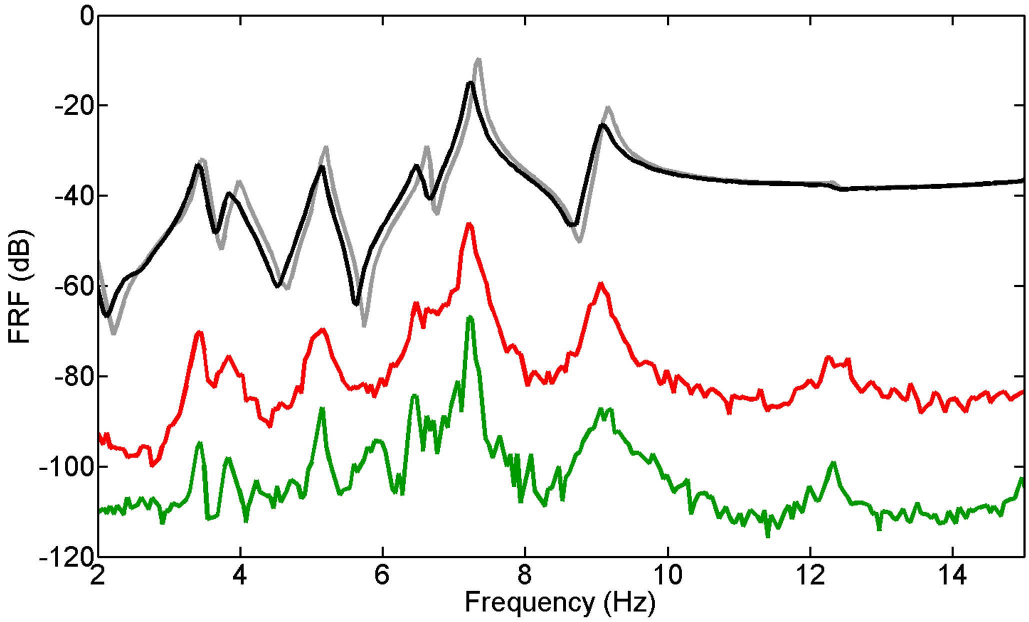

Figure 11: Results of the Robust Method with $M = 10$ and $P = 3$. Black = BLA, red = NL distortions, green = noise level, gray = BLA with a higher input power.

4 CONLUSIONS

Two methods were proposed, the fast and the robust one. The advantage of the Fast Method is the possibility to characterize the nonlinear distortion of the system, using a single experiment with one realization and several periods. The downside of this method is the decrease (less than half) of the frequency resolution due to the required presence of detection bins and the lower SNR that results from the short measurement time. To solve the SNR and resolution problems, the Robust Method can be used. This method requires more measurement time as multiple realizations of the input signals are measured. Another advantage of the Robust Method is the possibility to average the nonlinear distortion over the realizations. Averaging over the measured realizations yields a BLA estimate with a lower standard deviation. The authors advice the use of the combined method whenever this is possible. A higher measurement time is traded for a maximal knowledge about the nonlinear distortion of the wing-missile connection of the F-16 Fighter Falcon, and a better FRF is obtained.

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