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AN EFFICIENT APPROACH FOR IN-OPERATION MODAL ANALYSIS OF FLUTTER FLIGHT TESTS

B. Jacquier[1], P. Vacher[1], S. Leroy[2], D. Walker[2]

[1] Department of System Control and Flight Dynamics, ONERA
BP 4025, F-31055 Toulouse Cedex, France
beatrice.jacquier@onera.fr

[2] AIRBUS OPERATION SAS
316, Route de Bayonne, 31060 Toulouse, France
stephane.leroy@airbus.com

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Abstract: The certification of a new aircraft requires the modal analysis of the aircraft in a great number of flight conditions. However both the environmental conditions in which these tests are performed and the requirements for the processing algorithms are very specific as compared to conventional modal analysis. Namely, very short excitation signals like pulses are now used to reduce the duration and therefore the cost of the flight tests. In order to cope with these unusual conditions, a specific modal analysis procedure was devised. It is based on a deterministic output-only identification approach.

This procedure was developed in the framework of a research program in cooperation with Airbus. The goal was to develop a software to be implemented in the telemetry center for the near real-time processing of flight test data. The paper describes the specifications on the algorithms imposed by the operational context of flutter testing. It details the organization and the modules of the procedure designed to comply with these requirements. Modal analysis results are also presented both for Montecarlo benchmarks based on an elaborate aeroelastic model and for real flight test records.

1 INTRODUCTION

Modal analysis applies to a wide range of objects: vehicles (ground vehicles, trains, aircraft, satellites,...), civil engineering constructions (skyscrapers, historical monuments, bridges, ...) and various objects (wind turbines, electronic racks, home appliances, ...). Amongst this large gamut of applications, the analysis of aircraft is particular for special reasons. First, modal analysis is quite critical to ensure the safety of the flight tests of any new aircraft. The goal is to detect any tendency of the aeroelastic modes towards instabilities that might lead to the in-flight destruction of the airplane. This hazardous unstable phenomenon is called “flutter”. Second, the aircraft is analyzed in her operational environment which implies that the measurements are affected by noise and various other disturbances. The quality of the modal analysis also heavily rely on the manner the aircraft structure is excited. For modern transport airplanes equipped with fly-by-wire control systems, the structure is excited by injecting excitation signals into the actuators of the control surfaces. This approach not only avoids the installation of specific external excitation devices. It above all prevents possible alteration of the dynamic behaviour of the structure induced by these devices. However the main drawback is the limited level of excitation that can be achieved by this approach. This is due to the dynamic limitations of the control surface actuators and also to their locations on their aircraft which are indeed not guided by modal analysis considerations. The disturbances affecting the flight experiments and this lack of efficiency of the excitation lead to poor signal-to-noise ratios of the measurements. So the modal analysis of aircraft constitutes a challenging case for the processing methods.

The approach presented in this paper was motivated by an evolution of the flutter testing strategy at Airbus. In order to reduce the cost of flight testing, the great majority of the tests are now performed with pulse excitations instead of frequency sweeps. This new orientation results in a drastic decrease of the duration of each test from about two minutes down to ten seconds. But the consequence is the brevity of the data records and a marked deterioration of its quality. In the framework of a research project in cooperation with Airbus, two solutions were studied so as to improve the accuracy of the modal analysis:

- the design of more efficient approaches to excite the aircraft that preserve the shortness of the tests
- the development of a processing method dedicated to the processing of very short tests

The first topic, which has already been addressed in the articles [1-3], is briefly recalled in Section 2. The second point, *i.e.* the design of a procedure appropriate to the analysis of short data sequences is addressed in Section 3. This procedure, dubbed MATEST (Modal Analysis Tool for Extremely Short Tests), was conceived for the implementation in Airbus flight test center. Finally, Section 4 presents identification results on benchmarks based on an aeroelastic model and on real flight test data.

2 A NEW EXCITATION STRATEGY

2.1 Current flutter testing protocol

Flight tests are composed of several series of tests performed at stabilized test points that is to say points where the aircraft is maintained at a constant speed and a constant altitude. As the likelihood of flutter onset increases with airspeed, these points are explored in increasing order of speed.

At each of these test points, several tests are performed by applying excitations to the aircraft structure. The aircraft responses are measured by a set of accelerometers (100-150) installed all over the structure of the aircraft. These data are telemetered to the ground test center where the mode damping ratios are estimated as soon as the test is complete. The damping estimates obtained at each stabilized test point establish a trend as a function of airspeed which is used to evaluate the stability of the next higher speed point.

At Airbus, the current testing procedure is usually based on five tests performed at each test point. Each test consists in applying the excitation signal to the control surfaces in the five following way: symmetrical deflections of the ailerons, anti-symmetrical deflections of the ailerons, symmetrical deflections of the elevators, anti-symmetrical deflections of the elevators and rudder deflection (explicitly anti-symmetrical).

In order to shrink the overall duration of the flight tests, pulse signals are now mainly used for flutter testing. The aircraft response available for the modal analysis is about ten seconds long. This approach however leads to very poor signal-to-noise ratios in the measurements.

This situation gets even worse as the aircraft speed increases for two reasons. First, the level of the noise increases with speed. Second, the efficiency of the excitation also reduces because of limitations on the deflection of the control surfaces at high speeds. Thus the high speed tests which are the most critical for the appearance of flutter are associated with the poorest quality of data.

2.2 New testing protocol

In the flutter community, it is well-known that low excitation levels results in a large scatter in the damping values estimated from the measurements [4]. It is then essential to excite all the modes of interest with a sufficient energy level in order to assess their stability correctly. In the framework of this research project, two orientations were studied in order to improve the quality of the modal analysis:

- the coordinated excitation of several control surfaces of the aircraft,
- the design of more efficient excitation signals.

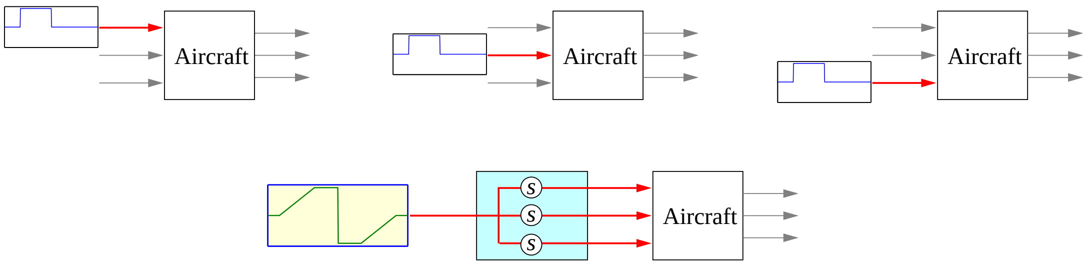

Figure 1: Improved excitation protocol

These two concepts are illustrated in Figure 1. Instead of performing tests on each control surfaces individually as depicted in the upper part of the figure, the proposed new protocol is based on the excitation of several control surfaces simultaneously with an appropriate sign for each surface (lower part of Figure 1). This has several beneficial consequences. First, the energy transferred to the aircraft is greater and this excitation energy is more evenly distributed over the different components of the structure. Second, as explained in [2,3], a reduced number of input combinations are sufficient to excite all the modes of interest of the aircraft. Finally, these combinations can be grouped in a single test and applied one after the other in a row. So the number of tests performed at each test point can be reduced from five in the present protocol to a single one thus entailing a radical reduction of test duration.

The design of more efficient excitation signals is based on variations on the doublet. This topic is developed in [1]. One of these improved signals called the “Mexican doublet” is depicted in Figure 1.As compared to a pulse, the signal was optimized in order to prevent any zeroing of its energy spectrum in the frequency band of interest for modal analysis.

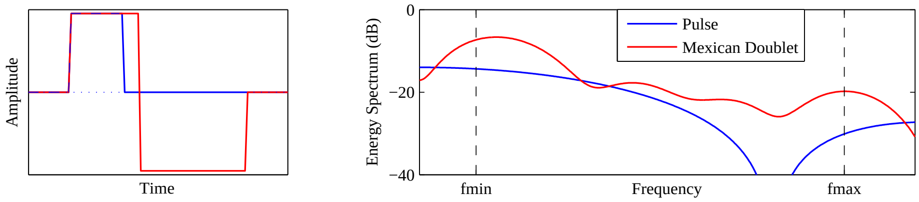

Figure 2: Comparison of a pulse and a Mexican doublet

3 THE MODAL ANALYSIS PROCEDURE

3.1 Specifications for the in-flight modal analysis

Performing the modal analysis in the very course of the flight tests implies specific requirements for the processing procedure. First of all, the algorithms must operate very quickly in order to deliver the clearance to the next test point as soon as possible. The second demanding requirement is that the modal analysis procedure should be fully automatic. Though aeroelastic models of the aircraft structural dynamics are available before the first flight, the identification of the structural modes of the aircraft cannot make use of this a priori information. The relevant modal parameters to detect possible structural instabilities are the mode damping ratios. These quantities are generally difficult to identify accurately. Moreover, as abovementioned, the modal analysis of flutter tests is carried out in unfavorable conditions. In addition, some of the modes may be quite close to each other, both in terms of frequency and damping ratio. In spite of this disadvantageous context, the estimation of the damping ratios should be robust to these various disturbances. It should also be accurate enough so that the test series underway would not be stopped unduly by inaccurate low value estimates of the damping ratios. But and above all, flutter should not occur unpredicted. Finally, the algorithms must deal with a great amount of measurements.

3.2 Strategy selected for the procedure

A common approach for flight test analysis is phase separation methods. These approaches are based on a sequence of two operations. In a first time, Frequency Response Functions (FRFs) are estimated from the flight test records. In a second phase, modal parameters are identified from the estimated FRFs.

Most approaches for FRF estimation requires sufficiently long data sequences in order to minimize the effect of noise by performing some averaging over several data windows. The duration of the tests used for flutter testing proves to be insufficient for this operation. So the modal analysis has to be performed directly on the raw Fourier transforms of the measurement samples.

Output-error methods are common in system identification. One could contemplate to use an output-error method for analyzing flutter tests by fitting a parametric model of the system transfer function to the raw Fourier transforms of the measurements. In the case of single-input systems, well-known algorithms are available to perform the identification [5-7]. These algorithms are reputed to be robust. An identification procedure based on these algorithms, dubbed MEFAS, has been developed and successfully implemented in Airbus telemetry center [8]. However, the extension of these methods to multivariate transfer functions is still a subject of academic research [9]. This extension leads to more complex and computationally costly algorithms with a level of maturity probably not sufficient for an operational implementation.

The single-input variant of the output-error approach is compatible with the current testing protocol which makes use of a single excitation signal. This method requires that the states at the beginning and at the end of the experiment be identical. In practice, this condition is fulfilled by considering a period long enough at the end of the test so that the system converges back to its original equilibrium. But, as explained in the following, a long time window is not always favourable for the quality of the data.

Output-errors methods are also based on the hypothesis of a linear relation between the inputs and the outputs of the system. On civil aircraft, the application of pulse-like signals on the control surfaces pushes the actuators to their limits. So the relation between the excitation signal and the deflection of the control surfaces is actually non-linear because of saturations occurring in the actuators.

So to overcome the above mentioned drawbacks, it was decided to perform the identification in a shorter data window than the one needed to estimate a transfer function of the system. The current and future testing protocols are based on short signals as illustrated by Figure 3 which depicts the simulated response of an aircraft to a succession of two Mexican doublets. As above-mentioned, the time zone where the excitation is active is affected by non-linearities. So the usable part of the data is the period after the excitation where the system returns to equilibrium. However, as suggested in Figure 3, it is also not necessary to consider a too long time period as, beyond a certain time, the level of the response will be comparable to the level of the noise. So, the procedure presented in this paper is based on the identification on a limited time window denoted Tid in Figure 3.

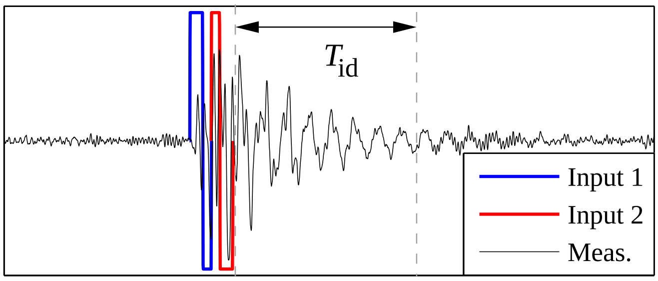

Figure 3: Time zone for the identification

3.3 Architecture of MATEST

The general organization of the processing in MATEST is depicted in Figure 4. The procedure is composed of four main modules detailed in Section 3.4. The inputs to this routine are the raw time-domain measurements of the flight tests and the frequency band for the modal analysis which depends on the excitation signals. The outputs are those of the identified modes which presumably correspond to structural modes.

The objective of the pre-processing phase (pale green box in Figure 4) is to carry out the selection of the good quality measurements and the determination of an appropriate identification time window. This function also prepares the data necessary for the identification. It is detailed in Section 3.4.1.

The identification module (pink box in Figure 4) carries out the identification of a model of a given order. It is based on a combination of two iterative algorithms described in Section 3.4.2: one is used for the determination of an initial model while the other is used for refining the identified parameters.

The monitoring module (yellow box in Figure 4) first manages the use of these two identification algorithms. It also scrutinizes the relevance of each identified mode so as to decide whether to keep it or discard it from the identified model. To perform this analysis, several intermediate identifications are launched. The objective is to identify the simplest model (i.e. a model with as few modes as possible) that fits the selected data. This function is presented in Section 3.4.3.

At this stage, the identified model reflects the overall behavior of the aircraft in the frequency band of interest. However this model might include modes that are not associated with the structural dynamics of the aircraft such as actuators modes, aerodynamic delays,... It is then necessary to select amongst the identified modes the sole modes which are presumably connected with the structural dynamics. This selection is performed in the final phase of the procedure (orange box in Figure 4). It is described in Section 3.4.4.

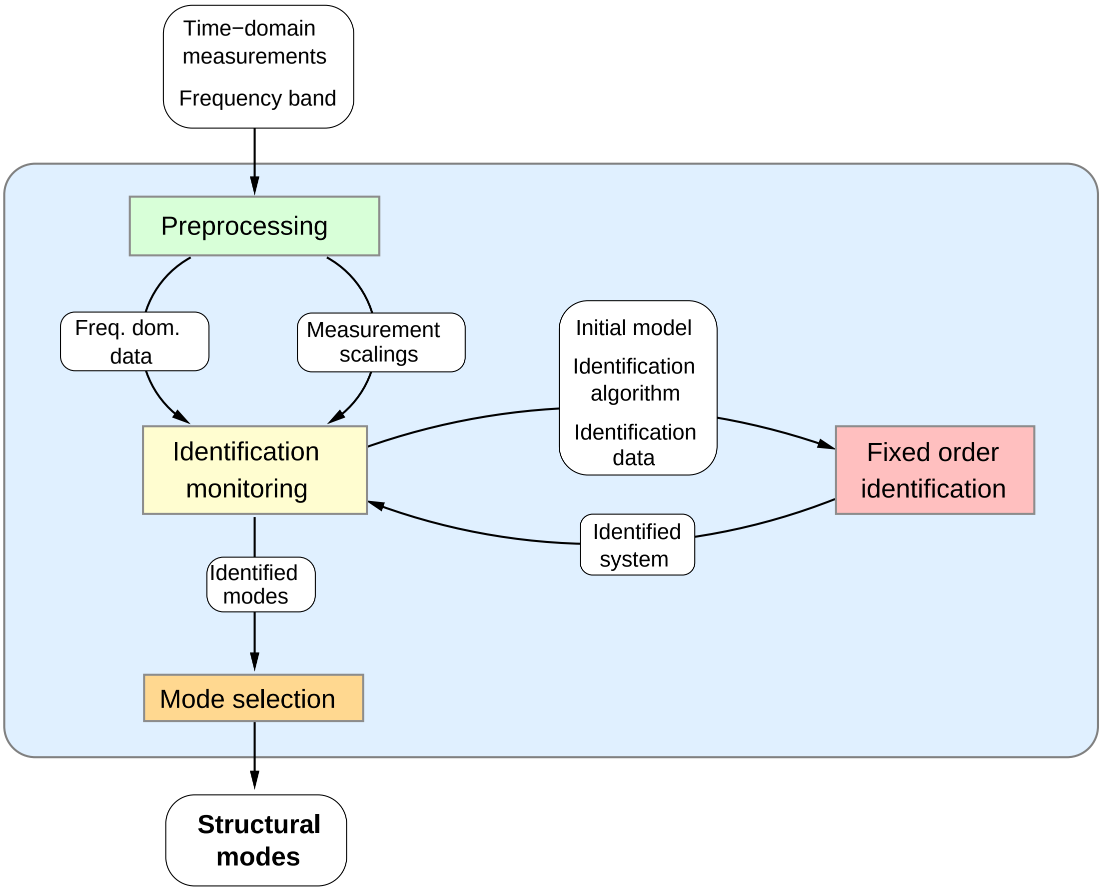

Figure 4: General organization of the MATEST procedure

3.4 Description of the MATEST modules

3.4.1 Pre-processing module

In this module, the succession of following operations is carried out:

1. computation of a quality degree for each measurement
2. selection of measurements with sufficient quality indices
3. computation of the identification time window for the selected measurements
4. computation of the Fourier transforms of the selected measurements limited to the identification time window
5. computation of measurement scalings for the identification algorithms

All these operations (except the Fourier transform of the measurements) actually make use of the quality criterions of the measurements computed in the first stage of this module. These quantities are specific signal-to-noise (S/N) ratios computed in the frequency domain.

Estimation of the back ground noise level is realized exploiting measurements in a time interval a few seconds long, before the application of the excitation, where the aircraft is stabilized with no action on the control surfaces

The selection of the measurements is simply performed by discarding the measurements with an estimated S/N ratio below a certain threshold.

The determination of the identification time zone follows the same line. An average S/N ratio is computed on a sliding window. This window is shifted rightwards till the associated S/N ratio falls below a threshold. For each measurement l, the last position of the window gives the length of the time zone T_l. The duration of the time zone retained for the identification Tid is taken as the mean of the values T_l for the selected measurements.

As detailed below, the identification routines are based on a non-linear least-square fitting in the frequency domain. They make use of the Fourier transform of the selected measurements on a time window. It is however necessary to apply scalings on the measurements in order to account for the various magnitudes and levels of quality. For each measurement l, the associate scaling ρ_l was defined in the following manner

$$\rho_l = \frac{\delta_l}{\sigma_{sl}} \tag{1}$$

where σ_{sl} is the estimated average magnitude of output l response signal; the objective of this coefficient is to equalize the magnitude of the aircraft responses for all the measurements. The factor δ_l reflects the quality of each measurement. It is defined, from the estimated S/N ratio, as a quantity bounded by 0 and 1.

3.4.2 Identification module

This module concerns the identification for a fixed order of the system. This order is denoted n_x.

The identification is carried out on a time period where the system converges back to the original equilibrium. Such experimental situations are called “free decays” in the modal analysis literature. Time-domain methods are usually used to analyze these tests.

In the MATEST procedure, a frequency-domain approach was devised in order to limit the identification to a frequency band. A model is thus needed to describe in the frequency domain the behaviour of a linear system during a free-decay test. In the literature, several articles [10-12] establish relations in the frequency domain between the inputs and the outputs of a system when its initial and final states differ. These relations include an additional transient term that accounts for this state difference. In the case of free decays, the behaviour of the system is then modeled by this sole transient term.

The model is chosen in continuous-time. Actually the data records satisfy the Nyquist-Shannon sampling condition because the data acquisition system integrates anti-aliasing filters. So the discrete Fourier transforms of the measurement samples are equal, up to a factor $1/\Delta t$, where Δt is the sampling period, to the continuous-time Fourier transform of the analog signals at the same frequency. Under these conditions, a continuous-time representation is at the same time more direct and also more accurate than a discrete-time model because this latter might introduce distortions on the estimated modes.

From the results presented in [12], the response of a system during a free decay test can be rewritten in the form of the product of a transfer function by a fictitious input. This transfer function is denoted H(s,θ) where s is the Laplace variable and θ is the vector of describing parameters to be identified. The outputs Y_f of the system at the frequencies f of F, the set of frequencies selected for the identification process, are therefore given by

$$Y_f = H(s_f\,,\theta)\,u_f \quad \text{with} \;\; u_f = \exp(j.\pi.\Delta t.f) \;\; \text{and} \;\; s_f = 2\pi.f.j \tag{2}$$

where $j = \sqrt{-1}$. The quantities u_f are the components of the Fourier transform of the fictitious input at the frequencies f. The great benefit of this expression of the system response is that one can make use of input-output approaches for the identification. In the rest of this article, the transfer function H(s,θ) will be modeled as the ratio of a numerator (vector of polynomials) by a scalar denominator (polynomial). Thus

$$H(s,\theta) = \frac{N(s,\theta)}{d(s,\theta)} \quad \text{with} \;\; \dim N(s,\theta) = n_y \text{ x } 1 \;\; \text{and} \;\; \dim d(s,\theta) = 1 \text{ x } 1 \tag{3}$$

where n_y denotes the number of outputs. The vector θ gathers the coefficients of the numerator and of the denominator. It is thus composed of $(n_y + 1)\,(n_x + 1)$ parameters.

The stochastic characteristics of the noise on the measurements are unknown. However, the measurements were selected in the pre-processing phase such that the effect of the noise be less dominant. For this reason, the identification problem was formulated as a deterministic output-error fitting. If Z_f denotes the components at frequency f of the Fourier transform of the measurements, the identification problem boils down to finding the set of parameters θ that minimizes the following criterion

$$J(\theta) = \sum_{f\in F} \left\| Q\left(Z_f - H_f(\theta)u_f\right)\right\|^2 \quad \text{with} \;\; H_f(\theta) = H(2\pi.f.j,\,\theta) \;\; \text{and} \;\; Q = \mathrm{diag}(\rho_l) \tag{4}$$

where ρ_l are the scalings defined in equation (1). As the vector θ is composed of the polynomial coefficients of the numerator and the denominator, the quantities N(2π.f.j, θ) and d(2π.f.j, θ) depend linearly on θ. However, $H_f(\theta)$ does not. So the optimization problem defined by equation (4) is non-linear.

During the development of the modal analysis procedure, it was found that a thorough optimization significantly improve the quality of the results. This precise optimization of the identification criterion implies the use of iterative algorithms. The MATEST procedure is based on two algorithms:

- the Sanathanan-Koerner method [5]: this algorithm is an iterative method where, in order to circumvent the non-linearity induced by the denominator, one utilizes the frequency values of the denominator $d_f\left(\tilde{\theta}\right)$ computed with the vector $\tilde{\theta}$ estimated at the previous iteration.
- the Whitfield formulation of the Gauss-Newton algorithm [7]: the basic principle of this method is to replace the non-linear terms by its the first order Taylor series at the values of the parameters of the previous iteration.

The Sanathanan-Koerner method offers two main advantages. First, it does not require any initial model. Second, it is known for its robustness to provide a fair estimation of the model parameters. However, no general proof of convergence was established. The evaluation of this method revealed that it does not converge to a local minimum of the identification criterion (4). On the other contrary, the Gauss-Newton algorithm requires a sufficiently precise initialization. But this method is very efficient to converge to a local optimum of the identification criterion if properly initialized. As detailed in Section 3.4.3 and similarly to the approach adopted in [13], one could benefit from the complementary of these two approaches by using them in combination.

The details of the implementation of these algorithms are not presented in this paper. Anyhow, it is worth mentioning that it is based on orthogonal polynomials [14, 15, 8] in order to improve the numerical stability and the execution speed of the codes.

3.4.3 Monitoring module

The goal of this module is to control the execution of the identification so that the modal analysis operates entirely automatically. It actually performs two successive operations:

- the search of an initial model with a deliberately oversized order n_{xmax}
- the elimination from this first guess of the modes that are not significant

Relevant quantitative indicators are required for the mode elimination process. Two criteria for assessing the adequacy of a model are thus introduced before the description of the mechanism of the mode elimination.

Search of an initial model

A first model is identified from scratch using the Sanathanan-Koerner algorithm in conjunction with the Gauss-Newton algorithm as in [13]. The philosophy is to use the Sanathanan-Koerner method as an “explorer” in order to find out a good initial guess for the identified model. For this reason, this method does not include any monitoring to ensure the decrease of the identification criterion between successive iterations. The best model obtained with the Sanathanan-Koerner algorithm, *i.e.* the one with the smallest value of J(θ), is then refined by the Gauss-Newton algorithm.

Model adequacy criteria

At this stage of the modal analysis procedure, the adequacy of an identified model is evaluated by the accuracy of the fitting between the identified model and the flight test data is analyzed. Two types of criteria are actually defined. First fitting criteria denoted $c_l(\theta)$ are defined for the measurements by

$$c_l(\theta) = \left(\frac{\sum_{f\in F} \left| z_{l,f} - h_{l,f}(\theta) u_f \right|^2}{\sum_{f\in F} \left| z_{l,f} \right|^2} \right)^{1/2} \tag{5}$$

where $z_{l,f}$ and $h_{l,f}(\theta)$ are the l^{th} components of Z_f and $H_f(\theta)$. Similarly, a normalized global criterion is defined by

$$C(\theta) = \left(\frac{\sum_{f\in F} \left\| Q\left(Z_f - H_f(\theta) u_f\right) \right\|^2}{\sum_{f\in F} \left\| Q\, Z_f \right\|^2} \right)^{1/2} = \left(\frac{J(\theta)}{\sum_{f\in F} \left\| Q\, Z_f \right\|} \right)^{1/2} \tag{6}$$

Mode elimination process

Let θ designates the identified parameters of the initial model of order n_{xmax}. The general idea is to test each mode of this model by analyzing whether, without this mode, the identification model still fits suitably to the data. The quality of the fit is evaluated by the two adequacy criteria described in equations (5) and (6). The measurement criteria $c_l(\theta)$ are used because some modes may have a local effect on the structure. So their contribution could be significant but limited to a small number of measurements. In this situation, the consideration of the sole global criterion C(θ) may result in the improper exclusion of such modes.

3.4.4 Final mode selection module

The identified system at the output of the monitoring function is a global model of the experimental responses of the aircraft to the excitations. As already mentioned, it is not limited to the sole aeroelastic dynamics of the structure. One has then to pick up the flexible modes from this model.

Without being allowed to use any a priori knowledge on the modes, the only available information for this selection is the frequency band of interest B and bounding values on the damping ratios. So the modes retained at this level have their natural frequencies in the band B and their damping ratios that lies between 0 and ξ_{max} where ξ_{max} is the upper limit expected for the damping ratios of the aeroelastic modes.

4 EVALUATION OF THE MATEST PROCEDURE

In this section, the procedure is evaluated both on simulated data and on real flight test data. The use of simulated data offer several advantages. First the evaluation can be performed by the Montecarlo approach and precise statistics can be computed to evaluate the accuracy of the identification. Second, new testing protocol such as the one described in Section 2.2 for which no real flight data is yet available can also be analyzed. The questionable aspect of this approach lies in the degree of realism of the simulated data. For this reason, it is also recommended to complete the evaluation by processing real flight test data. However, no statistical study is then possible.

The computational speed of the procedure is also an essential requirement for the integration in the flight test center. The evaluation of the efficiency of MATEST is presented at the end of this section.

4.1 Evaluation on simulated data

The simulation is based on an elaborate aeroelastic model derived from a finite element model. This model is in the form of a state-space representation which includes aeroelastic modes, aerodynamic delays, actuators dynamics and a Markovian model for the measurement noises. As described in [16], the parameters of the noise model are tuned by the spectral analysis of real flight test data. The state dimension of the model approaches 600. It includes nearly one hundred aeroelastic modes among which 29 lie in the frequency band B retained for the modal analysis. The model is used to simulate both the aircraft response to the stimulations applied on the control surfaces and the background noise.

The evaluation is based on the Montecarlo method. A set of 50 simulated tests are generated with different noise sequences but with the same response to the excitation signals. The modal analysis procedure is run on each test. Statistics are then computed from the series of 50 identification results. The computation of these statistics requires the pairing between the identified modes and the simulated ones. This mode association is based on a criterion that is a pole weighted version of an extension of the classical MAC criterion [17] (Modal Assurance Criterion) to complex modes. This dedicated criterion called MACXP is described in [18].

Two modes are considered associated if the value of their MACXP criterion is greater than 0.6.

This paper focuses on two quantities to analyze the performance of the modal analysis: the identification rate for each of the 29 aeroelastic modes of interest and the accuracy of the estimated damping ratios. The identification rate of a mode is simply the ratio between the number of times this mode is identified and the number of runs of the Montecarlo test. The accuracy of the damping ratios is only computed for the modes with an identification rate superior to 0.6. What matters for the surveillance of flutter test is actually the relative accuracy of the damping ratio. So the precision on the damping ratios is evaluated by the relative RMS (Root Mean Square) errors defined by

$$\mathrm{rel.RMS}\left(\overline{\xi}_i\right) = \frac{1}{\overline{\xi}_i}\sqrt{\frac{\sum_{k=1}^{N_i}\left(\xi_{i,k} - \overline{\xi}_i\right)^2}{N_i}} \tag{7}$$

where $\overline{\xi}_i$ is the true value of the damping ratio of the i^{th} mode, N_i is the number of times this mode is identified in the Montecarlo test and $\overline{\xi}_{i,k}$ are the N_i identified values of this parameter. Considering the three-sigma rule, the relative RMS on the damping ratio should be inferior to 33% in order to guarantee the stability of the identified modes.

Montecarlo evaluation results

Two tests are considered in this paper. They are performed in the same high speed flight conditions, but they correspond to extreme cases in terms of the quality of the modal analysis. The first one is the most unfavorable situation of all the tests currently performed on an aircraft (see Section 2.1): a symmetrical pulse on the ailerons. On the contrary, the second test was designed so that all the modes of interest for flutter surveillance could be identified with a sufficient accuracy on the damping ratios: Mexican doublets applied on two combinations. The analysis of these two tests is based on the same set of 38 measurements. The MATEST procedure was applied to these tests with the same settings except for the value of the initial system order n_{xmax}. The results appear in Figure 5 and Figure 6.

The upper part of these figures depicts the location of the identified modes for the 50 Montecarlo runs. These modes are plotted in the frequency-damping ratio plane together with the 29 modes of the simulation model which are indicated by the encircled numbers. The mode 13 does not appear on these figures because of the high value of its damping ratio. The axes of the figures are blind for confidentiality reasons. The identified modes that can be paired based on the value of the MACXP criterion with a simulation mode are plotted with red dots. The identification rates of the modes and the relative RMS errors on the damping ratios are depicted on the bar diagrams of the lower part of the figures.

For the pulse test (Figure 5), the real values of the signal-to-noise ratios defined in Section 3.4.1 vary widely from one measurement to another since this quantity covers the range -14.5 dB up to 26.0 dB. On average, 8.5 measurements are selected out of the 38 available. This number actually fluctuates between 1 and 16 amongst the runs of the evaluation. This clearly evidences that this stimulation on the ailerons is not efficient enough to excite the aircraft structure. The modal analysis procedure was initialized with n_{xmax}=16. Figure 5 reveals that 5 modes are identified with a rate greater than 0.6. One can also notice the limited efficiency of the pulse signal in the frequency domain since only modes in the first half of the frequency range B are identified. The relative accuracy on the damping ratios is rather coarse for this test since the average value of the relative RMS errors is equal to 38%.

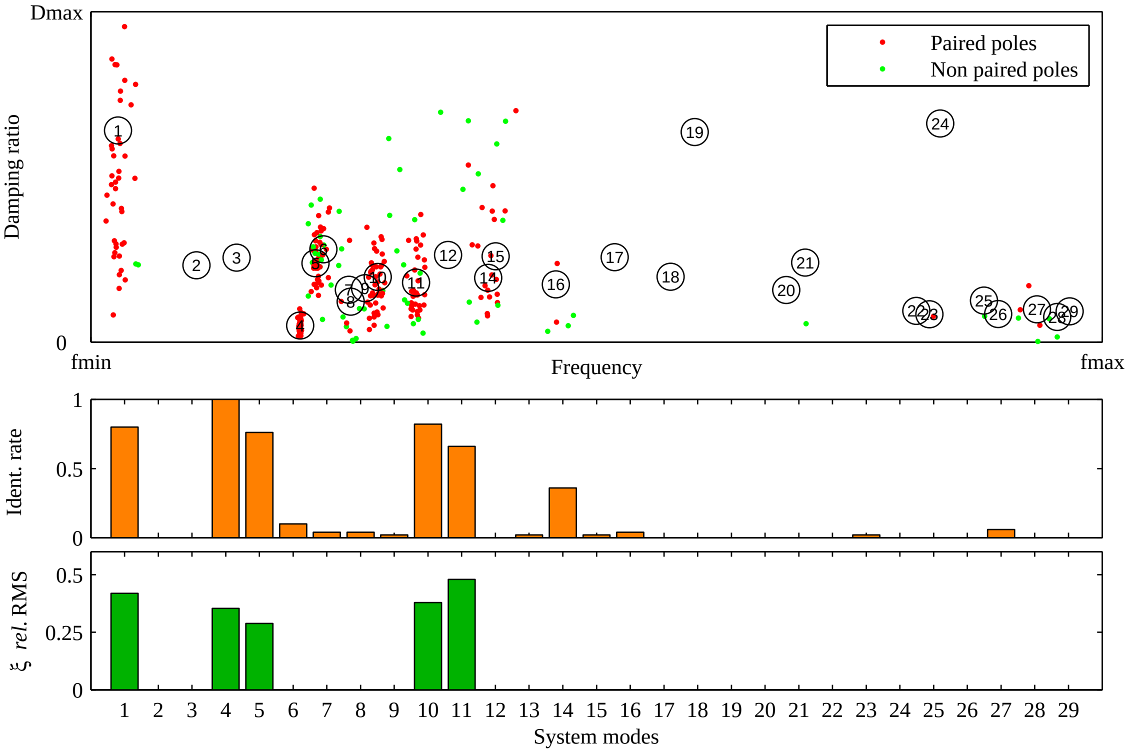

Figure 5: Montecarlo evaluation. Symmetrical pulse on the ailerons

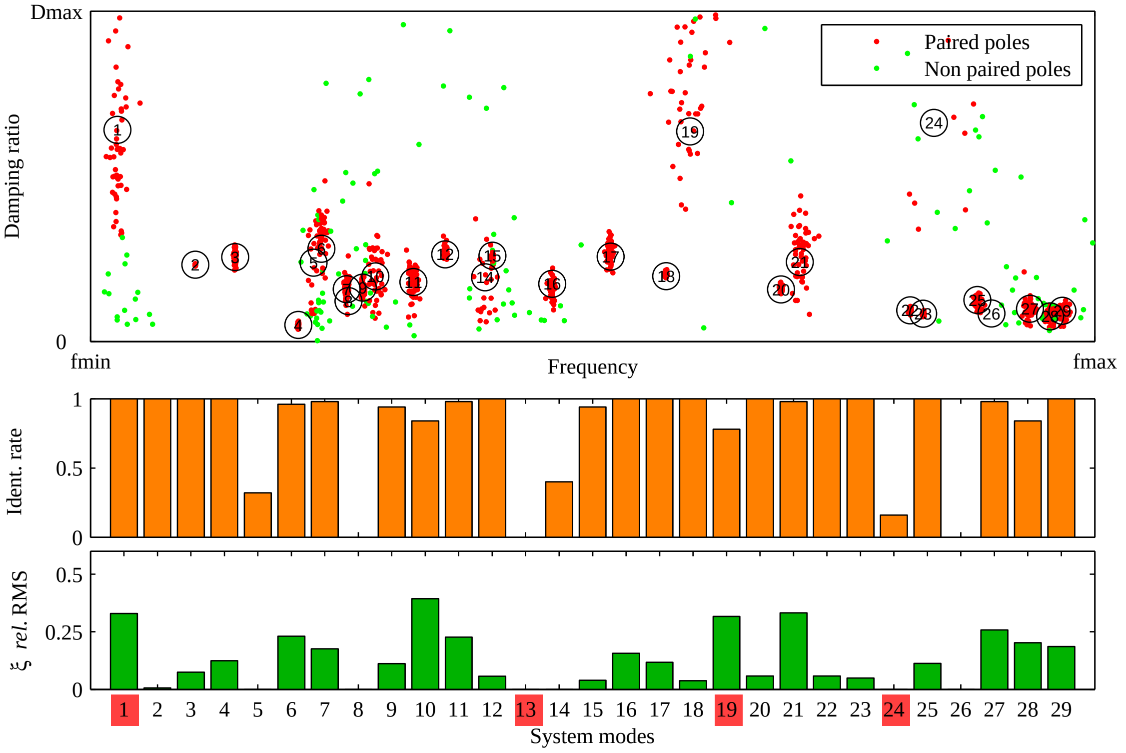

Figure 6: Montecarlo evaluation. Mexican doublets applied on two input combinations

In the test associated to Figure 6, the excitation was designed to stimulate only the modes susceptible to get unstable. So the heavily-damped modes (modes 1, 13, 19, 24) were not considered in the definition of the test. As introduced in Section 2.2, the excitation is based on the successive application of Mexican doublets on three combinations of control surfaces. The improved efficiency of the excitation is confirmed by the values of the signal-to-noise ratios on the measurements which range from 18.8 dB to 49.5 dB. It results that the 38 measurements are always selected in the 50 runs of the Montecarlo evaluation. The procedure was started with $n_{xmax}=60$. In Figure 6, it can be observed that 21 out of the 25 modes of interest are identified. A deeper examination reveals that the unidentified modes are all in the close proximity of another more dominant mode. Thus, either the identification of the former modes is perturbed by the nearby dominant mode (modes 5 and 14) or the procedure retains a single mode which lumps together the responses of two modes (modes 8 and 26). In the first case, only the shapes of the mode are perturbed. As the MAXCP criterion takes into account the mode shapes, the perturbed modes cannot be associated to any simulation mode in spite of correct estimated values for their frequencies and damping ratios. This explains the green dots at the proximity of these modes in Figure 6 and the poor score on the identification rate diagram. These phenomenons stem from the close proximity of the frequencies and damping ratios of two modes. If the distance between the modes of these pairs were to increase in the course of the flight, probably the two modes would be identified correctly. So the decrease of the damping ratios of one of the modes would likely be detected. The accuracy of the identified damping ratios is also quite satisfactory. The average value the relative RMS errors for the 25 modes considered is equal to 14%. These observations show that flutter surveillance could be achieved with a single test of about 10 seconds.

It is also interesting to analyze the ratio R_r between the number of frequency samples available for the analysis and the number of identified parameters. This quantity, called redundancy ratio, is equal to

$$R_r = \frac{2\, n_f n_y}{(n_y+1)(n_x+1)} \tag{8}$$

where n_x is the order of the identified model, n_y the number of selected measurements, and n_f the number of frequencies in the set F. The value of R_r is very low for this test since it ranges between 2.5 and 2.9. This result proves that the identification algorithm is able to extract much information from a small quantity of data.

This evaluation on simulated data demonstrates the versatility of the MATEST procedure to deal with various situations. It also shows its ability to extract fine details from the measurements and to produce fair values for the mode damping ratios.

4.2 Evaluation on real flight test data

The assessment of the MATEST procedure was performed by Airbus in two steps corresponding to the TRL4 and TRL6 milestones of the R&T project.

Algorithm Validation (TRL4)

For the TRL4 milestone, Airbus focused on the efficiency of the MATEST procedure from an algorithm standpoint. The purpose was to perform a back-to-back test between the previous version of the identification procedure, dubbed MEFAS [8], and the MATEST procedure (MEFAS V2) using standard single control surface excitations. The data presented in the

Table 1 originates from the A380 flight test campaign, which was chosen because of its high modal density which makes identification tricky. A 5° aileron symmetrical pulse performed at high speed and high Mach is presented below.

The MATEST algorithm was benchmarked versus the MEFAS procedure, used with or without automatic sensor selection.

	Frequency (Hz)							
MEFAS V2	0.973	1.97	2.01	2.16	2.4	2.91	3.55	3.82	4.69
MEFAS V1 without selection	/	1.96	2.04	/	/	2.91	/	/	/
MEFAS V1 with selection	/	1.95	2.02	/	/	2.9	/	/	/
	Damping (°/°°)								
MEFAS V2	110	6.5	33	28	48	19	35	6.6	13
MEFAS V1 without selection	/	11	12	/	/	26	/	/	/
MEFAS V1 with selection	/	10	9.8	/	/	18	/	/	/

Table 1: A380 flight test. Comparison of identification results.

The identification appears to be robust with respect to frequency assessment. Less modes are identified with MEFAS but it must be noted that MATEST was not tuned to be as selective as MEFAS in terms of pole selection. For the given example, the frequency identification of all three procedures was found to be consistent (Figure 7) even though the modes are located nearby (less than 0.1 Hz). On the contrary, the difficulty of separating such coupled modes is highlighted by increased scatter in the damping assessment (Figure 8). These results were subsequently compared with an analysis made by means of a commercial algorithm (LMS PolyMAX via the standard transfer function approach and the operational version). The difficulty of identifying this mode was confirmed.

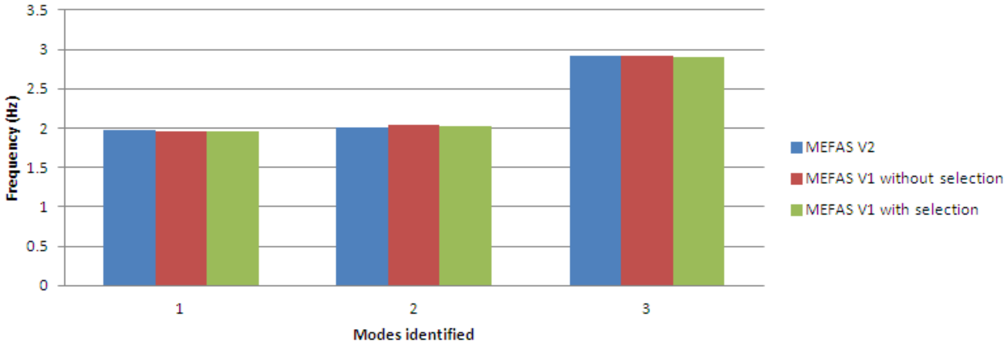

Figure 7: Results of the identification process for frequency

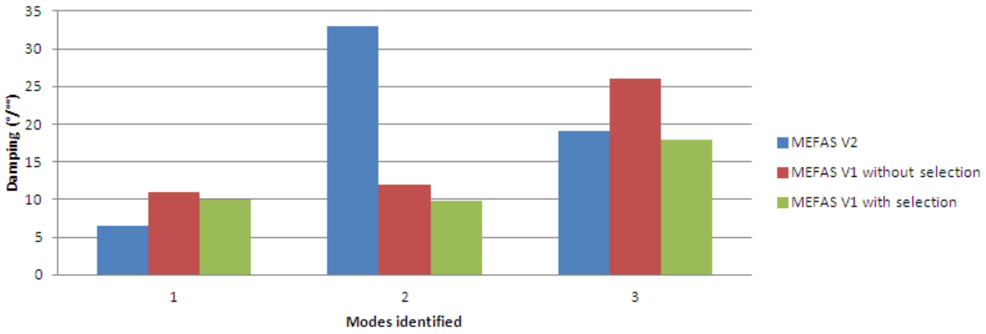

Figure 8: Results of the identification process for damping

Validation of combined control surface excitations (TRL6)

For the TRL6, a dedicated research flight was performed on an Airbus A320 flight test aircraft at the Airbus Flight Test Centre in Toulouse. A total of 35 pulse excitations were performed at the same high speed conditions, encompassing a number of control surface combinations and excitation specifications (duration / amplitude). The primary purpose of this test was to compare the identification rate and accuracy of single surface excitations with multi-surface, or combined, excitations. Two types of multi-surface excitations, acting simultaneously on the inner aileron and elevator control surfaces, have been tested: symmetrical (left/right in phase), and anti-symmetrical (left/right out of phase).

Of the two amplitudes tested, the signal-to-noise ratio of the measured signals was significantly improved by the higher amplitude. As a result, only 19 high amplitude pulses have been considered in the detailed analysis below.

Having been integrated into flight surveillance tools located in the Airbus Telemetry Room, the MATEST procedure was used to extract modal data during the flight. The sensors used for identification were selected automatically using a signal-to-noise threshold of 2. All analyses were performed using the same settings. The bar graphs below depict the identification rate of each mode and the relative standard deviation of the damping predictions for the single pulses and their corresponding combined pulse. Spurious modes have been removed during post-processing.

The efficiency of combined symmetrical pulses is confirmed in Figure 9, as the identification rate of all modes is high. Only mode three has a higher identification rate with single surface elevator pulses. In all cases, the relative standard deviation of damping predictions produced using combined pulses is less than 0.33.

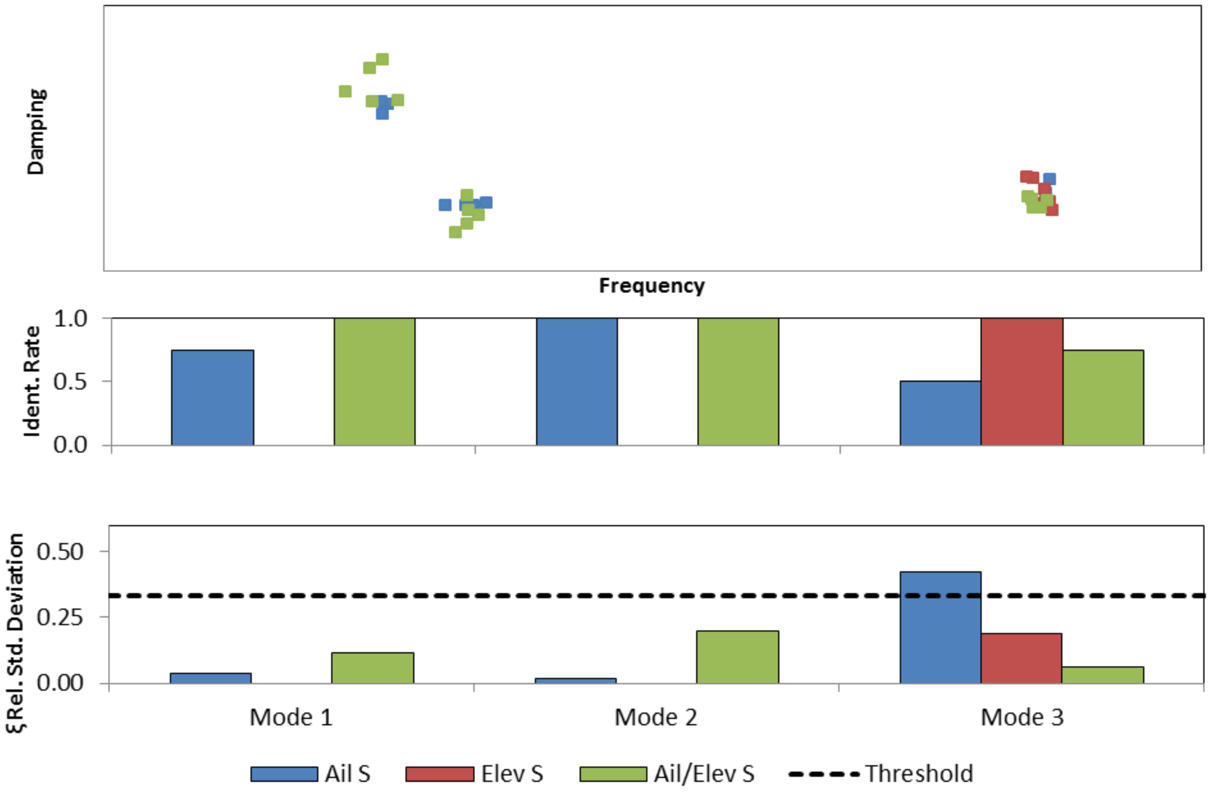

Figure 9: Flight test results. Symmetrical pulses on ailerons and/or elevators. A threshold for the ξ relative standard deviation is indicated at 0.33.

As shown in Figure 10, combined anti-symmetrical pulses have also increased the identification rate of most modes over corresponding single surface excitations. The sole exception is mode 2, which appears to be masked by the nearby dominant mode 1. This masking behavior was also identified by Montecarlo simulations (see Section 4.1). Improvements in damping relative standard deviation have also been observed for modes 1 to 4 using combined pulses.

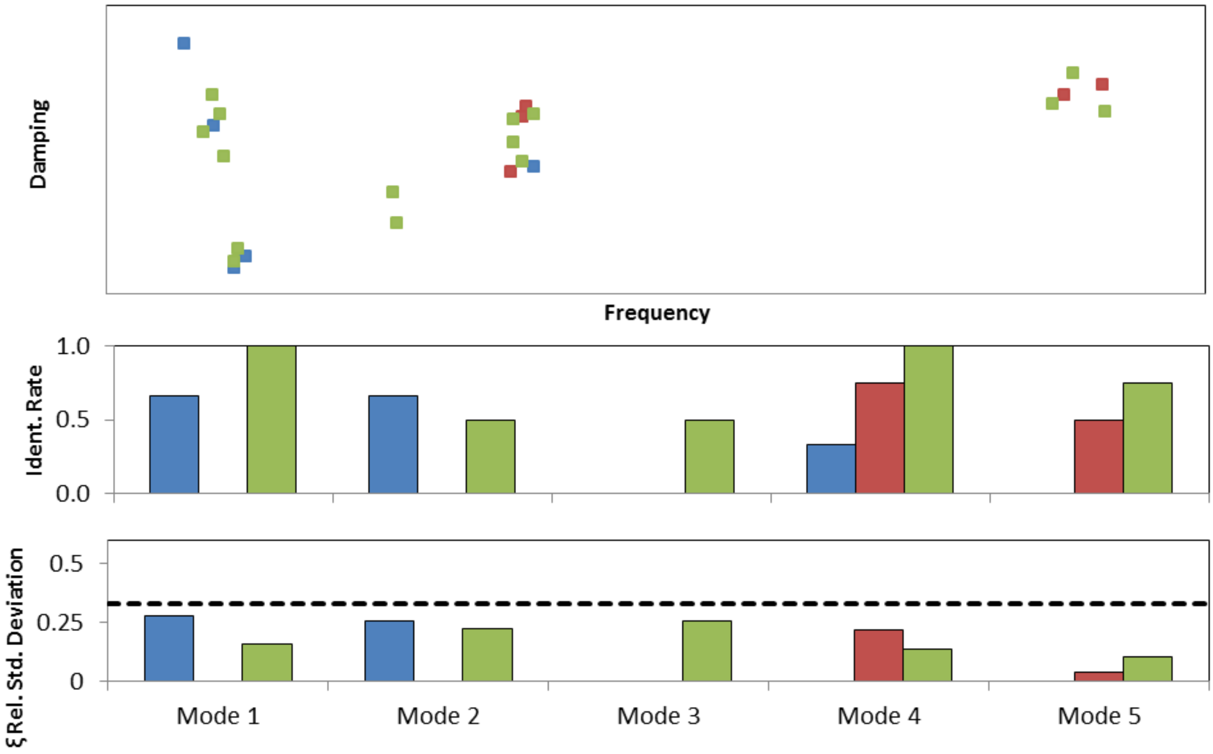

Figure 10: Flight test results. Anti-symmetrical pulses on ailerons and/or elevators. A threshold for the ξ relative standard deviation is indicated at 0.33.

4.3 Computational efficiency

The evaluation of the execution speed of the MATEST program was first performed on the second test of the Montecarlo benchmark presented in Section 4.1. This case was chosen because, as more modes are identified, it is much more computationally demanding than the pulse test.

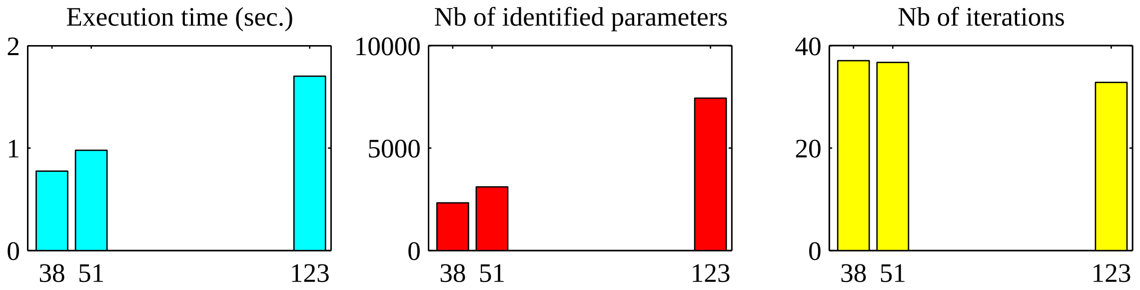

Figure 11: Evaluation of the computational efficiency

The MATEST procedure was fully implemented in the Matlab computing environment. The procedure was run for three bunches of sensors corresponding to 38, 51 and 128 measurements. For the three cases, the modal analysis was launched with an initial order n_{xmax}=60.

The MATEST program was run on a computer equipped with an Intel Core I5 processing unit with two cores and a 2.53 Ghz clock rate. The evaluation results are depicted in Figure 11. The first graph gives the mean value of the overall execution time for a single analysis. It can be observed that the program satisfies the specification. Only the last case exceeds the goal of one second specified in the requirements. The two following diagrams concern quantities which give an indication of the computational load:

- the mean number of parameters identified at each iteration, *i.e.* the size of θ
- the mean number of iterations of the identification algorithms (Sanathanan-Koerner and Gauss-Newton)

So the computational load is quite significant. Moreover it should also be noted that most of the iterations correspond to the Gauss-Newton algorithm.

This performance of the MATEST program was achieved by a careful implementation of the identification algorithms that makes full profit of the specificities of the problem. These results also show that an interpreted programming language such as Matlab can produce efficient operational codes.

During the validation on flight test data, it was confirmed that the computational efficiency of the MATEST procedure was improved. Indeed, for the same example below (an aileron pulse from the A380 flight test campaign), the computation time of the MATEST algorithm was four times faster than the latest version of MEFAS with automatic parameters selection.

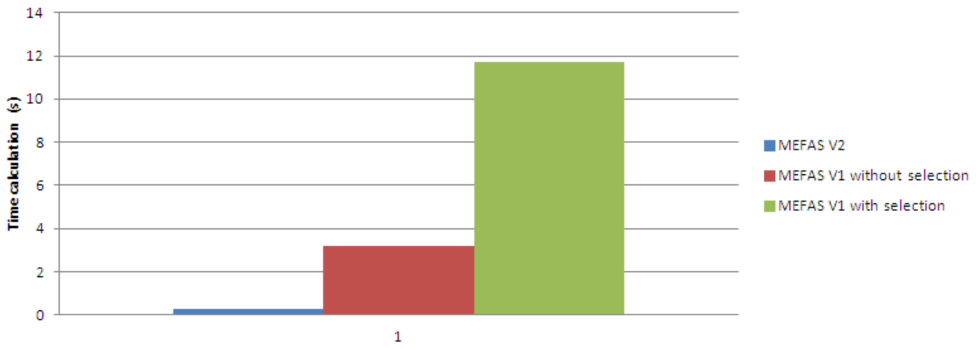

Figure 12: Time to get the identification results

5 CONCLUSION

In this paper, a procedure for the in-operation modal analysis of flutter flight test was presented. This procedure is dedicated to the processing of very short and highly disturbed data sequences. It was also designed to meet the requirements imposed by the operational conditions of flight tests.

The procedure is based on a free-decay identification approach in the frequency domain. Its development is based on simple principles. First a threefold selection is carried out on the experimental measurements in order to retain only good quality data for the identification of the modes. This selection is performed:

- in the frequency domain by the choice of a favourable frequency band

- in the time domain by the determination of window where the noise on the measurements is limited
- at the level of the outputs where only the measurements with a sufficient quality are retained

Second, a specific identification procedure in the frequency domain was developed to estimate the modes from the free response of the system. This procedure was designed to be at the same time sharp in order to extract the maximum information from the sometimes reduced selection of data and robust in order to be less sensitive to the residual noise in the data. As required by the specifications, the procedure also operates entirely automatically and delivers to the user a set of modes associated to the structural dynamics of the aircraft.

The detailed evaluation proved that the performance of the procedure is quite satisfactory taking into account the particular circumstances in which the analysis is carried out. It proved that the tool complies with the requirements imposed by this particular application and that it is able to accommodate a wide variety of levels of quality for the flight test records. The analysis of real flight data is consistent with the expected values for the modes and with the recorded behaviour of the aircraft structure. Finally the execution times are completely compatible with an operational use.

It was also shown in this paper that, with an improved testing protocol, the flutter surveillance could be achieved in a single and short test instead of the five tests currently performed at each test point. Airbus has decided to replace these 5 standard tests with two combined pulses, one symmetrical and one anti-symmetrical. These combined excitations make use of simple pulses. It is contemplated to go further using these new tools to perform modal analysis from short tests on evolutionary flight points, doing away with the costly requirement of realising stabilised tests points.

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