Probabilistic Robust Control Strategy for Gust Load Alleviation

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Abstract: The aeroelastic model for a wing in the time domain is constructed with continuous wind gust and structural uncertainties. The H_∞ optimal method and μ synthesis are applied to the robust control law design for gust load alleviation (GLA) problem. Specifically, with instability risk introduced in the uncertain model, the GLA factor can be largely increased. The instability probability for a uncertain system and nominal GLA factor are trade-off to get a higher performance and acceptable instability risk. At the same flow velocity, the stability robustness is decreased while the nominal gust load can be largely alleviated. Vise visa, though the nominal aeroelastic model is stable at all the velocities between 10 m/s to 30 m/s, the instability risk increased with the GLA factor ascending. Results indicate that with introducing a 0.2% instability risk, the nominal gust load can be alleviated by 24.5%.

1 Introduction

Gust load alleviation (GLA) active control is an effective tool to reduce the dynamic response due to gust, with a minor increasing of aircraft's weight[1][2][3]. The control law design is an important part in the process of GLA active technique[4][5]. Most of the researches about GLA control law design were concentrated in PID method and linear quadratic Gauss (LQG) theory[6][7][8]. H_∞ optimal control and μ synthesis are effective robust control methods to account for bounded random disturbance and variations in the mathematical model[9][10][11].

However, from the perspective of robust control theory, the demands on robust stability and nominal performance may be contradictory to each other. The μ synthesis, based on deterministic worst case design, is sometimes a conservative controller and thus it degrades the controller's efficiency. Instead of robust stability guarantee, the probabilistic robust control is a strategy to improve the performance by allowing a small risk of instability[12], which sacrifices the system's robustness. By specifying different levers of risk, the aeroelastic system can obtain different GLA efficiency and stability robustness. This can be a guidance of the controller design for different demand for performance.

At the beginning, the robust controllers based on H_∞ optimal control and μ synthesis are formulated. In this part, we emphasize on the weighting function chosen and uncertainty description. Then, the probabilistic robust controller is designed based on a modification of μ synthesis. Because this controller design method is easy to deal with structured uncertainty. The robustness of the closed system is analyzed by the μ method. The Monte Carlo Simulation method is applied to estimate the closed system's instability risk when the robustness cannot be guaranteed. A numerical example of a large aspect ratio wing is applied to validate the above framework.

2 Robust control law design for GLA

A. Problem formulation

The aeroelastic system with consideration of gust is described by the following equation [1]

$$\begin{bmatrix} \boldsymbol{M}_{\mathrm{qq}} & \boldsymbol{M}_{\mathrm{q\delta}} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{q}} \\ \ddot{\boldsymbol{\delta}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{\mathrm{qq}} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{\delta}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{\mathrm{qq}} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{q} \\ \boldsymbol{\delta} \end{Bmatrix} = \frac{1}{2}\rho V^2 \begin{bmatrix} \boldsymbol{Q}_{\mathrm{qq}} & \boldsymbol{Q}_{\mathrm{q\delta}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{q} \\ \boldsymbol{\delta} \end{Bmatrix} + \frac{1}{2}\rho V^2 \boldsymbol{Q}_{\mathrm{g}} \boldsymbol{w}_{\mathrm{g}} \tag{1}$$

Where $\boldsymbol{M}_{\mathrm{qq}}$, $\boldsymbol{C}_{\mathrm{qq}}$, $\boldsymbol{K}_{\mathrm{qq}} \in \mathbf{R}^{n_x \times n_x}$ describe the structural general mass, damping, and stiffness matrix, respectively. $\boldsymbol{w}_{\mathrm{g}}$ is the gust velocity. In this equation, $\boldsymbol{q} \in \mathbf{R}^{n_x}$ represent the modal displacement vector, and $\delta \in \mathbf{R}^{n_u}$ is the deflection angle of the control surface. ρ and V represent the flow density and flow velocity, respectively. $\boldsymbol{Q}_{\mathrm{qq}}$ 、 $\boldsymbol{Q}_{\mathrm{q\delta}}$ and $\boldsymbol{Q}_{\mathrm{g}}$ represent the generalized aerodynamic influence matrices due to modal displacements, deflection of control surface and gust.

The Dryden gust formulation is chosen to represent the static, continuous gust model. Its filter formation in the Laplace domain can be written as:

$$T_{\mathrm{g}}(s) = \frac{w_{\mathrm{g}}(s)}{\eta(s)} = \sigma_{\mathrm{wG}} \frac{\sqrt{3}\tau_{\mathrm{g}}^{-1/2} s + \tau_{\mathrm{g}}^{-3/2}}{\left(s + \tau_{\mathrm{g}}^{-1}\right)^2} \tag{2}$$

where η is a Gaussian white noise process of unit intensity and zero mean. σ_{wG} is the intensity of gust and $\tau_{\mathrm{g}}=L_{\mathrm{g}}/V$ is the scale of gust.

The indicative value to estimate efficiency for gust load alleviation in the time domain is given as:

$$\sigma = \frac{\sqrt{\sum_{i=1}^{n}(y_{oi})^2 - \sum_{i=1}^{n}(y_{ci})^2}}{\sqrt{\sum_{i=1}^{n}(y_{oi})^2}} \tag{3}$$

where σ is the load alleviation factor. y_{oi} is the acceleration response for the open-loop system in the time domain, and y_{ci} is the one for the closed-loop system. N denotes the data length in the time domain.

B. H∞ controller design

The objective of the controller design is to reduce the acceleration response at the wing tip. The block diagram for the GLA controller design is indicated in Figure 1.

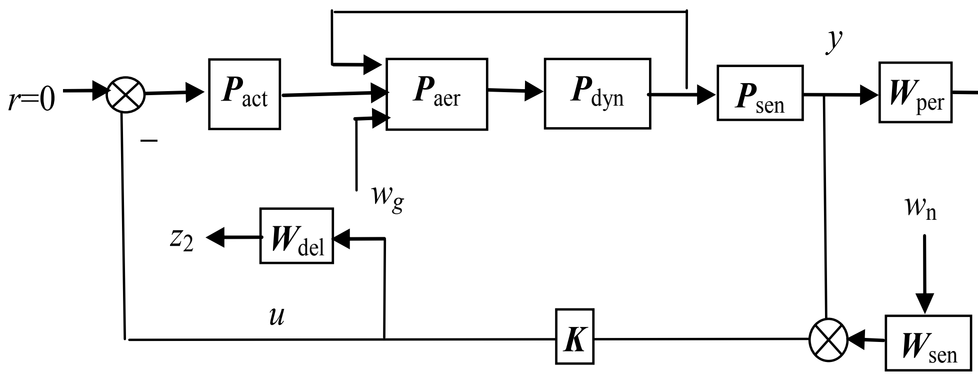


Figure 1: Block diagram for H_∞ controller design

In this diagram, the weighting function W_{del} is introduced to limit the deflection angles for the control surface. W_{per} is used to enforce the gust load alleviation specification, and W_{sen} represents the measured noise for acceleration. The choice of W_{per} is to satisfy a large static gain in the low frequency range. Their functions are chosen as below:

$$W_{\mathrm{per}} = P_{\mathrm{W}} \frac{s+800}{0.005s+1}, W_{\mathrm{del}} = P_{\mathrm{A}} \frac{s+200}{0.00001s+1}, W_{\mathrm{sen}} = 0.0001 \tag{4}$$

In the above equation, the performance weighting parameter P_{W} is introduced to balance the robust stability and varying levels of GLA performance. $P_{\mathrm{A}}=0.1P_{\mathrm{W}}$ is assumed to be fixed in the design process. By using the linear fractional transformation technique, the controller design problem shown is Figure 1 can be transformed to the standard H_∞ optimal one indicated in Figure 2.

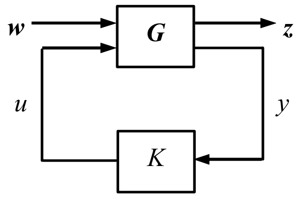


Figure 2: block diagram for standard H_∞ controller design

In this diagram, $\boldsymbol{G}$ is the plant and K is the controller. In this paper, single input and single output plant is considered. $\boldsymbol{w}=[w_{\mathrm{g}},w_{\mathrm{n}}]^{\mathrm{T}}$ is the input signals and $\boldsymbol{z}=[z_1, z_2]^{\mathrm{T}}$ is the output signals. H_∞ control law design is to find a controller which minimizes the infinitive norm of the transfer function $\boldsymbol{T}_{wz}$ from $\boldsymbol{w}$ to $\boldsymbol{z}$. That is

$$\min_{K\mathrm{stabilize}\boldsymbol{G}} \left\| F_l(\boldsymbol{G},K) \right\|_\infty \tag{5}$$

C. μ synthesis

The H_∞ controller in the above section provides gust load alleviation performance with random wind gust. However it cannot guarantee the robust stability of the closed-loop aeroservoelastic system, which is optimistic in application. In this section, the μ synthesis method is applied to GLA control law design with considering parameters' uncertainties in the mathematical model. In this study, the variations considered are these: the first four modal frequencies' variations, the actuator's perturbations and other unmodeled uncertainty.

The order of theoretical model for electro motor is assumed to be three. We can choose additive uncertainty type to characterize the difference of theoretical model and experimental frequency response. By data fitting method, we get the formulation of nominal model and weighting function for the motor, which are given that:

$$G_{\mathrm{act}}(s) = \frac{1}{1/\omega_1 \cdot s+1} \frac{\omega_2^2}{s^2+0.9\omega_2 s+\omega_2^2}, \omega_1 = 26\pi, \omega_2 = 22\pi \tag{6}$$

$$W_{\mathrm{act}}(s) = 4.5 \times \frac{\omega}{s^2+0.2\omega s+\omega^2}, \omega = 9.2\times 2\pi \tag{7}$$

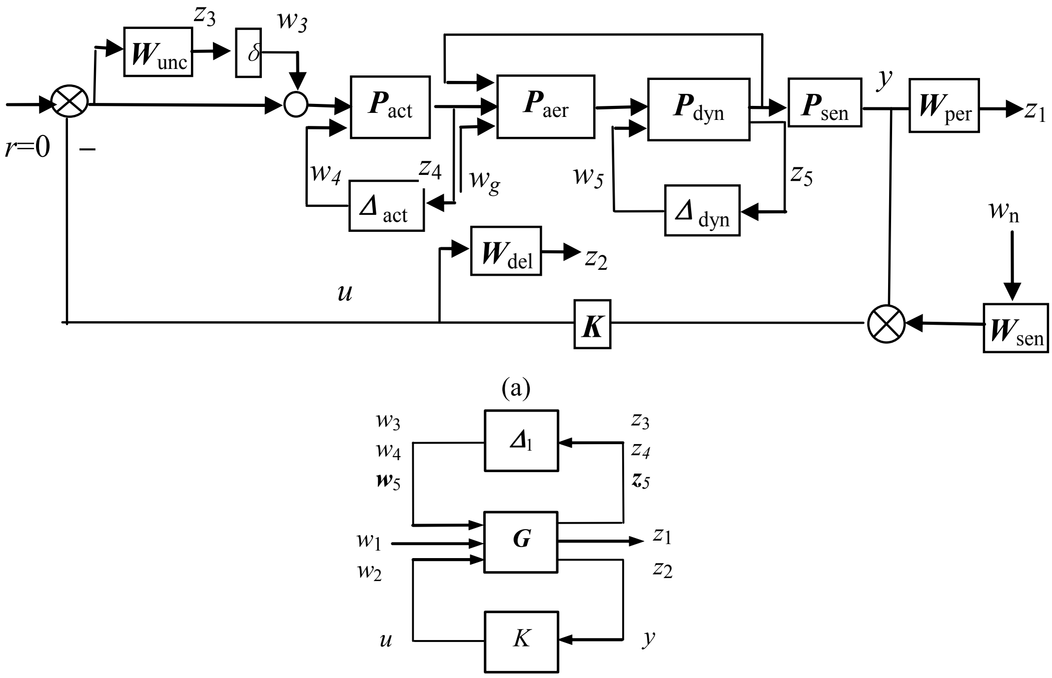
$$P_{\mathrm{act}} = G_{\mathrm{act}} + W_{\mathrm{act}} \times \Delta_{\mathrm{act}}, \Delta_{\mathrm{act}} \in C, \left\|\Delta_{\mathrm{act}}\right\|_\infty < 1 \tag{8}$$

A slight variation of the modal frequency is allowed between the finite element model and ground vibration test. 15% variations in each of the first four modal frequencies are described by four scaled real parameters' uncertainties. Meanwhile, the variations in the aerodynamic computation and sensor's location exit as well. A complex uncertainty is introduced to represent these variations, whose weighting function is written as:

$$W_{\mathrm{unc}} = 3\frac{s+1000}{s+10000} \tag{9}$$

This equation means that the model has a 30% errors in the low frequency range and 300% modeling errors in the high frequency range.

Controllers design with uncertainties can be recast into general μ synthesis framework. Fig 3(a) indicates the block diagram for μ synthesis, which contains the weight function for the whole uncertainties presented above. The standard framework shown in fig 3(b) can be obtained for μ synthesis, based on linear fractional transformation method. In this framework, the structured perturbation matrix is $\Delta_1 = diag(\Delta_{\mathrm{act}}, \Delta_{\mathrm{unc}}, \Delta_{\mathrm{dyn}})$. The input and output signals are $\boldsymbol{w} = \left[w_g, w_n, w_{\mathrm{act}}, w_{\mathrm{unc}}, \boldsymbol{w}_{\mathrm{dyn}}\right]^T$ and $\boldsymbol{z} = \left[z_g, z_n, z_{\mathrm{act}}, z_{\mathrm{unc}}, \boldsymbol{z}_{\mathrm{dyn}}\right]^T$.



(b)

Figure 3: Block diagram for μ synthesis

Augmented diagonal structured uncertainty matrix is written as Δ_g=diag(Δ_1, Δ_f), by introducing a fictious uncertainty block Δ_f. Then the robust performance design problem can be converted to a robust stability one. It is well known that no algorithm is available to compute the mixed μ in a general case. Though the D-K iteration cannot guarantee the global optimal controller, it was wide used in the engineering scope, which could give a not so bad controller.

3 Probabilistic robust control law design

The μ synthesis method can robustly stabilize the closed-loop under structured uncertainty set. However, the robust controller design may be conservative. A deterministic worst-case uncertainty seldom happens in the real world. The robust stability and nominal performance is contradictory with each other. If we just consider the robustness and ignore the probability of instability, the performance is expected to be degraded.

In this section, a small level of instability probability is allowed. In this case, a probabilistic robust controller is designed. A simple way to fulfill this is to reduce the design uncertainty radius. For a reduced uncertainty bound, the weighting parameter P_W is increased to achieve a better GLA performance. In this case, the reduced uncertain system set can guarantee robust stability. However, the real closed-loop may have a probability to go unstable. At different uncertainty radius $0<r_{syn}<1$, risk-adjusted controller is obtained to achieve varying performance level. The weighting function for reduced uncertainty bound is given as:

$$W_{act}(s) = r_{syn} \times 4.5 \frac{\omega}{s^2 + 0.2\omega s + \omega^2}, \quad W_{dyn} = r_{syn} \times 0.15, \quad W_{unc} = r_{syn} \times 3 \frac{s+1000}{s+10000} \tag{10}$$

4 Probabilistic robust analysis

For uncertainty radius adjusted controllers, the μ analysis is applied to estimate the system's stability robustness and a random method is employed to risk assessment.

The instability probability for aeroelastic system is given as

$$P_f(\rho) = \varepsilon = P(\det(\boldsymbol{I} - \boldsymbol{P}_{22}(j\omega)\boldsymbol{\Delta}(j\omega)) = 0, \omega \in R) \tag{11}$$

When the uncertainty parameters satisfied independent-uniform distribution, the probability formulation is that

$$P_f(\rho) = 1 - \frac{Vol\{\boldsymbol{S}_\Delta(\rho)\}}{Vol\{\boldsymbol{X}_\Delta(\rho)\}} \tag{12}$$

For a specified flight state and the real uncertainty bound, a random integer number is used to indicate stability or not

$$X = \begin{cases} 1 & \text{stable} \\ 0 & \text{unstable} \end{cases} \tag{13}$$

The Massart inequality gives the need numbers for sample experiments, which is given that

$$N > \frac{2(1-\varepsilon+\alpha\varepsilon/3)(1-\alpha/3)\ln(2/\delta)}{\alpha^2\varepsilon} \tag{14}$$

Where ε is the risk level, δ is the confidence parameter. $\alpha \in [0,1]$. N denotes the sample number. Eq.(14) can guarantee that $\Pr(\left|P_X - \frac{K}{N}\right| < \alpha\varepsilon) > 1-\delta$ and $P_x=1-\varepsilon$.

5 Numerical examples

The structure of a large aspect ratio wing with two spars is shown in Figure 4. This model has two control surfaces. While in this study, only the outboard control surface is used in the controller design. The areas for the wing and control surface are 0.72 m^2 and 0.056 m^2, respectively. The first four normal frequencies of this wing model are that: 1.76 Hz, 8.96 Hz, 16.59 Hz, 22.42 Hz. The modal shapes are first bending, second bending, first torsion and third bending. Considering the first four normal modal shapes, computational approaches, such as root-locus flutter analysis of this nominal model wing is performed, which predicts flutter to occur at the speed of 36.1 m/s with frequency of 5.6Hz. The V-ω plot shows that the flutter mechanism for the wing is second bending mode coupling with the first torsion mode typically. The Dryden wind gust model is applied. The gust intensity σ_{wG} is 1m/s and the gust scale L is 5 m.

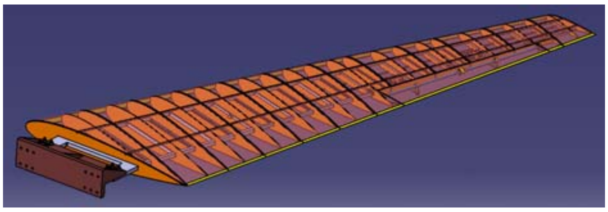
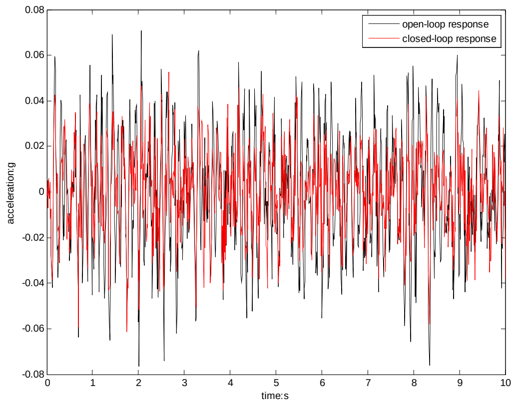


Figure 4: Structure of a wing model

In this application, H_∞ optimal method, μ synthesis and probabilistic method are performed to obtain different GLA performance at different risk levels. In the process of GLA controllers design, in order to reduce the controller's order, only four structural modes, two aerodynamic lag roots, three order of the motor are modeled for the open-loop aeroelastic system. In this situation, the order of the controller is 17. However, in verification of the performance and stability of the closed-loop system, a more complicated model is used to depict the true dynamics of the aeroelastic system, such as the number of structural modes is chosen as 22 and the aerodynamic lag root is four.

H_∞ is applied to design the nominal GLA controllers without uncertainty. In this case, the performance weighting parameter is chosen as P_W=0.05, P_A=0.026. And the acceleration response reduces 30.7%. The gain margin for the closed-loop is 8.5dB. The response for the open loop system and closed loop system in the time domain is shown in Fig. 5. The μ value by H_∞ optimal controller is shown in Fig. 6. From this figure, in this case, the controller cannot guarantee the closed-loop system's robust stability. Hence, we should consider the uncertainty in the aeroelastic model and to design a robust controller.



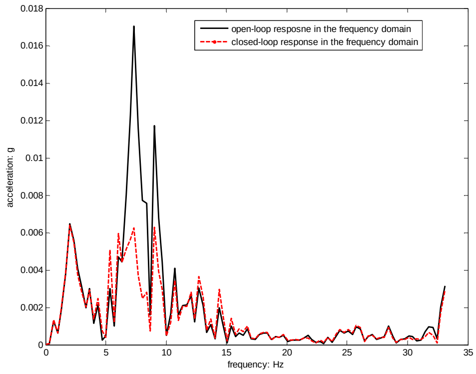


Figure 5: Response comparison of the closed-loop and open-loop systems in the time domain(left) and in the frequency domain (right)

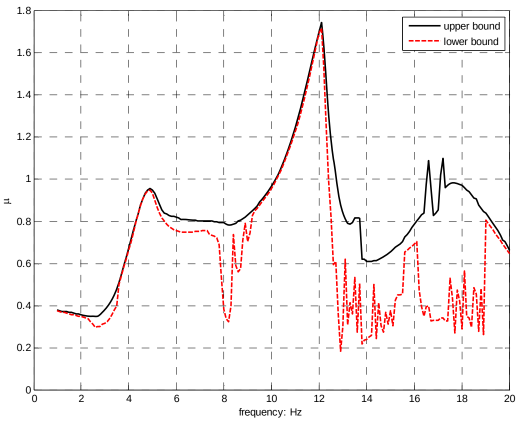


Figure 6: μ value of the closed-loop system by H_∞ controller

Hence, the robust μ controller is designed under different uncertainty radius, indicated in Table 1. From this table, when the uncertainty radius is less, the stability under a larger uncertainty range will be worse, while the gust load alleviate effect is more obvious. There is some stability risk under the uncertainty range, while a good alleviation effect is obtained.

Table 1: GLA performance and instability risk for the wing

r_{syn}	P_W	Gain(dB)	σ(%)	μ	Risk(%)
1/6	0.68	0.94	39.4	1.368	17.030%
1/3	0.4910	1.58	38.8	3.295	14.930
1/2	0.1320	2.03	32.6	2.412	8.663
2/3	0.0826	3.15	27.5	1.502	1.429
5/6	0.0665	3.82	24.5	1.216	0.192
1.0	0.0518	5.75	20.6	1.002	0.000

6 Conclusion

The state space aeroelastic model is constructed with continuous wind gust and structured uncertainties. The H_∞ optimal method and μ synthesis are applied to the robust control law design for gust load alleviation problem. Specifically, with instability risk introduced, the gust load alleviation performance can be largely enforced. At the same design condition, the stability robustness is decreased while the nominal performance can increased significantly. Simultaneously, the instability risk is increased with the performance ascending. This can provide an alternative idea to robust control law design.

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