SHIMMY OF WHEELS OF LANDING GEAR OF AIRCRAFT WITH LOCAL NONLINEARITIES

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Abstract: Local nonlinearities of airplane landing gears are considered. Analytical formulas and results of analysis of torsional stiffness and damping of a landing gear strut with local nonlinearities (free-plays, shimmy damper nonlinearities, seals friction) are presented.

Shown also the results of analysis of shimmy stability boundary and limit cycles oscillations under dry and quadratic nonlinear friction by means of methods based on Routh-Hurwitz procedure, solution of full eigenvalue problem of shimmy linear equations matrix, as well as on *D*-partition of torsional dynamic stiffness plane for landing gear strut.

1 INTRODUCTION

The analysis of the works published in the last 20 years on the problem of shimmy shows that this problem in our country and abroad is still relevant [1-7]. All types of landing gears with both cross wind undercarriage and steered and self-oriented wheels exposed shimmy. The occurrence of shimmy is dangerous because it can not only damage landing gear, but, as a consequence, leads to costly repairs and downtime of the aircraft. Therefore, the design and development of research methods shimmy is the relevant task.

The main reason for the existence of shimmy of the wheels of aircraft landing gear during stage flight tests, and in the course of their further exploitation is to mistakes made when the landing gear are being designed.

Construction landing gear are spatial, structural variable nonlinear dynamic systems [1]. One of the difficulties of analysis shimmy of such systems is the presence of nonlinearities in the design of almost all struts [4-9].

Some of non-linearity essentially depends from the wear of the structural elements of the strut, and it does the shimmy phenomenon is less predictable. Therefore, for accurate prediction of shimmy need to be able to estimate and model the nonlinearity and also to analyze the characteristics of shimmy taking into consideration range of changes of the characteristics of nonlinearities and to identify the most adverse combination of design parameters landing gear and operating regime of the aircraft.

In studies shimmy leading role belong to the computational methods, but for the reliable estimates of safety from shimmy of the wheels requires not only an adequate mathematical model of the shimmy phenomenon, but reliable methods for determining the parameters of this model [8]. This safety assessment from shimmy in the range of operating loads and speeds of the aircraft often carried out by means of a mathematical model, whose parameters are determined with insufficient accuracy and do not take into account nonlinearity in design of strut landing gear.

The most common research methods linear models shimmy are the methods based on application of the procedure of Routh-Hurwitz and the solution of the complete problem of eigenvalues of the matrix corresponding to equations [8-10]. Due to the variety of design schemes and technical solutions landing gear, control mechanisms wheels and means for damping of vibrations, due to the inherent substantial nonlinearities developed a linear mathematical model of shimmy for a typical landing gear is not always applicable. Application of frequency dynamic stiffness method [8] to calculate the boundaries of the region shimmy and parameters of limit cycles oscillations allows to take into account the above features.

2 THE MODELS OF THE NON LINEARITIES IN THE DESING OF THE LANDING GEAR OF THE AIRCRAFT

The main nonlinearity in the design of the landing gear [1-6] is a local nonlinearity, among which the most important are:

- bearing friction of the shock absorber and in the rotating clamp of the control system;
- nonlinear damping in the hydraulic module of the control system;
- nonlinearity in the design of shimmy dampers;
- friction and free-playes in the torque link and the attachment fitting of the strut to the airframe.

Analysis of the phenomenon of shimmy of the wheels of the landing gear of the aircraft taking into account the nonlinearities can be divided into two stages:

1. Development of nonlinear mathematical models of nonlinearities and study of their dynamic characteristics.
2. Study of the characteristics shimmy taking into account developed nonlinear models.

As is known, for prevent shimmy phenomena of cross wind undercarriage of the aircraft are being used hydraulic or friction dampers [5]. Therefore, development of mathematical models of dampers and study of their properties is an important task to ensure the safe operation of the aircraft.

Below we consider the mathematical model of hydraulic damper (DH), which, with some simplifications can be applied also for the analysis of the characteristics of the friction damper and the stiffness characteristics of the strut for torsion with free-play, with dry friction, etc.

Relatively detailed diagram of a hydraulic damper (DH) depicted in figure 1. In this scheme, three physical coordinates $z(t)$, $y(t)$, $\varphi(t)$ determines the dynamics of the following series-connected elements:

- springs with stiffness C_k, simulating the elasticity of the strut mount damper to the main structure and the elasticity of its body;

- piston, having area F, and cylinder, having volume V_0, filled with a compressible fluid with an equivalent modulus of elasticity ;
- springs C_S, that simulate the elasticity of the plunger of the damper and its mounting to the damped part of the structure.

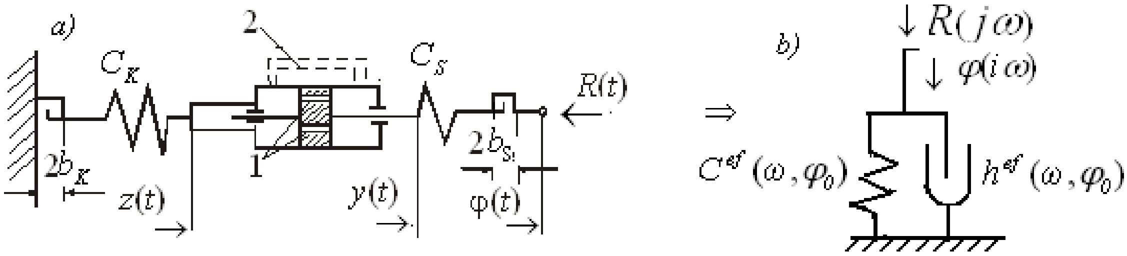


Figure 1: analytical model of a DH

Then the dynamics of DH can be described by the following system of nonlinear differential equations:

$$\left.\begin{aligned}
&m_K\frac{d^2z}{dt^2}+C_k f(z,b_k)-\Delta p(t)S=0;\\
&m_S\frac{d^2y}{dt^2}+C_S[y-f(\varphi,b_S)]+\Delta p(t)S=0;\\
&\Delta p(t)S=f(q,\frac{dq}{dt});\\
&q(t)=S(\frac{dy}{dt}-\frac{dz}{dt})-\frac{V_0+S(y-z)}{Eeq}\frac{d\Delta p}{dt};\\
&C_S[f(\varphi,b_S)-y]=R(t).
\end{aligned}\right\} \qquad (1)$$

Here the first two equations are expressing the condition of equilibrium of the forces acting upon fluctuations respectively on the mass m_k of the housing of the damper and the mass m_s of the piston area of S. In this case the nonlinear force $\Delta p(t)S=P(t)$ due to the pressure difference $\Delta p(t)$ on the piston DH arising from the flow of fluid from one cavity to another through the primary and secondary flow channels (1 and 2). Non-linear character of restoring forces in these equations are determined by the presence of free-play (b_K, b_S) in the places of attachment of the damper to the main structure.

View functions $f(\varphi,b_S)=\varphi_0\cdot f(\overline{\varphi},\overline{b}_S,\overline{P}_0)$ and $f(\varphi,b_k)=\varphi_0\cdot f(\overline{\varphi},\overline{b}_{\hat{e}},\overline{P}_0)$

for $\varphi_{10}=\varphi_0-b_S$, $z=\varphi=\varphi_0\overline{\varphi}$, $b_K=b_s=b$ shown in figure 2.

The third equation of system (1) is establishing the dependence of differential pressure on flow rate of working fluid through the channels of its flow. The theoretical description of this dependence is in general extremely difficult, therefore, in engineering practice is being used the following representation for the dependence of the resistance force of the damper from the flow rate $q(t)$ by its channels

$$\Delta p_q S = f_1 q + (f_2 q^2 + f_3)\,\mathrm{sign}\, q,$$

where f_1 and f_2 are coefficients, depending on fluid properties, the shape of the channels to overflow, and the effective area of the piston of the damper; f_3 is a coefficient that defines the dry friction force between the moving parts of the damper.

In this paper, we assume that the dependence of fluid flow from the differential pressure $\Delta p(t) = F(t)/S$ determined by experimental methods, and can be approximated by an exponential function (figure 2b).

$$q(t) = k_q (a + k|F(t) - F_e|)^{\gamma}\,\mathrm{sign}\, F(t). \tag{2}$$

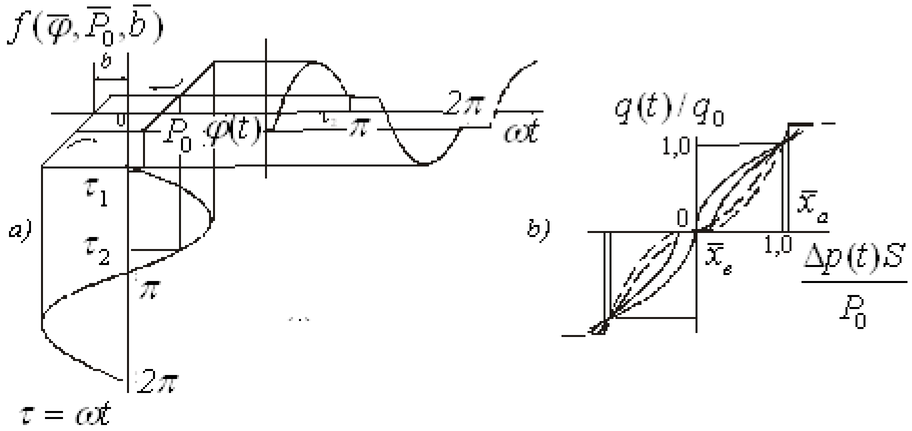


Figure 2: Nonlinearities of DH

In real structures of the hydraulic units the resistance force caused by the inertia of the fluid filling the channels of the ducts, can be significant if, for example, to enter the bypass channel 2 long length (figure 1). Then the inertial force of a fluids resistance to movement of the piston DH can be represented as: $\Delta\,\delta_m S = \mu \frac{dq}{Sdt}$, where μ is the reduced mass of liquid filling the channels of the ducts 1 and 2. The total resistance force of the fluid movement of the piston of the damper, obviously, is expressed by the equality: $\Delta pS = (\Delta p_q + \Delta p_m)S$.

The fourth equation of system (1) is a continuity equation of fluid flow with regard to its compliance, and the latter expresses the equality of the external force $R(t)$ to the force in the damper shaft.

Below are present the results of the study of the stiffness and damping properties of the DH, using the dynamic stiffness characteristics:

$$D(i\omega) = \frac{R(i\omega)}{\varphi(i\omega)} = C^{eff}(\omega, \varphi_0) + i\omega h^{eff}(\omega, \varphi_0) \tag{3}$$

The real part of this function will be called effective stiffness $C^{eff}(\omega, \varphi_0)$, and the coefficient $h^{eff}(\omega,\varphi_0)$ is the effective drag coefficient of the DH (figure 1b), ($i=\sqrt{-1}$, ω, φ_0 is the frequency and amplitude).

2.1 Model of linear hydraulic damper

In the simplest case, dynamic stiffness of linear DH with the drag coefficient h_d on an elastic foundation C_d can be obtained from equations (1 and 2) without taking the mass of the moving parts of the damper with $b_K = b_S = 0$, $\gamma = 0$, a=1. Then instead of equations (1) we have the equation, which in dimensionless form becomes:

$$\frac{d(\overline{\varphi}-\overline{F})}{d\,\tau} = \overline{F} - \overline{\mu}\frac{d^2(\overline{\varphi}-\overline{F})}{d\tau^2},\ \text{or}\ \overline{h}_d\frac{d\chi}{d\tau} + \overline{\mu}\frac{d^2\chi}{d\tau^2} + \overline{\tilde{N}}_d(\varphi-\chi) = 0\,, \qquad (4)$$

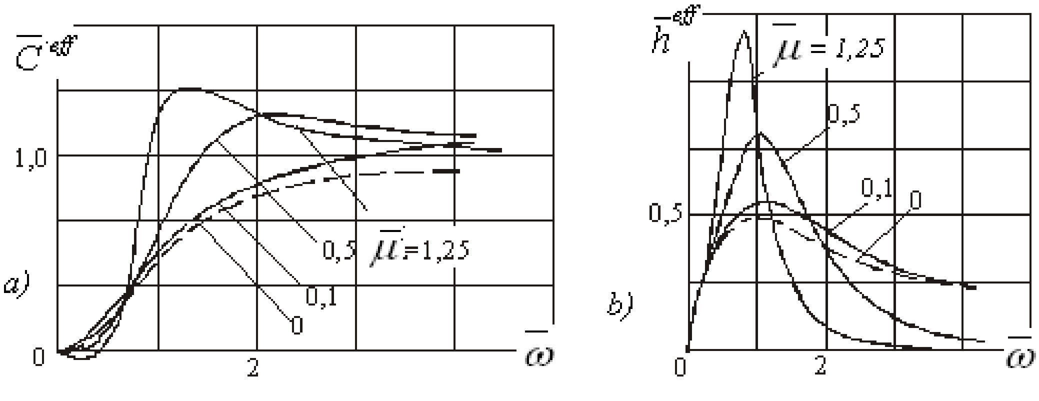
where: $\tau = D_L t$, $D_L = C_d / h_d$ – quality factor, $h_d = \dfrac{S}{K_q}$ — drag coefficient of damper, χ - displacement of the piston of the damper relative to its housing, K_q - the ratio of the flow characteristics of the damper, $1/C_d = 1/C_k + 1/C_S + 1/\tilde{N}_{eq}$; $C_{eq} = \dfrac{4E_{eq}S}{V_0}$.

In this case, equations (4) relationship of the components of the dynamic stiffness of the damper to determine its parameters is expressed by formulas (5) and graphically presented in figure 3.

$$\left.\begin{aligned} \overline{\tilde{N}}^{eff}(\overline{\omega}) &= \frac{\tilde{N}^{eff}(\omega)}{\tilde{N}_d} = \frac{\overline{\omega}^2[1-\overline{\mu}(1-\overline{\mu}\overline{\omega}^2)]}{\overline{\omega}^2+(1-\overline{\mu}\overline{\omega}^2)^2}; \\ \overline{h}^{eff}(\overline{\omega}) &= \frac{h^{eff}(\omega)}{\tilde{N}_d} = \frac{\overline{\omega}}{\overline{\omega}^2+(1-\overline{\mu}\overline{\omega}^2)^2}; \quad \overline{\omega} = \frac{\omega}{D_L} \end{aligned}\right\} \qquad (5)$$

Note here practical result: the maximum level of damping when μ=0 is achieving from the condition of equality of the stiffness: $h_d\omega = C_d$ - "damper resonance".

Dependency analysis in figure 3 shows that accounting for the inertia of the liquid μ can significantly change the elastic and damping properties of hydraulic damper.



(*a*) (*b*)

Figure 3: Effective stiffness (*a*) and effective drag coefficient of DH (*b*)

1.2 Model DH with nonlinearities

The results of the influence of the nonlinearities of the damper on its dynamic stiffness is made under the assumption that the inertia of the moving parts of the damper and inertia forces of resistance of the working fluid can be neglected. When considering the effect of nonlinear dependence of fluid flow from the pressure differential on the piston of the damper we assume that this dependence is determined from the experiment and is approximated by an exponential function (2) (figure 2b). Then study the dynamic properties of DH given the elasticity of attaching reduced to the study of solutions of the following nonlinear equations:

$$\left.\begin{aligned} \frac{d(\bar\varphi-\bar F)}{d\tau} &= (|\bar F|-\bar F_{\mathring{a}})^{\gamma}\, sign\bar F \quad of\ |\bar F|>\bar F_{\mathring{a}} \\ \frac{d(\bar\varphi-\kappa\bar F)}{d\tau} &= 0 \quad of\ |\bar F|\le \bar F_{\mathring{a}} \end{aligned}\right\} \qquad (6)$$

where: $\tau = D_N t$,

$$D_N = D_L \frac{\bar\varphi^{\gamma-1}}{(\bar F_a-\bar F_{\mathring{a}})^{\gamma}},\quad D_L=\frac{C_d q_a}{S};\ \bar F_{\mathring{a}}=\frac{F_e}{\tilde N_d\varphi_0},\quad \bar F_a=\frac{F_a}{\tilde N_d\varphi_0};\quad k=\frac{C_d}{C_d\,|_{E_{eq}=\infty}},\ \bar{\tilde o}_{\grave a}=\frac{\Phi_{\grave a}}{\tilde N_d\varphi_0},$$

$$\kappa=\frac{\tilde N_d}{\tilde N_d\big|_{\mathring{A}_{eq}=\infty}}.$$

The parameters $q_{\grave a}, \bar F_e, \bar F_a$ are the coordinates of the nodes of interpolation of experimental flow characteristics of the damper; D_N, D_L - quality factor of a nonlinear and a linear damper.

For two cases of the nonlinear equation (4), when the exponent of the approximating function in equation (4) equal γ=0.5 and 2.0, with help of harmonic linearization analytical are found the analytic expressions for the calculation of the components of the dynamic stiffness (3) [8]. When γ=0.5 equation (6) determines the dynamic properties of the hydraulicγ damper on an elastic foundation with a quadratic characteristic dependence of the resistance force on the speed of displacement of the strut relative to the housing. For this case the formula for the computation of components of the dynamic stiffness of DH is present below:

$$\left.\begin{aligned}
&\overline{\tilde{N}}^{eff} = \mathrm{Re}\overline{D}(\varphi_0, i\overline{\omega}) = \overline{D}_0^2;\\
&\overline{h}^{eff} = \mathrm{Im}\overline{D}(\varphi_0, i\overline{\omega}) = \overline{D}_0\sqrt{1-\overline{D}_0^{\,2}};\\
&\overline{D}_0 = \sqrt{\left(\frac{3\pi}{16}\overline{\omega}^2\right)^2 + 1 - \frac{3\pi}{16\overline{\omega}^2}};\\
&\overline{\omega} = \frac{\omega}{D_{\mathbf{N}}};\ D_N = D_L\sqrt{\frac{\overline{C}_d}{\overline{h}_\chi \varphi_0}},
\end{aligned}\right\} \qquad (7)$$

где D_N – the quality factor of DH with quadratic characteristic resistance force.

We note here that if DH with quadratic characteristic resistance force is fixed to a rigid base, the equivalent resistance coefficient of viscous friction damper is calculated by the known formula:

$$\overline{h}^{eq} \cong \frac{8}{3\pi}\overline{h}_\chi \overline{\omega}\,\varphi_0, \qquad (8)$$

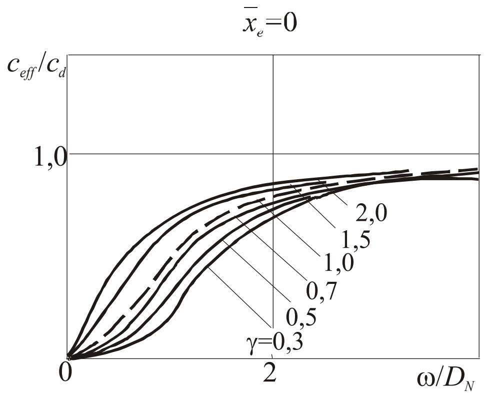
For other values of exponent γ of the approximating function in equations (6) calculation of the components of the dynamic stiffness was performed using the algorithm based on numerical integration of equations (6) by the Runge-Kutta method.

The modulus of dynamic stiffness D_0 and its phase ψ was determined from the following relationships:

$$D(\tau) = a_1 \cos\tau + b_1 \sin\tau\,;\quad D_0 = |D(\tau)| = \sqrt{a_1^2 + b_1^2}\,;\quad \psi = arctg\frac{a_1}{b_1}.$$

where: a_1, b_1 are the components of the basic harmonic of the Fourier series of steady-state periodic solution of equation (6) and represent the required components of the components of the dynamic stiffness of the nonlinear hydraulic damper.

The results of calculations at $\overline{x}_e$=0 and at different exponents of approximating function γ shown in figure 4.



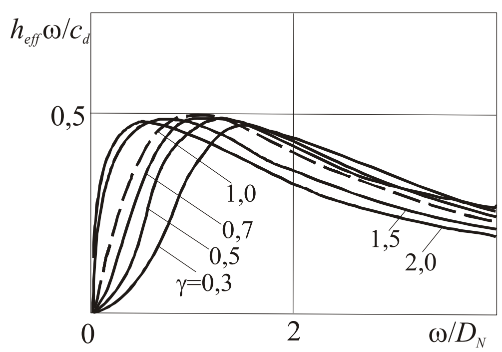


Figure 4: Influence of the exponent of the approximating function

2.3 The influence of free-play on dynamic stiffness DH

Analysis of the influence of the nonlinear function $f(\varphi, b_S, F_0)$ (1) due to the presence of free-play in the pivot joint of strut damper with an external load made in the simplest case, when the mathematical model of the damper is represented by the equation:

$$\frac{d}{d\tau}[\bar{f}(\bar{\varphi}, \bar{b}_S, \bar{F}_0) - \bar{F}] = \frac{\bar{F}}{\bar{\omega}}, \tag{9}$$

When $\bar{\varphi} = \sin\tau$, $\bar{F}_0 = \bar{D}_0$ a function $\bar{f}(\bar{\varphi}, \bar{b}_S, \bar{F}_0)$ can be represented by the method of harmonic linearization in the form $\bar{f}(\bar{\varphi}, \bar{b}_S, \bar{F}_0) = [g(\bar{b}_S, \bar{D}_0) + g'(\bar{b}_S \bar{D}_0)]\bar{\varphi}$, and then obtain the following characteristic equation for determining the modulus $\bar{D}_0$ and phase ψ of dynamic stiffness:

$$e^{-i\psi} = \frac{\bar{D}_0}{\omega}\left[\frac{(\bar{\omega} g - g') - j(\bar{\omega} g' + g)}{g^2 + g'^2}\right], \tag{10}$$

where the coefficients $g = g(\bar{b}_S, \bar{D}_0)$ and $g' = g'(\bar{b}_{S,} \bar{D}_0)$ are expressed through the parameters of the nonlinear function as follows:

$$\left.\begin{aligned} g(\bar{b}_S, \bar{D}_0) &= \frac{1}{\pi}[\pi + arcsin(1 - 2\bar{b}_{ø} - \bar{D}_0 - arcsin(1 - \bar{D}_0) + \\ &+ (1 - \bar{D}_0 - 2\bar{b}_S)\sqrt{1 - (1 - 2\bar{b}_S - \bar{D}_0)^2} - (1 - \bar{D}_0)\sqrt{1 - (1 - \bar{D}_0)^2}]; \\ g'(\bar{b}_S, \bar{D}_0) &= -\frac{4\bar{b}_S}{\pi}(1 - \bar{b}_S - \bar{D}_0). \end{aligned}\right\} \tag{11}$$

The solution of the transcendental equation (10) was performed by the graphic-analytical method, from which it follows (figure 5), that the emergence of a free-play in places of fastening of the hydraulic damper leads to a significant reduction of its stiffness and damping properties.

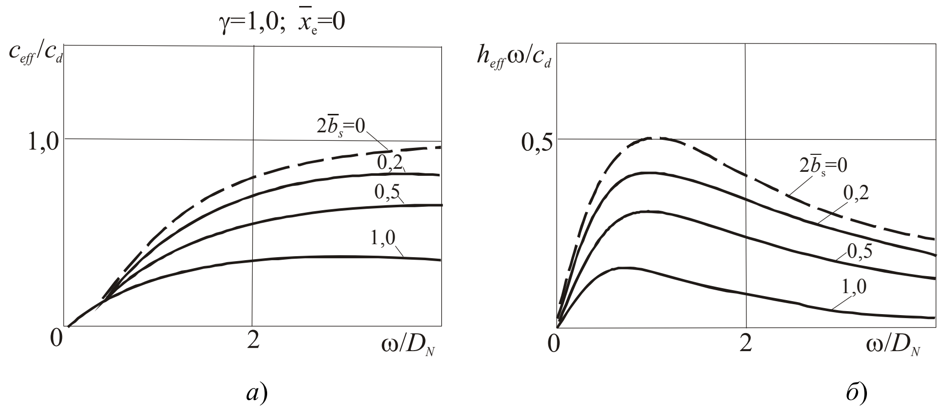


Figure 5: Influence of free-play on the dynamic stiffness of DH

We note here that at $\bar{\omega}$ the fluid flow in the channels of the duct damper, tending to infinity, is missing, and DH "turns" in the spring with free-play. In this case, the posting control with free-play in many cases can be represented by a model consisting of an elastic element with stiffness C_θ and free-play, size $2b$ (figure 6). This model, in particular, may show dynamic properties in torsion unmanaged wheel landing gear of the aircraft in the presence of free-play in the joints of the links of torque link.

In the general case of nonsymmetric nonlinearity obtained the formula (12) by the method of harmonic linearization to calculate the dimensionless effective stiffness of the elastic-loaded chain with a free-play [7].

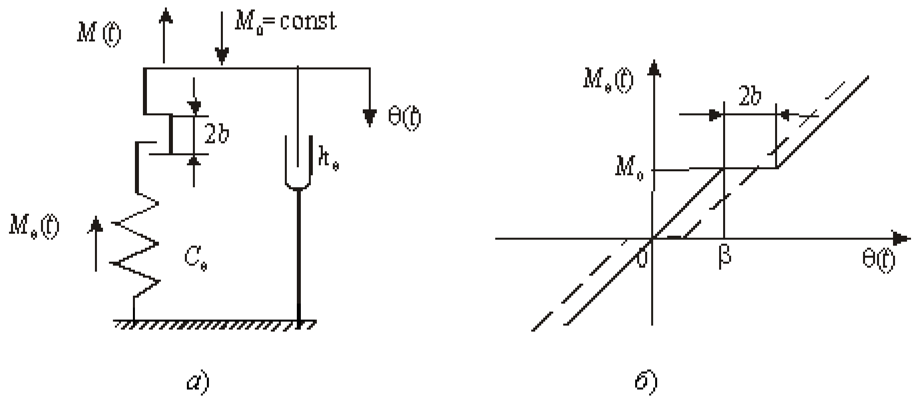


Figure 6: Scheme of the rigidity of the strut in torsion with free-play (indicated by a dotted line symmetric free-play)

$$\bar{N}^{eff} = \left| \begin{array}{c} 1 \ldots when \ldots \alpha \le \delta, \\ \frac{1}{2} + \frac{1}{\pi}(\arcsin x_1 + x_1\sqrt{1-x_1^2}) \\ when \ldots \delta < \alpha \le 1+\delta, \\ 1 - \frac{1}{\pi}(\arcsin x_2 - \arcsin x_3 + x_2\sqrt{1-x_2^2} - x_3\sqrt{1-x_3^2}) \\ when \ldots \alpha > 1+\delta. \end{array} \right\} \quad (12)$$

where $x_1 = 1 - \dfrac{2(\frac{\alpha}{\delta} - 1)}{\frac{\alpha}{\delta} + \sqrt{2\frac{\alpha}{\delta} - 1}}$, $x_2 = 1 - \dfrac{2(\alpha - \delta)}{\alpha + \sqrt{(\alpha-1)^2 + 2\delta}}$, $x_3 = 1 - \dfrac{2(\alpha - \delta - 1)}{\alpha + \sqrt{(\alpha-1)^2 + 2\delta}}$,

$$\alpha = \theta_0/2b, \quad \delta = \beta/2b.$$

From the analysis of formula (11) implies that the effective stiffness is determined by only two parameters α, δ, and plotted in the form of a one-parameter family of curves (figure 7). When δ=0 and $x_2 = -x_3 = \bar{\theta}_0^{-1}$ the relation (11) is identical to the correlation function, describing the “symmetric free-play” (figure 7, β=0).

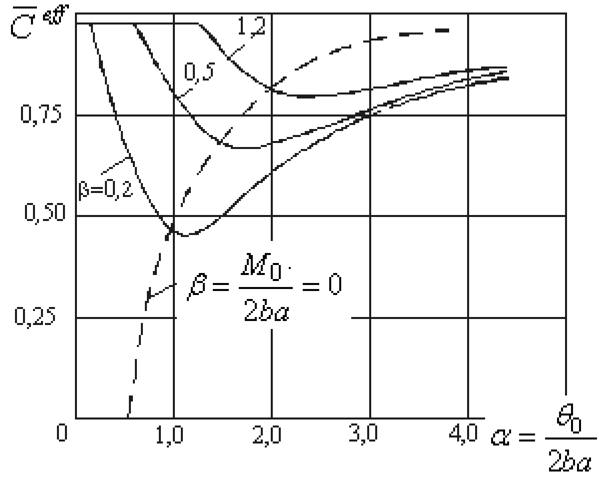


Figure 7: Dependence of effective stiffness of the strut torsion from the amplitude (indicated by a dotted line symmetric free-play)

Comparison of experimental and calculated data shows that the dynamic characteristics of hydraulic dampers can be with acceptable accuracy obtained by calculation [ ].

In summary, the above main results, it should be emphasized that the most important parameter determining the dynamic properties of DH is its quality factor, defined as the ratio of linearization of the flow characteristics of the damper (figure 2b):

The greatest damping effect of the hydraulic damper is manifested at frequencies of oscillation of the shaft close to the value of the quality factor of the damper.

The appearance of free-playes at the joints of the damper to the main structure can lead to significant reduction in the efficiency of the use of hydraulic dampers, especially, at frequencies $\omega > D$.

2.4 Dynamic stiffness of the friction damper

In some cases, to prevent shimmy of wheels of orientation are applying friction dampers (FD). Example of FD is the front landing gear of a light aircraft, where in a mathematical model the link between oriented and fixed part of the spring strut can be modeled in the form of a dry friction damper, attaching on an elastic foundation with stiffness C_θ and moment of friction M_{fr}. Then the components of the dynamic stiffness are calculated by the approximate formula (13).

$$\left.\begin{aligned} &C^{eff}(\theta_0) = \mathrm{Re}D(i\omega,\theta_0) = C_\theta[1-g(\overline{\delta}_s)],\\ &h^{eff}(\omega,\theta_0) = -\mathrm{Im}\frac{D(i\omega,\theta_0)}{\omega} = -\frac{C_\theta}{\omega}g'(\overline{\delta}_s),\end{aligned}\right]$$

$$where: \quad g(\overline{\delta}_s) = \frac{1}{\pi}[\frac{\pi}{2} + arcsin(1-2\overline{\delta}_s) + 2(1-2\overline{\delta}_s)\sqrt{\overline{\delta}_s(1-\overline{\delta}_s)}], \qquad (13)$$

$$g'(\overline{\delta}_s) = \frac{4\overline{\delta}_s}{\pi}(1-\overline{\delta}_s), \qquad \overline{\delta}_s = \frac{F_{bw}}{C_\theta\theta_0} = \frac{\theta_{bw}}{\theta_0}.$$

From formulas (13) and the graphs in figure 8 shows that the dry friction damper has a maximum damping coefficient when the vibration amplitude is twice the amplitude θ_{bw}, equal $\theta_{bw} = \dfrac{F_{bw}}{C_\theta}$, in which FD is included in the work.

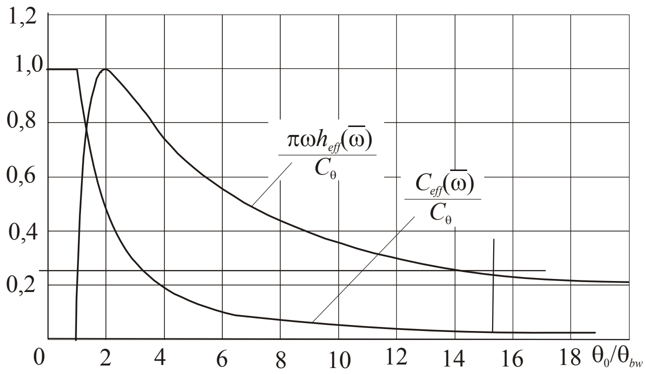


Figure 8: The dependence of components of the dynamic stiffness of FD on elastic foundation from the amplitude of vibration

3 THE INFLUENCE OF NONLINEARITIES ON SHIMMY OF WHEELS CROSSWIND OF UNDERCARRIAGE OF AIRCRAFT

As is well known [7-10], for the analysis of shimmy of wheels crosswind of undercarriage, the connection between the movable 1 and fixed 2 parts strut of landing gear is modeled using an elastic element C_θ and connected in series with perfect shimmy damper h_d (figure 9). In this case, the torque produced by shimmy damper, it is linearly dependent on the velocity $\dot{\chi}$ of the piston relative to the housing from deformation or ($\theta - \chi$) of the elastic element C_θ. In the case of quadratic dependence of the resistance force of the hydraulic damper on the speed of its piston, the pivot axis of the wheels, the corresponding equation has the form [9]:

$$\overline{h}_\chi \dot{\chi}^2 sign\dot{\chi} = \overline{C}_\theta(\theta - \chi), \qquad (14)$$

where $\overline{h}_d = \overline{h}_\chi = h_\chi / n\sqrt{am}\, r^2$;

a, m, r and n – tyre lateral stiffness, mass, wheel radius and wheels number (scaling coefficients).

For an approximate estimation of the effect on shimmy structural strength (dry) friction in the shock absorber landing gear, the dependence of the moment of these forces from the velocity $\dot{б}$ represented in the form of an ideal relay features:

$$Ì_{fr} = Ì_\theta sign\dot{\theta}. \qquad (15)$$

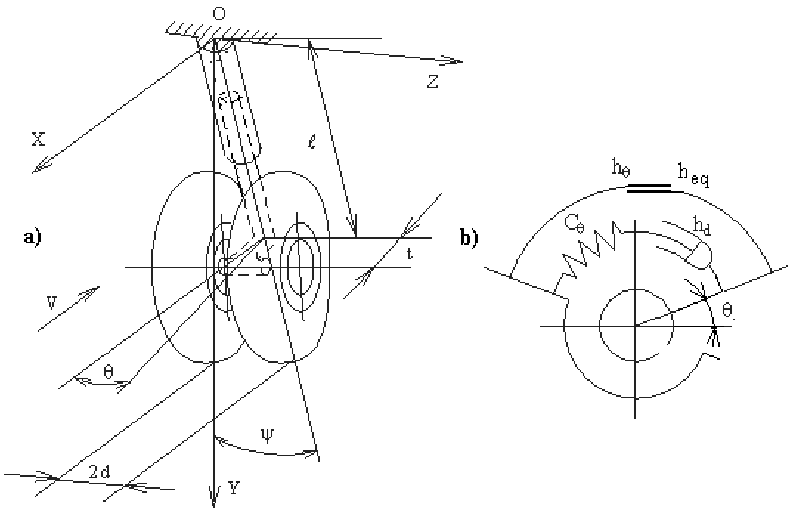


Figure 9: Scheme of a landing gear with steered wheel and wheel of crosswind

Then on the basis of relations (14) and (15) on known [10] the system of linear differential equations describing the motion of wheels of crosswind undercarriage may be in dimensionless form is represented as:

$$
\left.\begin{aligned}
&\bar I_x\ddot\psi + (\bar c_\psi \bar l^2 + \bar k \bar d^2)\psi + \bar I_{xy}\ddot\theta + \bar V \bar I_k\dot\theta - (1+\bar l)\bar\lambda = 0;\\
&\bar I_{xy}\ddot\psi - \bar V\bar I_k\dot\psi + \bar I_y\ddot\theta + \bar c_\theta + \bar M_\theta sign\dot\theta - \bar t\bar\lambda - \bar b\varphi - \bar c\bar d\xi = 0;\\
&\bar h_\chi \dot\chi^2 sign\dot\chi = \bar c_\theta(\theta-\chi);\\
&(1+\bar l)\dot\psi + \bar t\dot\theta + \bar V(\theta+\varphi) + \dot{\bar\lambda} = 0;\\
&\dot\theta + \dot\varphi - \bar\alpha\bar V\bar\lambda + \bar\beta\bar V\varphi - \bar\gamma\bar V\psi = 0.
\end{aligned}\right\} \qquad (16)
$$

In these equations $\psi, \theta, \lambda, \varphi$, respectively, the angles of rotation of the strut landing gear relative to the axes *Ox* and *Oy* (figure 9), the displacement of the center of tire contact from the diametrical plane of the wheel and the angle of rotation center of the contact patch.

Relation of dimensional and dimensionless quantities in (16) is defined by the following relations: $I_x = \bar I_x nmr^2$, $I_y = \bar I_y nmr^2$, $I_{xy} = \bar I_{xy} nmr^2$ are the moments of inertia of the fixing of the landing gear relative to the respective axes; $I_k = \bar I_e mr^2$ - the moment of inertia of the wheel about the axis of its rotation; $V = \bar V\sqrt{a/mr}$ - the speed of the aircraft; $k = \bar k a$, $b = \bar b ar^2$ - vertical stiffness and torsional stiffness of the tire; $C_\psi = \bar C_\psi na$, $C_\theta = \bar C_\theta nar^2$ - lateral and torsional stiffness of the landing gear mounting; $\alpha = \bar\alpha r^2$, $\beta = \bar\beta r$, $\gamma = \bar\gamma r$ – kinematic characteristics of the tire in the theory of rolling Keldysh; $t = \bar t r$ – the wheel axle offset; $l = \bar l r$ – the effective length of the strut. Approximate calculation of the amplitudes and frequencies of limit cycles oscillations (LCO) of nonlinear system (16) and evaluation of their stability in some cases can be made on the basis of results of calculation of the linear system.

Let's take the simplest example of such a system grapho-analytical method of determining the parameters of limit cycles using the results of the calculation of the stability of a linear system. Therefore, let us proceed from the condition that during oscillations of the landing gear on the border region shimmy with the frequency $\overline{\omega}_{sh}$ the moment of forces of dry friction approximately defined by an equivalent damping ratio $h_{\theta}^{eq} \cong \dfrac{4\grave{I}_{\theta}}{\pi\omega\theta_0}$, then, obviously, the values of the amplitudes of limit cycles oscillations θ_0^{LCO} can be determined from the equality of the calculated and specified values:

$$\pi\overline{h}_{\theta}^{sh}\overline{\omega}^{sh} = 4\overline{I}_{\theta} / \theta_0^{LCO}. \tag{17}$$

Graphic build path dependencies $\theta_0^{LCO} = f(\overline{V}, \overline{M}_{fr})$, based on equality (17), shown in figure 10.

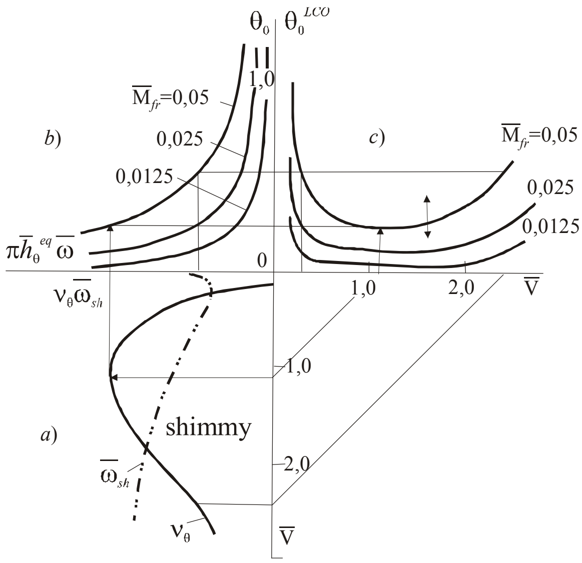


Figure 10: Calculation of parameters LCO NLG with friction damper

At this the figure in quadrant *a*) shows a boundary region shimmy in the parameter plane $\pi\overline{h}_{\theta}^{sh}\overline{\omega}^{sh}$ and the speed of rolling of the wheels $\overline{V}$, and the frequency of the shimmy $\overline{\omega}^{sh} = \omega^{sh} / \sqrt{a/m}$, which are obtained from the solution of the equations (16) for χ=0 and replacing a summand $\overline{M}_{\theta}\, sign\,\dot{\theta}$ on the summand $\overline{h}_{\theta}\dot{\theta}$. In quadrant *b*) is dependence $\pi\,\overline{h}_{\theta}^{eq}\overline{\omega}$ from the amplitude θ_0. Then, on the basis of equality (17) in quadrant *c*) depicts the desired dependence of the amplitudes of the LCO from the speed and moment of dry friction .

From the presented results it is seen (figure 10c) that the limit cycles due to dry friction forces, are unstable, since the oscillation of the strut with the amplitudes $\theta_0 > \theta_0^{LCO}$ these forces produce in the strut landing gear damping torque, less torque required for rolling stability of the wheels.

In the case when to prevent shimmy at the strut of landing gear is set DH with quadratic characteristic (14), take in (16) the friction moment is equal to zero. Then, replacing the nonlinear part of the third equation of this system with linear part, equal $\overline{h}_d\dot{\theta}$, for the corresponding linear system of equations can determine the boundary of the stability domain in the parameter $\overline{h}_d / \overline{\omega}^{sh}$ plane and another independent parameter, e.g. a parameter $\tilde{n}_{\theta}$.

Then, from the equation:

$$\bar{h}_d^{sh} = \bar{h}_{eq}^{sh}, \qquad (18)$$

where $\bar{h}_{eq}^{sh} \cong \frac{8}{3\pi}\bar{h}_\chi \overline{\omega^{sh}}\chi_0^{LCO}$ is the equivalent resistance coefficient of viscous friction damper, it is not difficult to obtain the dependence of the amplitudes of limit cycles χ_0^{LCO} from parameter $\tilde{n}_\theta$.

In the more general case, when the movement of the crosswind wheels is determined by the system of nonlinear equations (16), having two nonlinear terms, the application of the graphical method is time consuming task.

Significant advantages when performing parametric calculations shimmy based on local nonlinear dependencies, as indicated above, the use of dynamic stiffness method (DSM) [ ]. In this method the equations shimmy with the help of the Laplace transform is written a single matrix equation. The matrix of this equation will contain an element that when setting a variable of the Laplace $s = i\omega$ is frequency response dynamic torsion stiffness of the strut $D(i\omega, \theta_0)$, which can be used to explore shimmy with a variety of models of this stiffness. Examples of computation of this function is discussed above and in [7-9].

Method DSM, first, provide a much higher performance when computing the boundary of the stability domain in comparison, for instance, with the method of stability analysis, based on the Routh procedure, in combination with the solution of the complete problem of eigenvalues. Secondly, this method allows to investigate shimmy of the wheels with regard to more than two local nonlinearities and used in the calculations of the experimental frequency response. Third, the DSM method provide an opportunity to investigate the stability of the movement struts not only with crosswind wheels, but also other types of struts wheeled landing gear (managed, unmanaged) and consider the variety of relationships between the moving and fixed parts by means of the frequency characteristics of dynamic stiffness.

Following this method [8], the system of equations (16) is represented in the frequency domain in the form of a nonlinear system of homogeneous algebraic equations:

$$K(\theta_0, i\overline{\omega})\theta(i\overline{\omega}) = 0 \qquad (19)$$

where $\theta(i\overline{\omega}) = [\psi(i\overline{\omega}), \theta(i\overline{\omega}), \bar{\lambda}(i\overline{\omega}), \varphi(i\overline{\omega})]^T$ is a complex vector of generalized coordinates.

Matrix $\hat{E}(\theta_0,\ i\omega)$ for equations (16) can be represented as the following structure:

$$K(\theta_0, i\overline{\omega}) = \begin{bmatrix} (-\bar{I}_x\overline{\omega}^2 + \bar{c}_\psi\bar{l}^2 + \overline{k}\overline{d}^2); & (-\bar{I}_{xy}\overline{\omega}^2 + i\overline{\omega}\overline{V}\bar{I}_{0k}); & (1+\bar{l}); & 0 \\ (-\bar{I}_{xy}\overline{\omega}^2 - i\overline{\omega}\overline{V}\bar{I}_{0k}) & [-\bar{I}_y\overline{\omega}^2 + \overline{D}(\theta_0, i\,\overline{\omega})]; & \bar{t}; & \bar{b} \\ i(1+\bar{l}); & \overline{V} + i\overline{\omega}\bar{t}; & i\overline{\omega}; & \overline{V} \\ -\bar{\gamma}\overline{V}; & i\overline{\omega}; & -\bar{\bar{\alpha}}\overline{V}; & (\bar{\beta}\overline{V} + i\overline{\omega}) \end{bmatrix} \qquad (20)$$

It is obvious that the summand $\overline{D}(\theta_0, i\,\overline{\omega})$ of element $K_{22}(i\overline{\omega})$ of matrix $K(\theta_0, i\overline{\omega})$ is the frequency characteristic of the dynamic stiffness of the strut in torsion and in the present case, crosswind wheels is the sum of the dynamic stiffness of the nonlinear hydraulic damper $D_d(\theta_0, i\omega)$ and dynamic stiffness $D_t(\theta_0, i\omega)$ due to dry friction forces:

$$\overline{D}(\theta_0, i\overline{\omega}) = \frac{\overline{M}_{fr}(i\overline{\omega}) + \overline{M}_d(i\overline{\omega})}{C_\theta \theta_0} = \overline{D}_{fr}(\theta_0, i\overline{\omega}) + \overline{D}_d(\theta_0, i\overline{\omega}), \quad (21)$$

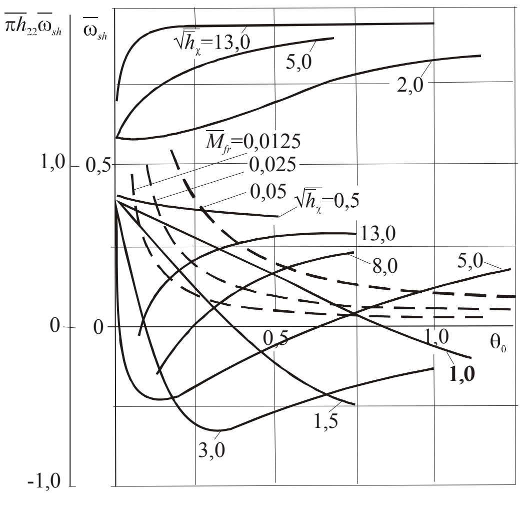
where $\overline{D}_{fr}(\theta_0, i\overline{\omega}) = i\overline{\omega}\overline{h}^{eq}_{\theta} = i\frac{4\overline{M}_{fr}}{\pi\theta_0}$; and components of the dynamic stiffness $\overline{D}^{eff}(\theta_0, i\overline{\omega}) = \overline{N}^{ef}(\overline{\omega}, \theta_0) + i\overline{\omega}\overline{h}^{ef}(\overline{\omega}, \theta_0)$ are calculated by the formulas (5).

After calculating the matrix $K(\theta_0, i\overline{\omega})$ by equations (20) and (21), take $\overline{D}_t(\theta_0, i\overline{\omega}) = 0$ (dry friction is absent), then using the method of dynamic stiffness can calculate for the stability of motion of the wheels the required values of the damping ratio $\nu_\theta = \pi \mathrm{Im}\Delta k_{22}(i\overline{\omega}^{sh})$, which are determined by complex value of the increment to the imaginary part of element $k_{22}(i\omega)$ of matrix and calculated by the formula:

$$\Delta K_{22}(i\overline{\omega}) = -I_y \overline{\omega}^2 + \frac{det' K(\theta_0, i\overline{\omega})}{det\, A_{22}(\overline{\omega})} \quad . \quad (22)$$

where $det' K(\theta_0, i\overline{\omega})$ is the determinant of the matrix $K(\theta_0, i\overline{\omega})$ for $K_{22}(i\overline{\omega}) = 0$; $\det A_{22}(i\overline{\omega})$ - algebraic addition to element $K_{22}(i\overline{\omega})$ of the matrix $K(\theta_0, i\overline{\omega})$.

It is obvious that the intersection points of the calculated function $\nu_\theta = \pi \mathrm{Im}\Delta K_{22}(i\overline{\omega}^{sh})$ with the specified function $\pi \mathrm{Im}\overline{D}_{fr}(\theta_0, i\overline{\omega}^{sh}) = 4\overline{M}_{fr}/\theta_0$ (figure 14) determine the amplitude and frequency of LCO of the testing system depending on the change of the preset parameters of nonlinearities $\overline{M}_{fr}$, $\sqrt{\overline{h}_\chi}$ and speed $\overline{V}$ (figure 11).



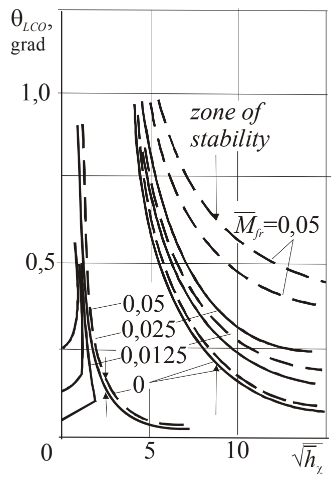


Figure 11: The influence force of the quadratic *a*) and dry friction on LCO amplitude (- - - numerical integration).

From the analysis of these dependences shows that the nonlinearity of the type of dry and quadratic friction significantly change the picture of the emergence and development of shimmy. In the absence of forces of structural friction the movement of the crosswind wheels may be accompanied by sustained oscillations, while the level of external disturbances will not cause vibration of strut with amplitude exceeding the amplitude of the unstable limit cycle. With the growth of the forces of dry friction the amplitude of the unstable limit cycle increases, and steady, on the contrary, decrease, i.e. dry friction in the strut can significantly delay the occurrence of shimmy. Increasing the fluid flow flowing in the damper (reducing the coefficient of resistance $\bar{h}_\chi$), you can also boost safe level of external disturbances.

4 SELF-OSCILLATION OF MLG WITH FREE-PLAY IN THE ATTACHMENT FITTING OF TORQUE LINK

The method of DSM were calculated parameters of limit cycles oscillations of the wheels of the main landing gear of a transport aircraft with the use of models in torsional stiffness with free-play (12). The results of the calculation of the amplitude of self-oscillations for the cases of symmetric and asymmetric free-play is shown in figure 12 for two values of speed $\bar{V}$ of rolling of the wheels. Analysis of these results shows that the oscillations of the wheels of MLG with "symmetric" free-play will be damped at all levels of initial angles of twist of the strut θ_0, if the damping ratio $\bar{h}_\theta > \bar{h}_\theta^{sh}$. When $\bar{h}_\theta < \bar{h}_\theta^{sh}$ the wheel oscillation will be damped only when the initial deviation is smaller than the amplitude of the unstable LCO: $\theta_0 < \theta_0^{LCO}$.

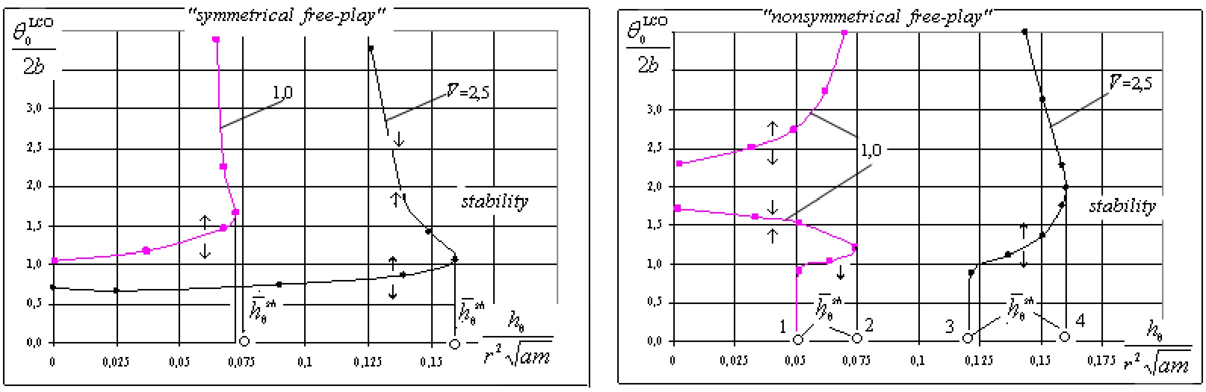


Figure 12: Dependence of the amplitudes of the LCO from damping coefficient of the support in torsion with "symmetric" and "asymmetric" free-play

For "asymmetric" free-play parameters LCO depend on the speed of rolling of the wheel. At low speeds and when $\bar{h}_\theta > \bar{h}_\theta^{sh}$ the calculations at shimmy in the time domain, performed in [ ] show that the accuracy of an approximate method of harmonic linearization models of nonlinearities is quite satisfactory for engineering calculations of the parameters of limit cycles of self-oscillations of the wheels of the landing gear.

5 CONCLUSION

The results of the studies discussed above demonstrate the importance of taking into account nonlinearities in the study of shimmy and show a significant influence on the parameters of limit cycles oscillations of the wheels.

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