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STIFFNESS DISTRIBUTION AND AEROELASTIC PERFORMANCE OPTIMIZATION OF HIGH-ASPECT-RATIO WINGS

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Abstract: Based on aeroelastic optimization of global stiffness design for high-aspect-ratio wings, the relationship between stiffness distribution and aeroelastic performance for a beam-frame model and a 3-D model is studied. The sensitivity information of wing spanwise stiffness distribution with respect to the twist angle at wing tip, the vertical displacement at wing tip and the flutter speed is obtained based on sensitivity method for both models. Then the relationship between stiffness distribution and aeroelastic performance is summarized to guide the design procedure. By using the genetic/sensitivity-based hybrid algorithm, an optimized solution satisfying the strength, aeroelastic and manufacturing technology constraints is obtained. It is found that the summarized guidance is well consistent with the optimal solution, so it can provide valuable design advice with efficiency. The study also shows that the aeroelastic optimization based global stiffness design procedure can obtain the optimal solution under multiple constraints with high efficiency, thus having strong practicality in engineering.

1 INTRODUCTION

With the rapid development in aircraft design technologies, the application of advanced material, and the increase in aircraft size, wing deformation and aeroelastic effects increase significantly, which has great influence on aerodynamic characteristics[1]. Therefore, a rational design of wing stiffness distribution becomes the priority of the conceptual design of aircraft which takes aeroelastic effects into consideration. Instead of the strength design method, the stiffness design method has been introduced for the wing design, namely deformation control[2].

In the preliminary design stage, the stiffness distribution of wing can be obtained by empirical formulas, based on a currently existing reference wing. This conventional design method may cause unnecessary reiteration in the design procedure because of its lack of precision. A more precise and rational stiffness distribution can be obtained by optimization method with multiple constraints[2,3], such as deformation, flutter and strength constraints. However, the relationship between stiffness distribution and aeroelastic performance needs to be studied so that general laws could be summarized to give comprehensive suggestions for the design. Moreover, because of the lack of adequate information in the preliminary design stage, only the beam-frame model can be built to perform optimization procedure which can not take strength constraints into consideration. So, it is also necessary to find out what influence the strength constraints have on the stiffness distribution.

For the abovementioned problems, a beam-frame model and a 3-D model of a high-aspect-ratio wing are introduced. Based on sensitivity analysis and genetic/sensitivity based hybrid optimization algorithm, the importance information of wing stiffness and skin thickness with respect to different constraints is analyzed for two models, respectively. The relationship between stiffness distribution and aeroelastic performance of high-aspect-ratio wings is investigated. Furthermore, the influence of different constraints on stiffness distribution is determined by comparing the optimization results with different key constraints for the 3-D model.

2 METHODOLOGY

2.1 Static aeroelastic response analysis

The basic equation for static aeroelastic response analysis is generally represented as follows[4]:

$$(\mathbf{K}_{aa} - \bar{q}\mathbf{Q}_{aa})\mathbf{u}_a + \mathbf{M}_{aa}\ddot{\mathbf{u}}_{aa} = \bar{q}\mathbf{Q}_{ax} + \mathbf{P}_a \tag{1}$$

where ***K*** is the structure stiffness matrix, ***u*** is the displacement vector, ***Q*** is the aerodynamic influence coefficient matrix, $\bar{q}$ is the dynamic pressure, ***M*** is the structure mass matrix, and ***P*** is the vector of applied loads. The subscript a denotes the displacement vector set of *a*-set, namely analysis set, and the subscript *x* denotes the displacement vector set of *x*-set, namely additional aerodynamic points set.

2.2 P-k method of flutter analysis

The fundamental equation for modal flutter analysis by p-k method is generally stated as follows :

$$\left[\left(\frac{V}{b}\right)^2 p^2\mathbf{M}_{hh} + \frac{V}{b}p\mathbf{B}_{hh} + \mathbf{K}_{hh} - \frac{1}{2}\rho V^2\left(\mathbf{Q}_{hh}^R + \frac{p}{k}\mathbf{Q}_{hh}^I\right)\right]\mathbf{u}_h = 0 \tag{2}$$

where *V* is the velocity, *b* is the reference length of semi-chord, *p* is the eigenvalue, ***B*** is the damping matrix, and *k* is the reduced frequency. The subscript *h* denotes the model analysis set of *h*-set, the superscript *R* denotes the real part, and *I* denotes the imaginary part.

2.3 Aeroelastic sensitivity analysis

Design sensitivity is introduced to calculate the variation rate, or first derivative, of a particular response quantity with respect to a variation of a given structural parameter, or design variable. Through sensitivity analysis, key design variables that have significant effects on the design objective can be determined to decide the design scheme[5]. Analytical and finite difference methods are two common sensitivity analysis methods. According to the characteristics of aeroelastic issues [6, 7], a hybrid analysis method based on analytic and finite difference methods is introduced for sensitivity analysis.

The sensitivity of multiple constraints with respect to every design variable can reflect their relative importance[8]. Generally, the importance of different design variables is different for the same constraint. Moreover, importance of a design variable for different constraints is also different[9]. The importance information of every design variable to different constraints is calculated by Eq. (3) in this study.

$$V_{IPij} = \frac{\bar{S}_{ij}}{\max\{|\bar{S}_{i1}|, |\bar{S}_{i2}|, \cdots |\bar{S}_{in}|\}} \tag{3}$$

where $i = 1 ,2 , \ldots, m$; $j = 1 ,2 , \ldots, n$. *VIPij* represents the importance information of the *j*th design variable with respect to the *i*th function, $\bar{S}_{ij}$ represents the design sensitivity of the *i*th function to the *j*th design variable. Since the design sensitivity is varied in the whole design space, several design points can be randomly generated. Then, the corresponding design sensitivity can be figured out and $\bar{S}_{ij}$ is considered as the average value. Moreover, *m* denotes the sum of objective functions and constraint functions, while *n* denotes the sum of design variables.

2.4 Optimization method

The aeroelastic optimization problem can be written as a standard optimization problem, that is, searching for a set of design variables to minimize the objective function $F(v)$, and meet the conditions as follows:

$$g_j(\boldsymbol{v}) \leq 0 \quad j = 1, \ldots, n_{\mathrm{con}} \tag{4}$$

$$(\boldsymbol{v}_i)_l \leq v_i \leq (\boldsymbol{v}_i)_u \; i = 1, \ldots, n_{\mathrm{dv}} \tag{5}$$

where v is the design variable vector and n_{con} denotes the number of constraints in Eq. (4) which are the aeroelastic performance constraints or strength constraints. Furthermore, in Eq. (5), v_i is the *i*th design variable, $(v_i)_l$ denotes the lower boundary of the *i*th design variable, $(v_i)_{\mathrm{u}}$ denotes the *i*th upper boundary of the design variable, and n_{dv} denotes the number of design variables.

A hybrid optimization algorithm based on genetic and sensitivity algorithms is introduced for aeroelastic optimization, because of its excellent adaptability and high efficiency[10,11]. The flowchart of this optimization procedure is showed in Figure 1.

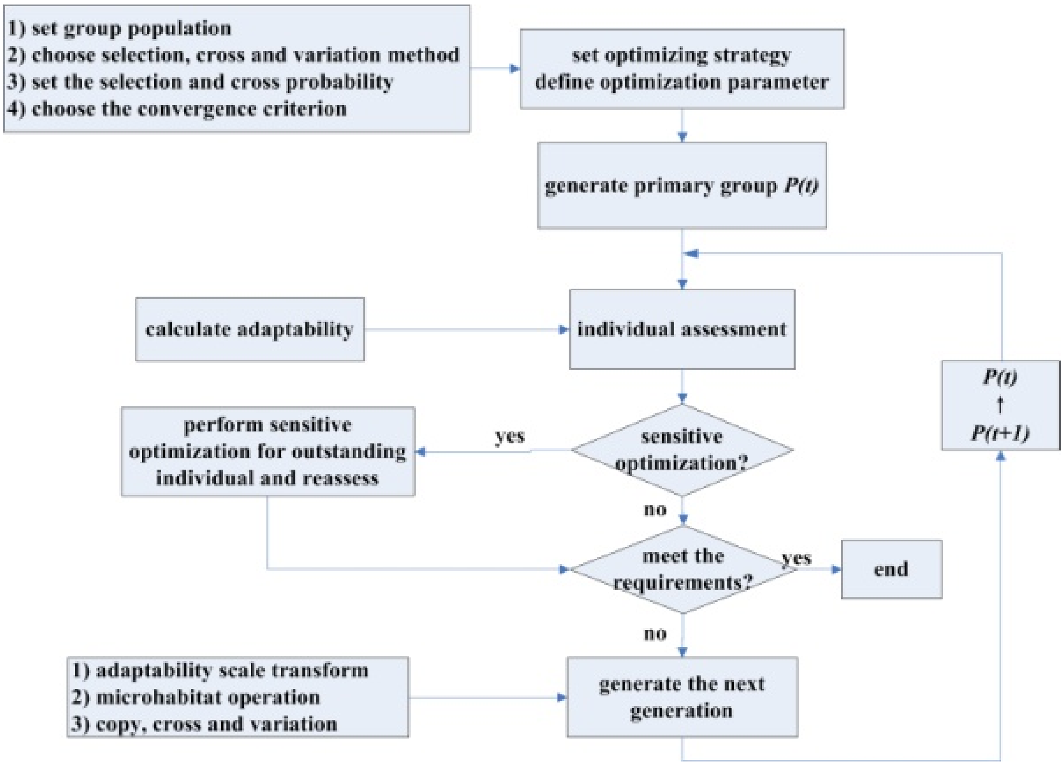

Figure 1: Flowchart of optimization procedure

3 NUMERICAL MODEL

3.1 Structural model

Beam-frame models and 3-D box models are often used to design the stiffness distribution of the wing. A beam-frame model can conveniently and directly represent the stiffness characteristics while detailed structure information is not available. But the strength constraints may not be considered for this model. However, a 3-D model can comprehensively take multiple constraints into account, especially the strength constraints. But the modeling and analysis process is correspondingly complicated. Both the abovementioned finite element models are built, as shown in Figure 2 and Figure 3.

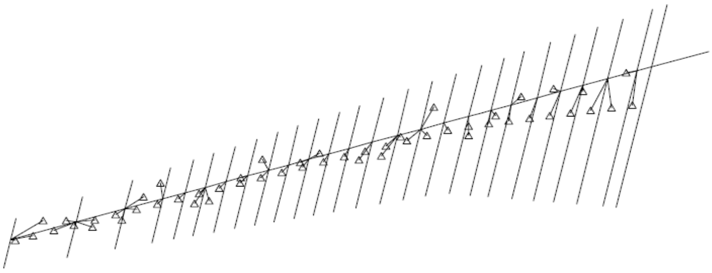

Figure 2: Beam-frame model

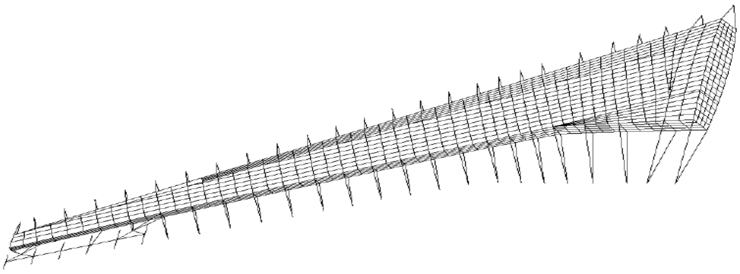

Figure 3: 3-D model

The subsonic double lattice method is used for static aeroelastic analysis to calculate steady aerodynamic force and for flutter analysis to calculate unsteady aerodynamic force. The aerodynamic model has 6 aerodynamic surfaces which is shown in Figure 4

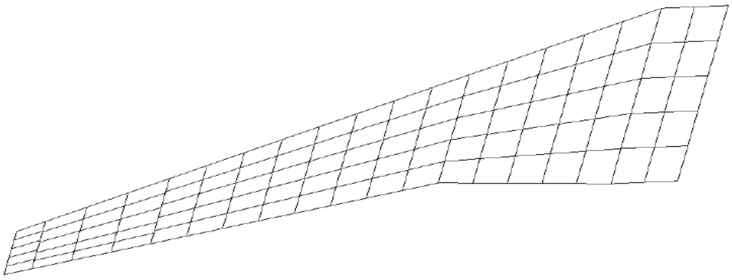

Figure 4: Aerodynamic model

3.2 Objective function

The weighted sum of torsional and vertical bending stiffness at each station is introduced as the objective function to be minimized for the beam-frame model, and the objective function to be minimized for the 3-D model is the total structural weight.

3.3 Constraint conditions

Because the strength constraints could not be considered in the beam-frame model, only aeroelastic constraints are considered in its optimization. In accordance with the design requirements, the load cases and constraints are specified as follows:

1. lift trim under 2.5 g pull-up with $u/l \le 14\%$ and $|\varphi| \le 2.5^\circ$, where φ is the twist angle at the wing tip, u is the vertical displacement at wing tip, and l is the length of half wingspan.
2. flutter speed at sea level is not less than 320 m/s.

For the 3-D model, besides the abovementioned constraints, the strength constraints should also be met, that is, the maximum stress of the upper and lower skin cannot exceed the allowable values. Meanwhile, the minimum size of defined skin cannot be lower than the allowable value due to the fabrication requirement [12].

3.4 Design variables

For the beam-frame model, the main beam is divided into 26 parts. The initial values of torsional and vertical bending stiffness at each position of the wing are estimated as baseline values based on a reference wing, which is a currently existing similar type [2,3]. During the optimization, torsional and vertical bending stiffness of wing at each position are respectively described as product of their baseline values and scaling coefficients called scaling coefficient of torsional or bending stiffness. Accordingly, these 52 scaling coefficients are adopted as the design variables for the beam-frame model. The domain of these design variables are defined from 0.9 to 1.3.

The material of the 3-D model is metal. The upper and lower skin are divided into 25 zones, respectively. The thicknesses of these 50 zones are made design variables for the 3-D model.

4 SENSITIVITY ANALYSIS AND OPTIMIZATION RESULTS OF THE BEAM-FRAME MODEL

4.1 Aeroelastic response of the wing with baseline stiffness

For the initial model with baseline stiffness, the twist angle at wing tip is 2.63° and the vertical displacement at wing tip is 13.8% of the half wingspan at the limit load case (2.5 *g* pull-up lift trim). The flutter speed of the wing with root fixed is 305.3m/s at sea level, and the coupled modes are wing first bending and first torsional modes, in which wing first torsional mode is the traversing branch in *v-g* plot.

Therefore, it is indicated that the responses of the twist angle at wing tip and the flutter speed cannot meet the optimization constraints.

4.2 Importance of various design variables with respect to the twist angle at wing tip

For the convenience of analysis, the sensitivity information is given at every 10% of the half wingspan from wing root to wing tip, and the wing is artificially divided into three regions. The inner region is defined as the region between the wing root and the position at 40% of half wingspan, the outer region is defined as the region between the position at 80% of half wingspan and the tip, while the middle region is defined as the region between the positions at 40% and 80% of half wingspan.

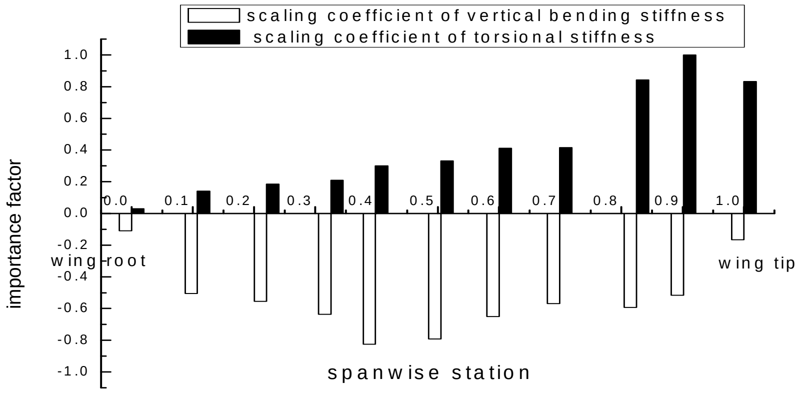

Figure 5: Importance of various variables of beam-frame model with respect to

twist angle at wing tip

The importance of various variables to the twist angle at wing tip for the beam-frame model is shown in Figure 5. It is indicated that the twist angle at wing tip is mainly affected by the vertical bending stiffness of the middle region and the torsional stiffness of the outer region. It is negatively correlated with the vertical bending stiffness and positively correlated with the torsional stiffness.

Because the elastic axis of a swept wing is swept aft, when a bending deformation is induced by upward loads, the section in the direction of the streamwise airflow exhibits a torsional deformation with an upward deformation of the trailing edge relative to the leading edge. Thus, reducing the vertical bending stiffness will increase the bending deformation and the twist angle at wing tip. However, since the aerodynamic center is located in front of the elastic axis, the aerodynamic moment leads to an upward deformation of the leading edge relative to the trailing edge. So reducing the torsional stiffness will decrease the twist angle at wing tip. Compared to the twist angle caused by bending deformation, the component caused directly by torsional deformation is relatively small.

For all regions of the wing, the importance of vertical bending stiffness to the twist angle at wing tip doesn't differ that much like torsional stiffness, but the value of stiffness in middle and outer regions is much lower than the inner region. Therefore, in order to satisfy the same constraint of twist angle at wing tip, a lighter structural weight can be obtained by increasing the vertical bending stiffness of the middle and outer regions rather than that of the inner region.

4.3 Importance of various design variables with respect to the vertical displacement at wing tip

The importance of various variables to the vertical displacement at wing tip for the beam-frame model is shown in Figure 6, which indicates that the vertical displacement at wing tip is negatively correlated with both of the vertical bending and torsional stiffness. It is mainly affected by the vertical bending stiffness of inner and middle regions, while not so much by the torsional stiffness.

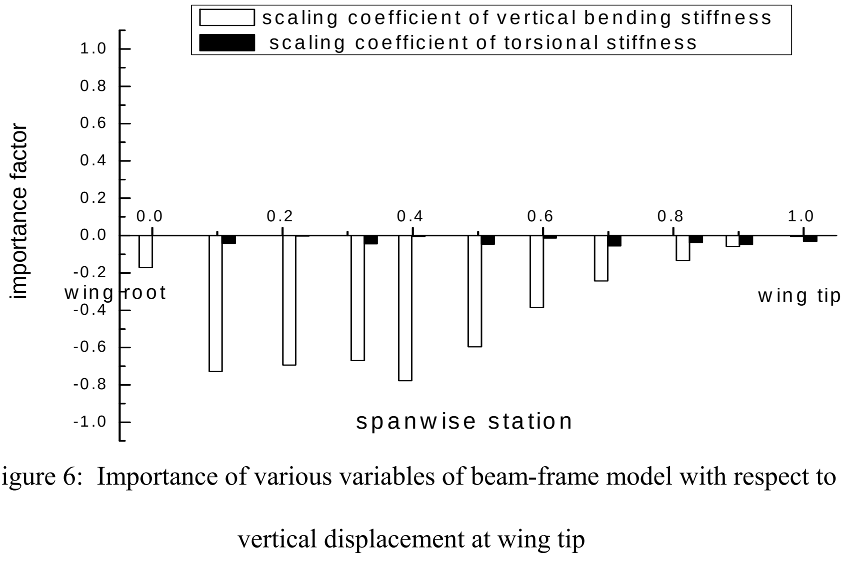

Figure 6: Importance of various variables of beam-frame model with respect to

vertical displacement at wing tip

4.4 Importance of various design variables with respect to the flutter constraint

Figure 7 shows the importance of various design variables with respect to the g value of the flutter traversing branch in v-g plot at 320m/s, which should be reduced to less than zero to meet the flutter constraint. It can be seen from the figure that the g value is positively correlated with the vertical bending stiffness and negatively correlated with the torsional stiffness. Furthermore, the torsional stiffness of the middle and outer regions has the most important influence on the g value. An efficient and economical way to improve flutter characteristics is increasing the torsional stiffness of the middle and outer regions.

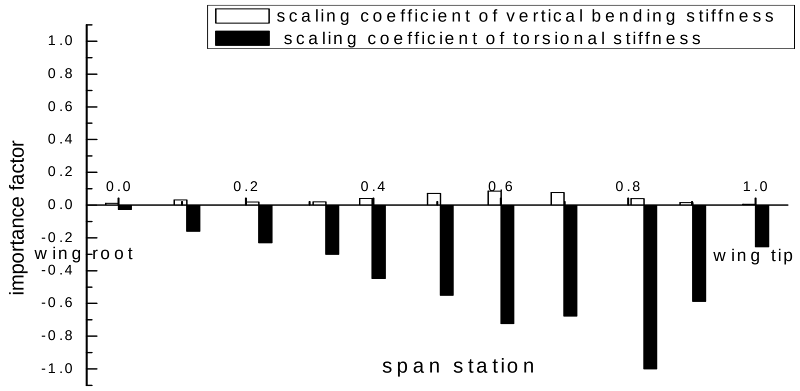

Figure 7: Importance of various variables of beam-frame model with respect to *g* values of flutter branch in *v-g* plot at 320m/s

4.5 Design guidance for the baseline model

Aeroelastic analysis for the baseline model in section 4.1 show that the twist angle at wing tip and flutter speed of the baseline model don’t satisfy the constraints. According to conclusions made from section 4.2 to 4.4, by increasing the vertical bending stiffness of the middle region or decreasing the torsional stiffness of the outer region, the twist angle at wing tip can be decreased; by increasing the torsional stiffness of the middle and the outer region, the flutter characteristics can be improved.

4.6 Optimization results

The scaling coefficients of torsional and vertical bending stiffness of the optimal solution are displayed in Figure 8. It is indicated that these scaling coefficients are almost close to the lower boundary of their domain in the inner region of the wing. At the middle region of the wing, these scaling coefficients raise to their upper boundary gradually. At the outer region, the scaling coefficients of bending stiffness are still close to the upper boundary, while the scaling coefficients of torsional stiffness decrease for the sake of reducing the twist angle at wing tip.

It can also be found that the aeroelastic characteristics of the wings are mainly affected by the stiffness of the middle and outer regions.

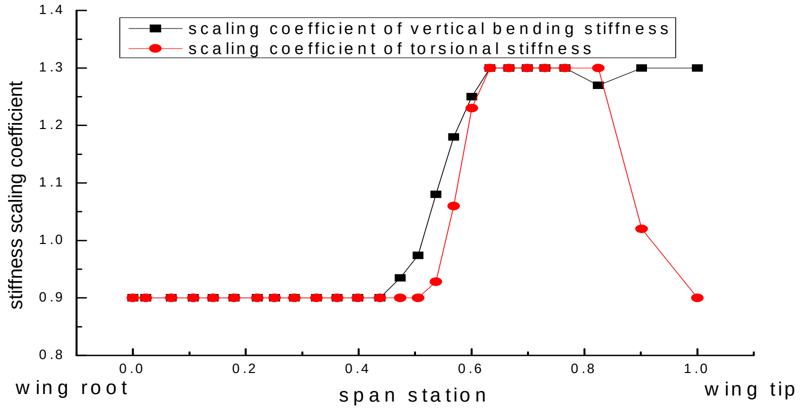

Figure 8: Stiffness scaling coefficients of optimal solution for beam-frame model

The optimal stiffness distribution is well consistent with the design guidance, thus proving the sensitivity analysis and optimization procedure correct and effective.

5 SENSITIVITY ANALYSIS AND OPTIMIZATION RESULTS OF THE 3-D MODEL

In order to further study the influence of aeroelastic constraints on structure design of various wing parts, a 3-D box model is built to take the strength constraints into consideration. The correlative investigation is performed by sensitivity analysis and comparison of optimization results with and without aeroelastic constraints.

5.1 Importance of various design variables with respect to the twist angle at wing tip

The importance of various variables with respect to the twist angle at wing tip for the 3-D model is shown in Figure 9. It is indicated that the twist angle at wing tip is mainly affected by the skin thickness of the middle region, and negatively correlated with the skin thickness of the inner and middle regions and positively correlated with the outer region. To decrease the twist angle at wing tip, the most efficient and economical way is to increase the skin thickness of the middle region. The trend reflected in Figure 9 is basically consistent with that in Figure 5.

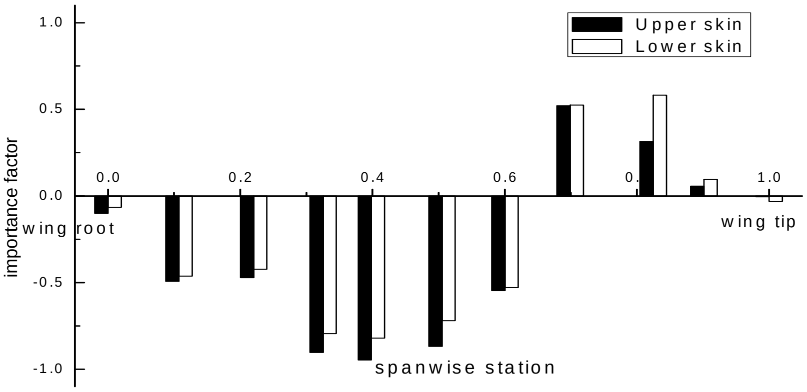

Figure 9: Importance of various variables of 3-D model with respect to twist angle at wing tip

5.2 Importance of various design variables with respect to the vertical displacement at wing tip

The importance of various variables with respect to vertical displacement at wing tip for the 3-D model is shown in Figure 10, which indicates that the vertical displacement at wing tip is negatively correlated with skin thickness. It is mainly affected by the skin thickness of the inner and middle regions.

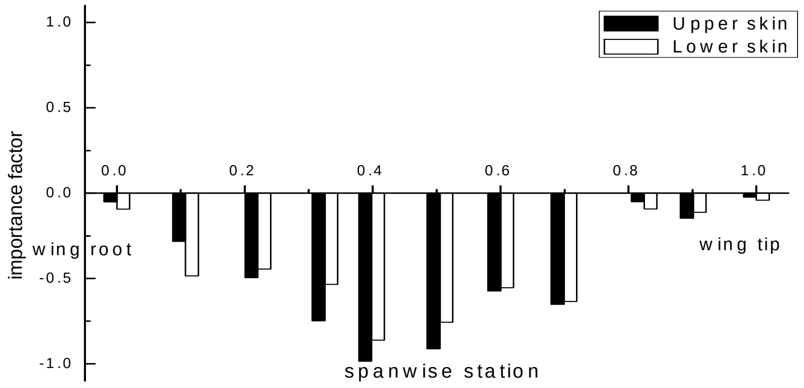

Figure 10: Importance of various variables of 3-D model with respect to vertical displacement at wing tip

5.3 Importance of various design variables with respect to the flutter constraint

Figure 11 shows the importance of various design variables with respect to the g value of flutter traversing branch in v-g plot at 320m/s. It can be seen from the figure that the g value is negatively related with skin thickness and mainly affected by the skin thickness of the middle region. The results are well consistent with that of the beam-frame model.

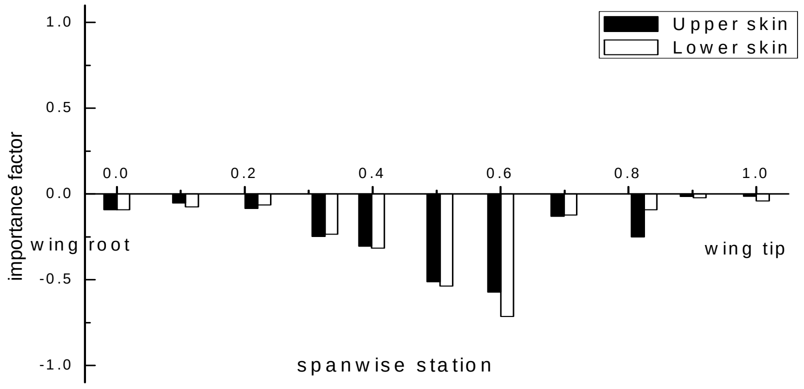

Figure 11: Importance of various variables of 3-D model with respect to g values of flutter branch in v-g plot at 320m/s

5.4 Comparison of optimization results with and without aeroelastic constraints

By using genetic/sensitivity hybrid algorithm based optimization procedure, two optimization cases are carried out. Only strength constraints is considered in the first case, while both strength and aeroelastic constraints are considered in the other one. Comparison between results of two cases is carried out.

Comparison between the two results in terms of aeroelastic response is shown in the Table 1. It is indicated that the design requirement of the twist angle at wing tip and the flutter speed cannot be met by just considering the strength constraints. The stiffness of middle and outer regions should be increased to meet these requirements according to the conclusions drawn by Figure 5 and 7.

	Strength constraints only	Both strength and aeroelastic constraints
Displacement at wing tip（percentage of half wingspan）	11.8%	10.4%
Twist angle at wing tip	3.0°	2.5°
Flutter speed	312.5m/s	321.0m/s

Table 1: Comparison of aeroelastic response of optimization results with different constraints

Figure 12 and 13 show comparison between the results of two cases in terms of skin thickness distribution. Compared with the one only considering the strength constraints, an obvious increase of the skin thickness in the middle region is obtained by taking aeroelastic constraints into consideration, while the difference is small in the inner and outer regions. It is illustrated that the strength constraints play a major role in determining the skin thickness of inner region, while aeroelastic constraints have major influence on skin thickness of the middle region. In the outer region, the main restrict comes from manufacturing technology constraints. The optimal wing stiffness distribution is also well consistent with the former conclusions made by sensitivity analysis.

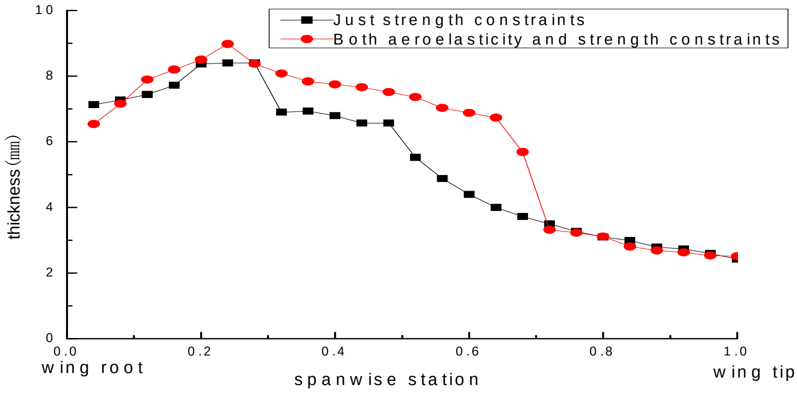

Figure 12: Comparison of thickness distribution of upper skin

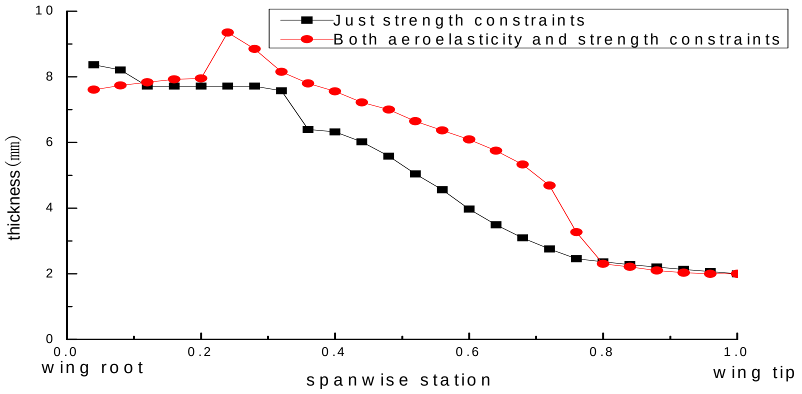

Figure 13: Comparison of thickness distribution of lower skin

6 CONCLUSIONS

The relationship between stiffness distribution and aeroelastic performance for a beam-frame model and a 3-D model is studied based on aeroelastic optimization of global stiffness design for high-aspect-ratio wings. Through sensitivity analysis and optimization comparison, the following conclusions could be drawn:

1. The twist angle at wing tip is mainly affected by the vertical bending stiffness of the middle region and the torsional stiffness of the outer region. By increasing the vertical bending stiffness of the middle region or decreasing the torsional stiffness of the outer region, the twist angle at wing tip can be decreased. The vertical displacement at wing tip is mainly influenced by the vertical bending stiffness of the inner and middle region of the wing. By means of increasing the vertical bending stiffness of the middle region, the vertical displacement at wing tip can be decreased without gaining too much weight. The flutter speed is mainly influenced by the torsional stiffness of the middle and the outer region, thus increasing the torsional stiffness in these regions can increase the flutter speed.

2. The stiffness distribution in the middle region of the wing is mainly influenced by aeroelastic constraints such as vertical displacement at wing tip, twist angle at wing tip and flutter speed. As for the structure design in the inner region of the wing, great attention should be paid to the strength constraints.

3. Stiffness distribution design laws summarised from sensitivity analysis for high-aspect-ratio wings and the results from aeroelastic optimization have good consistency, thus proving the design laws and the optimization procedure convincible. Either the beam-frame model or the 3-D model can be chosen to perform aeroelastic optimization based on the amount of given information, the requirement of efficiency and the type of constraints. The optimal stiffness distribution is more reasonable than the conventional estimation method.

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