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INFLUENCE OF STRUCTURAL DAMPING ON AIRPLANE DYNAMIC LOADS AT FLIGHT IN TURBULENCE AND AT RUN

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Abstract: Structural damping is important in dynamic loading of airplane. In order to show it, this paper provides a brief description of the developed technique. It is based on engineering schematization of airplane structure, modeled by the system of crossed non-uniform beams, acting in bending and torsion. These beams carry the distributed masses and the concentrated ones and the inertia moments. Thus, displacements of airplane are arranged in a number of the products of natural mode shapes by the generalized coordinates. The aerodynamic forces are defined based on the theory of trailing vortex sheet by S.M. Belotserkovsky method.

Structural damping is introduced as an additive component into a diagonal of aerodynamic damping matrix.

As an example, the regional aircraft weighing about 40 tons and with two engines on the wing is considered. Special attention is given to vertical and lateral accelerations acting at the engine center of gravity. The action of the continuous turbulence is considered. The loading at run on aerodrome cyclic irregularities is explored also. This is done by varying the values of structural damping, including those obtained in the ground vibration tests. The loads maxima are determined.

1 INTRODUCTION

The airplane while in service is often treated to act of dynamic loads, i.e., the loads caused by elastic structure vibrations. It concerns, first of all, to flight in turbulent air, to landing and run on aerodrome irregularities. It is possible to indicate as well of a blade loss of the jet engine, leading to its imbalance and also causes dynamic response of structure.

Loads gained thus frequently are defining general strength of an airplane, and also that is especially important for structure fatigue life, i.e. for its resource.

Indicated cases are especially important for heavy airplanes, therefore, they are introduced as rated cases to the deeds regulating strength of transport category airplanes CS-25 (Europe), FAR-25 (USA) and AR-25 (Russia).

Elastic vibrations basically decay under influence of two forces of various natures. First of all, it is air forces caused by velocities of elastic deformations. There are many theories (stationary, nonsteady, subsonic, transonic, etc.) and their reliability is justified enough.

The nature of the second damping force is related to a friction in a material and in various joints in the structure. Here, also, there are a number of theories, but, according to the author, their justification leaves much to be desired. Therefore, at calculations of dynamic loads it is necessary to take interior friction as equivalent viscous friction (i.e. proportional to deformation's velocity), and to take the damping coefficients from regulation documents or to update them on the basis of experiment.

The purpose of this paper is to display how various levels of damping influence on dynamic loads and how it is possible to reduce their maxima by means of refinement of this damping.

2 DESIGN PROCEDURE

The airplane structure is schematized by a system of elastic crossed beams which can twist and bend in two planes – so called "stick model". They bear the distributed masses and the inertia moments. Engines, landing gear and a number of other heavy objects are modeled by lumped objects having six elastic degrees of freedom.

The natural frequencies and shapes of the structure are determined on the basis of the engineering theory. The solution for linear z and rotational ϑ displacements is given in form

$$z(y,t)=\sum_{i=1}^{N} f_i(y)q_i(t), \qquad \vartheta(y,t)=\sum_{i=1}^{N}\varphi_i(y)q_i(t), \tag{1}$$

where $f_i(y)$ and $\varphi_i(y)$ - joint natural mode shapes of bending and torsion;

$q_i(t)$ - generalized coordinates with respect to time;

N – number of modes used in the calculation.

Using the Lagrange equations, it is possible to gain combined equations

$$\ddot{q}_i + 2h_i\dot{q}_i + p_i^2 q = \frac{1}{c_{ii}}Q_i \quad i=1,2,\ldots,N\,, \tag{2}$$

where h_i – structural damping coefficient for the i-th mode,

p_i– natural frequency,

c_{ii} – generalized mass,

Q_i – generalized force that represents the external forces work to the virtual displacements, i.e. the natural mode shapes.

In what follows, we consider the case of a flight in turbulence, which is used for definition of aerodynamic forces by S.M. Belotserkovsky method. According to this method, lifting surfaces are broken down into a number of elementary panels. Inside them the singularities in the form of bound vortices are arranged.

Determine the distribution of aerodynamic forces from gusts and vibrations of the structure on each mode, we can find the generalized forces Q and come to the equations normally used in solving problems of aeroelasticity

$$\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{H}\dot{\mathbf{q}} + \mathbf{B}\mathbf{q} + \mathbf{K}\mathbf{q} = \mathbf{R}_U U(t), \tag{3}$$

where **D** and **B** - full matrices of aerodynamic damping and stiffness,
H and **K** - matrices of structural damping and structural stiffness, along the diagonals of which damping coefficients and squares of natural frequencies are,
$\mathbf{R}_U$ - right hand side vector, defined as the work of forces from the gust.

Equations (3), written in the time domain, correspond to the case of gust action. Integration of this system of equations (3) with respect to time (for example, by the Runge-Kutta) allows to get the changes of generalized coordinates **q** and their first and second derivatives and, thus, to determine dynamic loads in the transition process. For example, the dynamic vertical load factor in a cross section *y* has view as

$$N_z(y,t) = \frac{1}{g}\sum_{i=1}^{N} f_i(y)\,\ddot{q}_i(t)\,, \tag{4}$$

where g - acceleration of gravity.

In the case of flight in continuous turbulence it is required to find modules of transfer functions (amplitude-frequency characteristics), i.e. to find a steady response to a unit harmonic gust $U(t)$ with frequency ω in the form of:

$$U(t)=1\cdot e^{i\omega t}\,. \tag{5}$$

The solution is located in a complex kind

$$\mathbf{q}(t) = \left[\overline{\mathbf{q}}(i\omega) + i\overline{\overline{\mathbf{q}}}(i\omega)\right]e^{i\omega t}\,, \tag{6}$$

here $i = \sqrt{-1}.$

After substitution of expression (6) in (3) we have a system of algebraic equations

$$\begin{bmatrix} \mathbf{B}+\mathbf{K}-\mathbf{E}\omega^2 & -\omega(\mathbf{D}+\mathbf{H}) \\ \omega(\mathbf{D}+\mathbf{H}) & \mathbf{B}+\mathbf{K}-\mathbf{E}\omega^2 \end{bmatrix}\begin{bmatrix} \overline{\mathbf{q}} \\ \overline{\overline{\mathbf{q}}} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{R}}_U \\ \overline{\overline{\mathbf{R}}}_U \end{bmatrix}. \tag{7}$$

The solution to this system for each frequency is performed by known methods, for example, by Gauss method. Knowledge of amplitude-frequency characteristics of each of generalized coordinate $q_i(\omega)$ and its derivatives $\dot{q}_i(\omega)\,,\ddot{q}_i(\omega)$ allows to find amplitude characteristics of loads. In particular, for the vertical load factor we have

$$|\,N_z(y,\omega)\,| = \frac{1}{g}\sqrt{\left(\sum_{i=1}^{N} f_i(y)\overline{q}_i(\omega)\right)^2 + \left(\sum_{i=1}^{N} f_i(y)\overline{\overline{q}}_i(\omega)\right)^2}\,. \qquad 8)$$

These modules are then used to find spectral densities of loads

$$Sd_N(\omega) = U_\sigma \Phi(\omega)\,|\,N(\omega)\,|^2\,, \tag{9}$$

where $\Phi(\omega)$ and U_σ - defined in the regulating documents the spectral density of turbulence and its intensity.

Integrating (9), we obtain the increment of the dynamic load

$$\Delta N = U_\sigma \int_0^{\omega_{end}} \Phi(\omega) \, | \, N(\omega) \, |^2 \, d\omega \,, \qquad (10)$$

where ω_{end} - the upper limit of integration usually taken a little larger then the frequency of the highest mode of elastic vibrations.

According to the given procedure the PC-based computer program is written. How calculations correspond with experiment, see [1-2].

3 ABOUT STRUCTURAL DAMPING

As mentioned in the introduction, enough credible theory of structural damping for current aircraft practically is insufficient. Therefore, the basic information we receive from experiment and especially from Ground Vibration Tests (GVT).

Figure 1 shows structural damping coefficients of the first ten modes, obtained in such tests of a number of airplanes. Also values recommended for use in calculating gust dynamic loads are marked. In the Russian AR-25 [3] g = 0.016 (logarithmic decrement θ = 0.05), whereas in the European Regulations CS-25 [4] g = 0.03 (θ = 0.094) that indicates almost twice the value above.

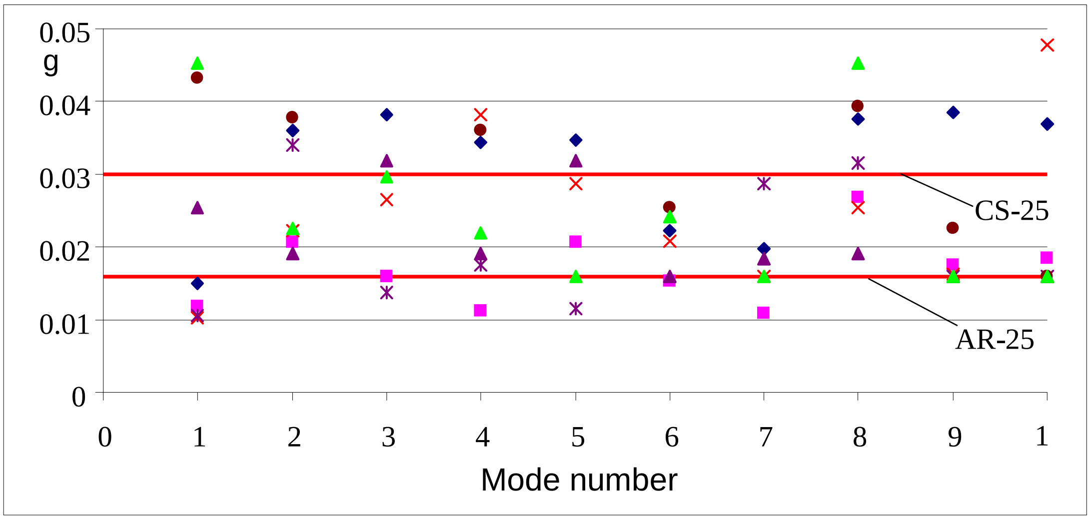


Figure 1: Structural damping of several airplanes

As follows from this figure, the values of g represent a star-shaped field.

The question is: what damping to take into consideration? Let's take an indication of the CS-25 (book 2 AMC 25.341):

"Damping Model Validation. In the absence of better information it will normally be acceptable to assume 0.03 (i.e. 1.5 % equivalent critical viscous damping) for all flexible modes. Structural damping may be increased over the 0.03 value to be consistent with the high structural response levels caused by extreme gust intensity, provided justification is given."

If we assume that information from vibration tests is reliable, then it follows from just given, we must take into account only those factors that are more than 0.03. And if some of them are less than 0.03? Formally, we need to stop at a value 0.03.

But to author's opinion to do so is incorrectly, it is necessary to take tests effects for all modes.

Below on an example of one regional airplane it is shown as various damps influence on load factors of the engine at flight in turbulence. This case gives the most critical dynamic loads for wing-engine mounting strength.

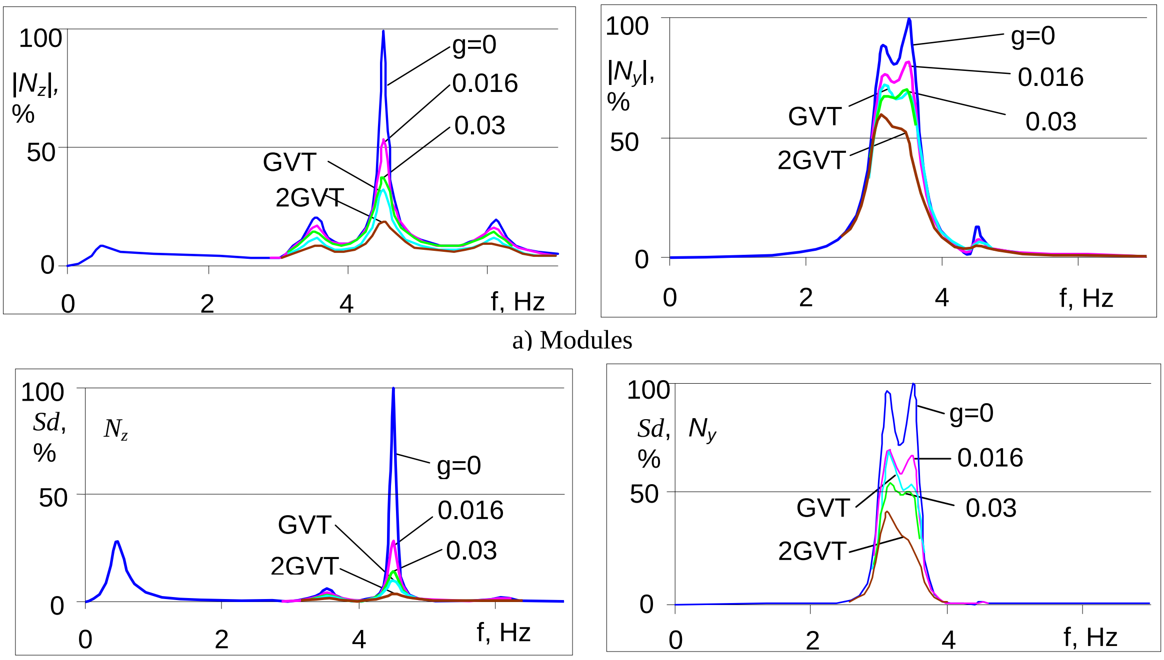
4 EXAMPLE OF CALCULATION

Calculations carried out taking into account the movement of the plane as a rigid body, and the first ten modes of elastic vibrations. It was made for the five variants of structural damping values:

1. At zero damping, i.e. for all modes it was taken $g=0$.
2. At damping $g = 0.016$, taken as a lower bound for the Russian AR-25.
3. At damping $g = 0.03$, satisfying recommendations of European CS-25.
4. At the damping taken from GVT. In following figures this case is labeled as GVT.
5. At damping corresponding to twice the coefficients obtained in vibration tests. This hypothetical variant is labeled as 2GVT, designed to track the trend of loads, if tests give higher (say twice) levels of damping.

In order to obtain a relative assessment of influence of parameters, below all curves in the figures referred to the respective maxima obtained at zero structural damping, or this zero version is taken for 100 %.

Figure 2 shows results for the engine vertical load factor N_z – on the left side and for the lateral N_y – on the right side. In the upper part of Figure 2 there are modules of transfer functions, and in lower part - the spectral densities of loads calculated by formula (9).



a) Modules

b) Spectral densities

Figure 2: Modules and spectral densities of engine loads

It is seen that the damping growth significantly reduces the peaks of curves. Note that in this example, calculations at g=0.03 and at g taken from GVT give almost confluent curves (green and turquoise). It is attributed to the fact that the second and the third flexible modes, matching vertical and lateral engine vibrations and making the main contribution to the engine loading, have coefficients of 0.027 and 0.024, i.e. close enough to set in CS-25.

More clearly the influence of structural damping can be seen in Figure 3, which shows results of integration of spectral densities shown in Figure 2. These integrals are taken by (10) and divided by those received at g=0.

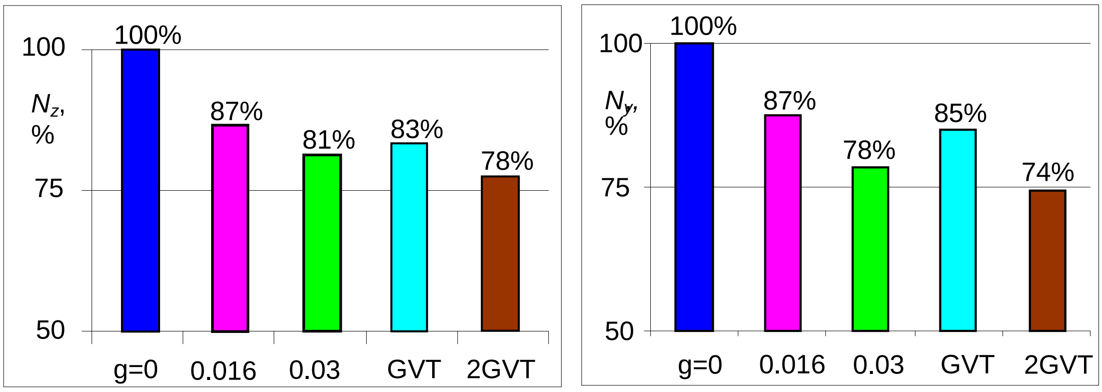


Figure 3: Structural damping effect on maxima of engine loads

It can be seen that introduction of sufficiently small damping 0.016 leads to decrease of engine loads maxima by 13%. On further increase in g the lateral load reacts more strongly, dropping another 9%.

If we compare calculations with g=0.03 and with g taken from GVT, the difference for N_z is only 2%, which is explained in the proximity of spectral density curves. For N_y the difference is a little bigger – 7%, Note that, if at vibration tests we received twice g, then the lowering of loads would be more significant, for lateral engine load up to 36%.

However, not always the introduction of structural damping leads to appreciable decrease of loads. It is noticed that wing bending moments react poorly to g. Figure 4 may be an explanation to it, where percentage of structural damping h_{ii} and aerodynamic damping d_{ii} in diagonals of matrices **H** and **D** of equations (3) is shown. (It is known, these diagonal terms make the main contribution in vibration attenuation of observed modes.)

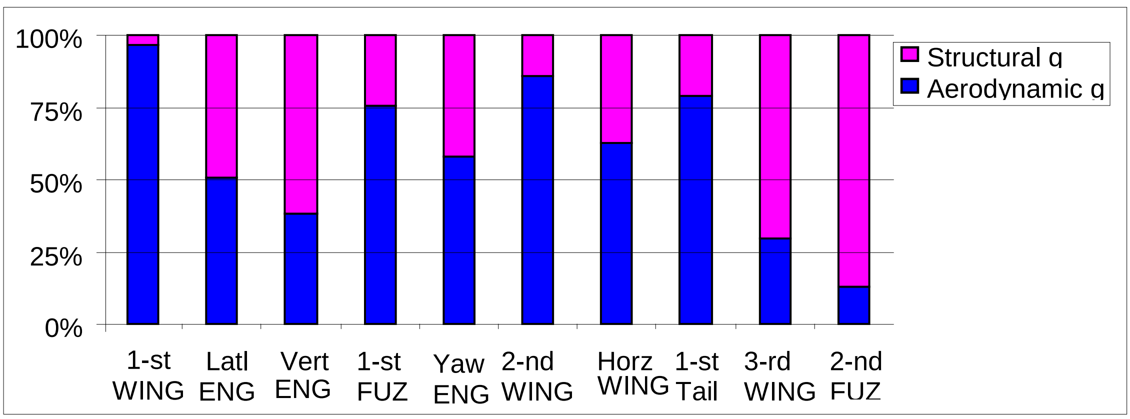


Figure 4: Percentage of structural damping and aerodynamic damping for each vibration mode

The ordinate of this figure has consistently given short names for normal modes.

It can be seen for the first mode the structural damping is only a few percent of the total damping. And since this mode plays a major role in the formation of dynamic bending moments of the wing, it becomes clear why wing moments practically do not react to changes in the relatively low structural damping.

Other is observed for the second and the third modes associated with flexible lateral and vertical engine vibrations. Here, structural damping holds a predominant part. So, its increase immediately affects the reduction of engine loads.

6 CROSSING THE CYCLICAL IRREGULARITIES

Above we considered the case of flight in turbulence, which involved both structural and aerodynamic damping. In order to emphasize the role of only the first one we consider the run on the airfield at relatively low speeds, when aerodynamic forces practically are not formed.

In CS-25 (Book 2 AMC 25.491 p. 5) there is a case that requires the calculation of dynamic loads when an airplane crosses two sequentially going cyclical irregularities (bumps) – Figure 5. The length of each of them is equal to the baseline of landing gears L and also equals the doubled baseline $2L$.

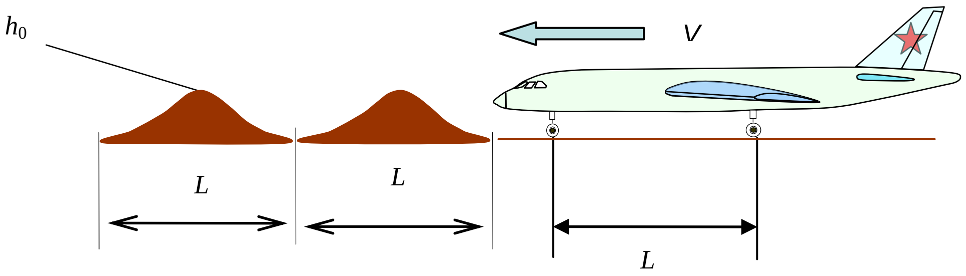


Figure 5: Bumps crossing

The height of each bump h_0 in mm is given by

$$h_0 = 30.5 + 0.116\sqrt{L}\,, \tag{11}$$

where L is determined also in mm. (For current heavy airplanes h_0~50 mm.)

The profile of bump is

$$h(x) = \frac{h_0}{2}\left[1 - \cos(2\pi x / L)\right]. \tag{12}$$

Run is to be observed at velocities from 37 km/h (20 knots) to the maximum velocity of run on aerodrome V_R.

Because of smallness of these velocities in comparison with flight ones, aerodynamic damping is practically negligible. Therefore, attenuation of exited flexible vibrations is realized mainly by structural damping and attenuation of rigid body movements – by the hydraulic force created in landing gears amortization.

This hydraulic force is non-linear; it depends on compression velocity $\dot{s}$ as

$$Q_{\mathrm{hydr}} = \nu\,(s)\;\dot{s}^2\,, \qquad (13)$$

where $\nu(s)$ – the coefficient of hydraulic resistance, which is generally a function of compression (piston stroke) s and takes different values on direct and reverse motion.

The force of gas resistance Q_{gas} caused by gas volume changes is non-linear also

$$Q_{\mathrm{gas}}=p_0 S_p \,/\, (1 - s/H)^{\,\mathrm{k}}, \qquad (14)$$

where p_0 - initial pressure in the air chamber of absorber,
S_p is the area of the piston,
κ - polytropic index.

These two main forces of the amortization are included in the right-hand side of equation (2) and together with the equation of motion of wheel (gear moving parts) form a system of equations that is solved numerically by Runge-Kutta method.

7 EXAMPLE OF CALCULATION

Parametric calculations of crossing bumps by the same airplane as when flying in turbulence were carried out. Parameter was the airplane run velocity, referred to a maximum airplane ground velocity V_R.

$$\overline{V} = V \,/\, V_R \qquad (15)$$

In Figure 6 the dynamic loads on the landing gear divided by static values are shown.

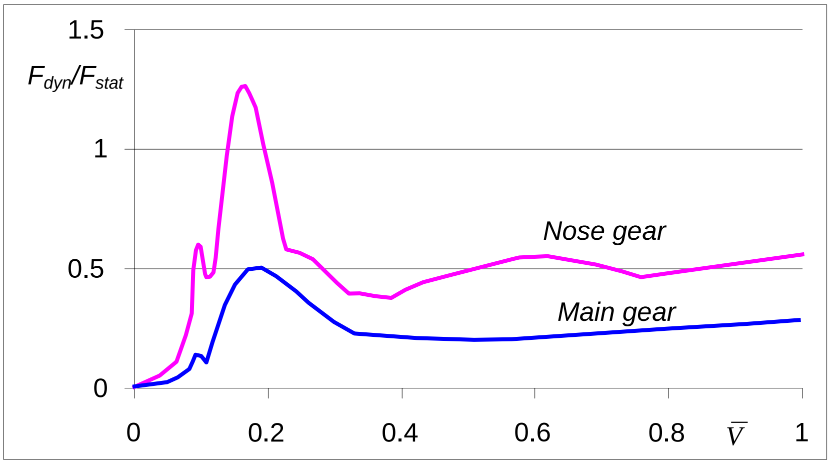
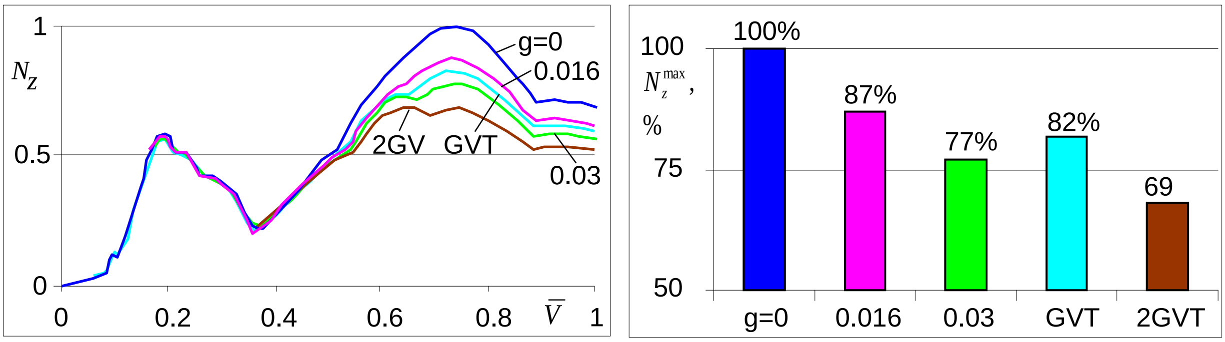


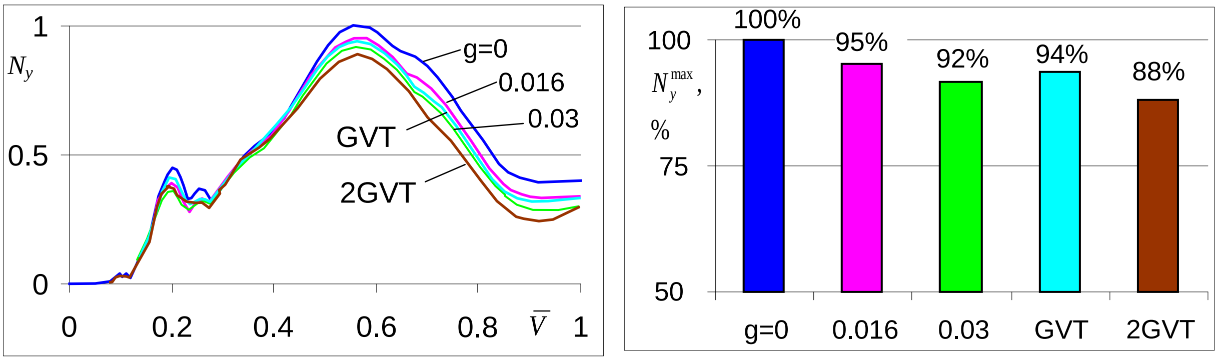
Figure 6: Forces on landing gear

Despite the fact that structural damping varied the forces on landing legs are remained invariable. It is no wonder as flexible structure deformations in sections of attachment of legs are much smaller then displacements of an airplane as rigid body. It may be noted two maxima on curves in Figure 6 which correspond to a swing of an airplane on modes of its vertical and pitch movements.

But engine loads behave differently, as it can be seen in Figure 7.



a) Vertical load



b) Lateral load

Figure 7: Engine loads at bums crossing

The vertical load reacts at changing of structural damping substantially. Note that values g taken from vibration tests lead to smaller load than at g=0.03. Significantly it is reduced by damping corresponding to twice damping from vibration tests.

More compactly curves are settled describing the dependence of the lateral load on the engine. Their maxima occur at lower velocity than that of the vertical load. This corresponds to the excitation of lower modes - engine lateral vibration modes. The gain in reducing the lateral load is also noticeable.

Thus, once again it shows that the increase of structural damping that obtained by vibration tests can lead to the desired reduction in the airplane loading.

8 CONCLUSION

The technique is developed and investigation of structural damping influence on airplane dynamic loads is conducted for cases of flight in turbulence and run.

It is shown that the increase of structural damping at higher vibration amplitudes can essentially reduce the design loads.

It is recommended to use existent methods and to develop new ones and tools for structure excitation at GVT to receive higher amplitudes of various vibration modes.

9 ACKNOWLEDGEMENTS

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