

# FLUTTER AND BUFFETING OF LONG SPAN SUSPENSION BRIDGES IN FULLY ERECTED AND PARTIALLY ERECTED CONDITIONS

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**Abstract:** The work presents a 3D aeroelastic model of a long-span suspension bridge with streamlined deck equipped with leading- and trailing-edge flaps. The objective is to devise a model for the computation of the bridge stability (flutter and torsional divergence) and forced response (buffeting), yet of manageable size to allow for modern control design. The Humber Bridge (UK) is used as a design example. Both fully erected and partially erected conditions are considered. The comparison with previous published data and the results of a preliminary robust controller aimed at increasing the flutter critical velocity and reducing the buffeting are included.

## 1 INTRODUCTION

It is now widely accepted that long span suspension bridges may be prone to aeroelastic phenomena such as flutter and buffeting and an accurate wind resistant design has become the standard [1]. In the quest for super long suspension bridges the use of movable winglets for the suppression of wind induced vibrations has been recently investigated [2],[3]. However most of the published papers on the subject employ basic control schemes and the potential of such strategy may be largely unexplored. In order to leverage the potential of a modern control approach it is necessary to devise a control oriented aeroelastic model of the bridge, i.e. a model of a manageable size and yet sufficient accuracy.

In this work we present a 3D bridge aeroelastic model which build upon previous works on 2D models [4] and show preliminary results on a feedback control based on leading and trailing edge winglets aimed at increasing the flutter critical speed and reducing the buffeting response while maximizing robustness against model uncertainties. A numerical example based on the Humber Bridge (UK) is reported, and the results in terms of flutter critical speed and expected RMS (buffeting response) are compared with the experimental results reported in [5] and [6] for model validation. The model is then used to investigate different partially erected configurations in terms of flutter stability.

In more detail, the structural model is a modified version of the finite element model reported in [7], the cable model is from [8] and allow to account for load distribution of a partially erected deck, the unsteady aerodynamics model is based on the thin aerofoil theory [9],[10] (thus the model targets modern streamlined closed box deck) with a rational approximation of the lift-deficiency (Theodorsen) function. The problem is cast in a state space formulation,

thus the eigenvalues (e.g. the flutter) can be computed directly (i.e. without the iteration of the so-called p-k methods) and the buffeting response results from the combination of the different modal responses.

## 2 STRUCTURAL MODEL

The structural Finite Element (FE) model follows the simplified approach proposed in [7], which provides a significant reduction of the computation burden because it accounts for the effect of the main cables and hangers for every deck element indirectly. In addition, the lateral, vertical and torsional structural modes of vibration are considered uncoupled (of course the aerodynamic forces will introduce a coupling between modes, see next section). In the pure vertical modes, all points of a given cross section move by the same amount and remain in phase. In the torsional case the bridge section rotates about its centre point. For the lateral motion, each cross section swings in a pendulum fashion with an incidental upward movement of the cables and of the suspended structure. Further assumptions include: the initial dead load is carried out by the main cables, the cables take a parabolic profile under dead load (instead of catenary, reasonable assumption when small sag to span ratio as in long-span bridges), the hangers are inextensible and are regarded as continuous, warping is neglected (reasonable assumption with box girders), towers are considered rigid, displacements and rotations during vibration are assumed small. The cable configuration (which is related to the local hangers length) and the horizontal dead load, which are needed to evaluate the stiffness matrix of the suspension bridge finite element, are computed following the three-steps approach suggested in [8]. The horizontal dead force of the inextensible cable is computed first, assuming the sag in inextensible conditions. Second, the increase in the sag and reduction in horizontal dead force related to cable elasticity (stretching) is considered. Finally, the effect of loading due to the bridge deck (transmitted through the hangers) is introduced and the related increase in cable sag and tension is computed. Few iterations of the assumed sag in inextensible conditions are necessary to find the actual sag (with stretching and loading) which is a data usually available. An alternative approach is to start with the length of the cable in inextensible conditions, if available, or as the guess parameters for iteration. This procedure allows to easily account for partially erected conditions, where only a limited section of the cables is loaded. The resulting mass matrix  $M$  and stiffness matrix  $K$  as well as the equations for cables are fully reported in [11]. The structural damping is assumed proportional to the mass and stiffness matrix of the structure:  $C = a_M M + a_K K$ . The coefficients  $a_M$  and  $a_K$  are chosen in order to have a damping ratio of 1% for the first vertical and the first torsional mode.

The vibration modes of the FE model are compared in with the experimental findings reported in [12] for the case of the Humber bridge **Figure 1**. The agreement is good. It is expected that the most important modes for flutter would be the first symmetric torsion (TS) and the first vertical symmetric (VS), while for buffeting the first vertical symmetric (VS). The lateral vibration modes are usually less important. Actually they are not relevant at all when using thin aerofoil theory, which generates only lift and moment terms. However there are cases where these modes are important, see e.g. for the truss-decked Akashi Bridge [13].

Vertical	Exp. [12] (Hz)	Model (Hz)	Lateral	Exp. [12] (Hz)	Model (Hz)	Torsion	Exp. [12] (Hz)	Model (Hz)
VS	0.117	0.122	LS	0.056	0.082	TS	0.311	0.314
VA	0.154	0.116	LA	0.141	0.148	TA	0.482	0.458
VA	0.177	0.179	LA	0.309	0.295	TS	0.650	0.696
VS	0.218	0.213	LS	0.418	0.426			
VA	0.240	0.242	LS	0.518	0.535			
VS	0.310	0.313	LS	0.632	0.630			

Table 1: Vertical (V), Lateral (L) and Torsion (T) modes, Symmetric (S) and Anti-symmetric A).

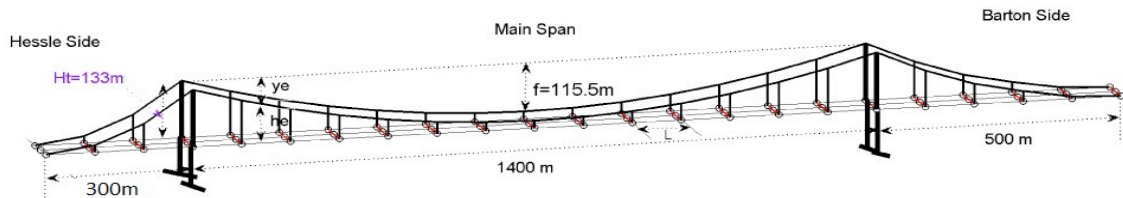


Figure 1: Basic dimensions of the Full Humber Bridge FE model

The deck erection phase is modelled using half of the torsional rigidity while the flexural stiffness is neglected altogether in order to account for the temporary character of the connections [14]. The ratio of the first torsional mode to the first vertical mode is consistent to the findings reported in [14],[15], see Figure 2 (where also the case where the torsional stiffness is not reduced is included).

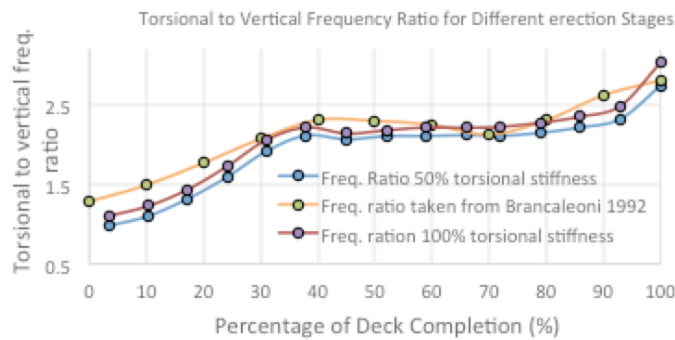


Figure 2: Torsion to bending frequency ratio as a function of deck completion percentage for both the cases of half and full deck torsional stiffness.

### 3 AERODYNAMIC MODEL

The aerodynamic forces consist of the motion dependent (self-excited) forces  $F_{se}$  and the motion independent (buffeting) forces  $F_b$ . The system is written in the standard form:

$$M\ddot{x}_s + C\dot{x}_s + Kx_s = F_{se} + F_b \quad (1)$$

where  $M$ ,  $C$ ,  $K$  are the mass, damping and stiffness matrix of the structure and  $x_s$  contains the structural variables, i.e. the displacements and rotations of the FE model.

The expressions of the self-excited forces are from the thin aerofoil theory [9],[10], and depend on the irrational lift-deficiency (Theodorsen) function  $C(k)$ :

$$L_{se} = -\pi\rho b^2 \left[ U\dot{\alpha} + \ddot{h} \right] - 2\pi\rho U b C(k) \left[ U\alpha + \dot{h} + \frac{\dot{\alpha}b}{2} \right] \quad (2)$$

$$M_{se} = -\pi\rho b^2 \left[ \frac{bU\dot{\alpha}}{2} + \frac{b^2\ddot{\alpha}}{8} \right] + \pi\rho U b^2 C(k) \left[ U\alpha + \dot{h} + \frac{b\dot{\alpha}}{2} \right] \quad (3)$$

$$C(k) = \frac{J_1(k) - iY_1(k)}{(J_1(k) + Y_0(k)) - i(Y_1(k) - J_0(k))} \quad (4)$$

where  $\rho$  is the air density,  $b$  the deck half-chord width,  $U$  the wind velocity,  $h$  and  $\alpha$  are the heave and pitch,  $J_0, J_1, Y_0, Y_1$  are Bessel function of the first and second kind respectively,  $i$  is the imaginary number and  $k$  is the reduced frequency  $k = \omega b/U$ , with  $\omega$  the circular frequency. These expressions should be used only for streamlined deck sections (e.g. Humber Bridge, Severn Bridge, Great Belt Bride), while for truss-decked bridge the flutter derivatives approach introduced in [16] is more appropriate.

It is worth noting that the non-circulatory components of forces (i.e. those not depending on  $C(k)$ ) do not pose particular issues when introduced into the structural FE model. Indeed they result in an aerodynamic mass and an aerodynamic damping matrices which subtract from the structural ones. On the contrary the circulatory terms depends on the irrational function  $C(k)$ , which does not allow to derive a standard state space formulation with frequency independent matrices. The problem is overcome when  $C(k)$  is replaced by a Rational Function Approximation (RFA). In this work the fourth order RFA of  $C(k)$  reported in [11] is used:

$$C(k) = \frac{\sum_{n=0}^4 b_n (ik)^n}{\sum_{d=0}^4 a_d (ik)^d} \quad (5)$$

Employing the concepts of analytic continuation,  $ik$  can be replaced by  $s' = sB/U$  in Eq.( 5 ), where  $s$  is the Laplace variable. In practice, the expression derived for oscillatory motion is extended to use with arbitrary motion. The problem can now be cast in state space terms with frequency independent matrices. The formulation is now suitable for stability analysis, forced analysis and modern controller design:

$$\begin{cases} \dot{x} = Ax + BF_b \\ y = Cx \end{cases} \quad (6)$$

where the state vector  $x$  consists of the structural states  $x_s$ , their derivative (because the model is written in a first order form), and the additional aerodynamic states  $x_a$  related to the RFA. Since there is a different  $C(k)$  for each of the  $n_e$  element of the FE model and the RFA is

fourth order, there are  $n_e \times 4$  additional aerodynamic states. It is worth noting that when using the flutter derivatives approach with fourth order RFAs of the related terms, there are  $n_e \times 4 \times 2$  additional aerodynamic states (in the formulation depending only on heave and pitch), because heave and pitch have now different circulatory aerodynamics (the additional states are even more when including the effect of the sway motion in the flutter derivative formulation). Further analysis is included in [11].

It is noted that it has been implicitly assumed that the self-excited forces on the  $i$ -section depend only the flow field at the same  $i$ -section (in practice the model is the combination of several 2D section models). This is the so-called “strip-theory”, which is a standard and widely accepted assumption when it comes to self-excited forces. The same assumption is often employed also for buffeting forces, however in this case it is known that the assumption holds valid only when the length scale of turbulence is much larger than the width of the deck [17]. Indeed the coherence of buffeting lift forces is larger than the coherence of turbulence velocities [18],[19], but this effect can be neglected when the (spanwise) length scale of turbulence is much larger than the deck chord. It has been also pointed out that this may not be true with (the lower end of) the range of length scales that are typically experienced by long-span suspension bridges [20]. However a recent investigation [21] showed that when considering sections with large aspect ratios (such as those typical of long-span suspension bridges), the increase in the coherence of buffeting forces is balanced by the reduction in their magnitude. As a result, in these cases (section with large aspect ratios) the “strip theory” can be used regardless the length scale of turbulence, i.e. even if the ratio of the length scale of turbulence to the deck width is not large.

The general expressions for buffeting lift and moment take the following form [1]:

$$L_b = \frac{1}{2} \rho U^2 (2b) \left( 2C_L \chi_{Lu} \frac{u}{U} + (C_L' + C_D) \chi_{Lw} \frac{w}{U} \right) \quad (7)$$

$$M_b = \frac{1}{2} \rho U^2 (2b)^2 \left( 2C_M \chi_{Mu} \frac{u}{U} + C_M' \chi_{Mw} \frac{w}{U} \right) \quad (8)$$

where  $u$  and  $w$  are the fluctuations of longitudinal and vertical velocities related to turbulence,  $C_L$ ,  $C_M$ ,  $C_D$  are the lift, moment and drag coefficients,  $C_L'$  and  $C_M'$  the derivatives of the lift and moment coefficients, and the  $\chi$  terms are the aerodynamic admittances which account for the reduction of forces related to unsteadiness. In the case the thin aerofoil theory is employed also for buffeting forces, it is  $C_L=C_M=C_D=0$ ,  $C_L'=\pi$ ,  $C_M'=\pi/2$  and  $\chi_{Lw}=\chi_{Mw}$  consists of the irrational Sears function [22],[10] or its widespread Liepmann approximation [23].

The spectral response is computed with the following approach, which is repeated at all simulated velocities. Firstly the modal formulation is obtained from Eq.( 6 ), using the eigenvalues/eigenvectors already computed for the stability analysis:

$$\begin{cases} \dot{q} = \Lambda q + B_q F_b \\ y = C_q q \end{cases} \quad (9)$$

where  $q$  consists of the modal coordinates and  $\Lambda$  is the matrix with eigenvalues. Since the response is usually dominated by the first  $m$ -modes, only the first  $m$ -eigenvalues/eigenvectors are retained to derive Eq.( 9 ). Secondly the forced response is derived:

$$q = (I\omega - \Lambda)^{-1} B_q F_b \quad (10)$$

where  $\omega$  is the forcing frequency. Thirdly the spectrum of the modal coordinates  $q$  is obtained as a function of the spectrum of buffeting forces  $S_{FF}$ :

$$S_{qq} = (I\omega - \Lambda)^{-1} B_q S_{FF} B_q^H (I\omega - \Lambda)^{-H} \quad (11)$$

The spectrum of the observed variables is computed from the spectrum of the modal coordinates:

$$S_{yy} = C_q S_{qq} C_q^H \quad (12)$$

Note that this is the multimodal response.

The spectrum of forces  $S_{FF}$ , which is necessary to compute  $S_{qq}$  and thus also the spectrum of displacements  $S_{yy}$ , is now derived. The buffeting forces can be written in terms of the nodal velocities  $u$  and  $w$ , when assuming a certain shape function for the distribution of the  $u$  and  $w$  within the element:

$$F_b = \frac{1}{2} \rho U^2 (A_u u + A_w w) \quad (13)$$

where  $A_u$  and  $A_w$  are the FE buffeting matrices having as many rows as the  $A$  in Eq.( 6 ) and as many columns as the number of nodes of the structure. The spectrum of buffeting forces is computed from:

$$S_{FF} = \left( \frac{1}{2} \rho U^2 \right)^2 \left( A_u S_{uu} A_u^H + A_u S_{uw} A_w^H + A_w S_{wu} A_u^H + A_w S_{ww} A_w^H \right) \quad (14)$$

which clearly depends on the spectrum of velocities on the FE nodes, namely the spectrum of longitudinal velocity  $S_{uu}$ , the spectrum of vertical velocity  $S_{ww}$  and the co-spectrum  $S_{uw}=S_{wu}^*$  (usually neglected).

The most important spectral component is  $S_{ww}$  (which is the sole necessary when assuming thin aerofoil behaviour). The (double sided) Busch-Panofsky spectrum [1],[24] is herein used:

$$S_w(\omega) = \frac{2\pi}{\omega} u_*^2 \frac{3.36 \left( \frac{\omega z}{2\pi U} \right)}{1 + 10 \left( \frac{\omega z}{2\pi U} \right)^{5/3}} \frac{1}{4\pi} \quad (15)$$

where  $z$  is the height from ground,  $u_*$  is the friction velocity,  $\omega$  is the circular frequency. The expression can be alternatively written in term of the variance  $\sigma_w$  of (turbulent) vertical velocities. It is easily found by integration that  $\sigma_w^2 \approx 1.67 u_*^2$ . However different alternative formulations exist in the literature. It is noted that the roll off rate for  $\omega \gg 1$  is  $-5/3$ , which is related to the so-called Kolmogorov inertial subrange [25],[26] where the turbulence can be considered isotropic and the viscous dissipation can be neglected, being the anisotropy mainly related to large eddies (i.e. small wavenumbers) and viscous dissipation mainly related to very small eddies (i.e. very large wavenumbers). Typical models for  $S_{uu}$  are the Hino's and von Karman's [1]. Anyway  $S_{uu}$  is neglected in this work for simplicity.

The correlation of the turbulent vertical velocities is modelled with an exponential function [1],[27], and thus the cross-spectrum of wind components between two points is:

$$S_{w_1 w_2} = S_w e^{-cL_{12} \frac{\omega}{2\pi U}} \quad (16)$$

where  $S_w$  is from Eq.( 15 ),  $c$  is the correlation coefficients and  $L_{12}$  is the distance between the two points considered. Note that the diagonal of  $S_{ww}$  in Eq.( 10 ) consists of  $S_w$  in Eq.( 15 ), while the out-of-diagonal terms consist of Eq.( 16 ).

The expected response RMS is finally computed by integration of the (double-sided) spectrum as:

$$\sigma_y = \sqrt{\int_{-\infty}^{+\infty} S_{yy} d\omega} \quad (17)$$

#### 4 STABILITY: FLUTTER & TORSIONAL DIVERGENCE

The stability analysis is performed by computing the eigenvalues of the state space matrix  $A$  in Eq.( 6 ). The computation is repeated at different wind velocities. The system is stable as long as all eigenvalues have negative real part. An obvious essential requirement is that the bridge must be stable for speeds greater than the ones it will experience when in place.

Note that other traditional approach such as the so-called “k-method” and the “p-k-method” [28] require repeated evaluation of  $C(k)$ , and one must seek for one resonance at a time while distinguishing between single and coupled modes. The use of an RFA is therefore particularly convenient, since it allows for an effective and efficient multimodal analysis of the system.

The left hand side of **Figure 3** depicts the root locus of the Humber Bridge, with the wind velocity swept from 0 to 85m/s. At 0m/s the aeroelastic modes consist of the structural modes (magenta) and are purely vertical, purely lateral and purely torsional. As the wind velocity increases the vertical and torsion modes couple as a result of the aerodynamic forces, while the lateral modes remain unaffected (because the thin aerofoil theory produces only lift and pitching moment). In particular the first structural torsional mode reduces its frequency and includes heave components as the wind velocity increases. The damping first increases (the complex conjugate eigenvalues move towards the left) and then reduce its damping becoming unstable at 65m/s (flutter critical speed), in agreement with the experimental estimation of [5]. At 74m/s there is a second instability: the torsional divergence, which is the result of the (negative) aerodynamic moment fully balancing the (positive) structural elastic moment. Note that the torsional diverge is non-vibrating and therefore presents itself as a pure real eigenvalue. The inspection of the (complex) flutter mode shape shows that its components consist mainly of the first torsion symmetric mode, the first vertical symmetric mode and the second vertical symmetric mode. The right hand side of **Figure 3** shows the critical flutter velocity and torsional divergence velocity for different percentage of deck erection, with comparison with the findings reported in [14]. The erection sequence starts from the mid-point of the suspended span and progresses symmetrically sideways, which is the most frequent technique used in practice for suspension bridges.

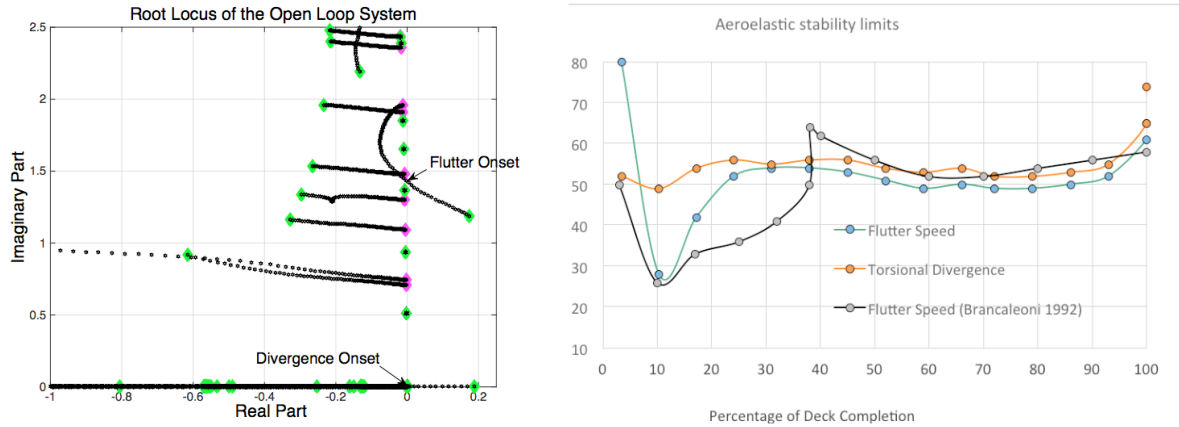


Figure 3: Left: root-locus of the Humber Bridge for wind velocities from 0 to 85m/s. Right: flutter critical wind speed for different erection stages.

### 5 FORCED RESPONSE: BUFFETING

Buffeting is the forced response induced by the wind turbulence on a structure: the wind turbulence generates aerodynamic forces that excite the structural vibration modes which in turn result in vibrations. Assuming wind turbulence as a random process, we make use of a spectral approach in which the structural vibration can be considered as the combination of three main elements: the turbulence spectrum, the aerodynamic admittance and the mechanical admittance (i.e. structural response).

Figure 4 shows the comparison between the maximum vertical RMS  $\sigma_z$  (normalized by the bridge deck width  $2b$ ) of the simulated response and experimentally identified response reported in [6] (which fitted the results of [29]). The variance of the vertical turbulence has been assumed  $\sigma_z/U=0.07$  (i.e. turbulence intensity of 7%),  $C_L'$  has been reduced from the aerofoil value  $2\pi$  to the value 2.5, identified in [5]. The agreement is again good.

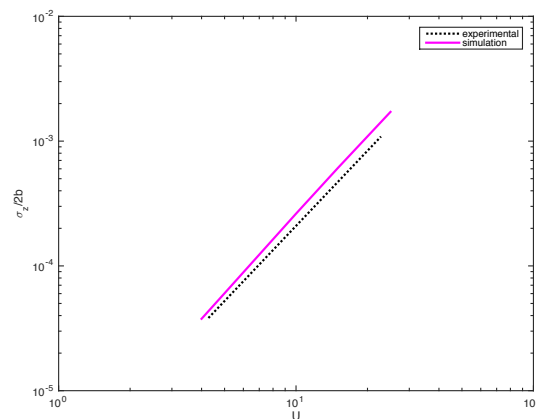


Figure 4: Simulated and experimental [6] buffeting vertical RMS for wind velocities from 4 to 25m/s.



## 6 BRIDGE CONTROL WITH MOVABLE FLAPS

The flutter and buffeting analysis above showed that the numerical model, although implementing the simple thin aerofoil theory, is in good agreement with the experimental findings. Therefore the effect of leading- and trailing-edge flaps can be easily accounted for after the wing-aileron-tab combination [30] is transformed into a flap-deck-flap combination [11]. These movable flaps can be used to increase the flutter critical velocity and/or improving the buffeting response. From a different perspective one can envision the possibility of using a more economical structure while maintaining a sufficiently high flutter critical velocity using movable flaps, or increasing the length of the suspended span while balancing the reduction in the critical velocity using the flaps. The use of movable flaps has been initially proposed in [31]. Other studies are reported in [32]-[34]. Both active and passive (i.e. not requiring power supply) solutions have been proposed. However the problems related to the robustness of the devised controllers against model uncertainties have been rarely addressed in the previous works related to bridge control using movable flaps.

In this paper we show the results of a preliminary controller which increases the flutter critical speed up to the torsional divergence while maximizing the robustness against (multiplicative) perturbation of the model [35]. In this preliminary study we the following 2<sup>nd</sup> order compensator  $K(s)$  is optimized, which generates a flap angle  $\beta$  from the deck pitch angle  $\alpha$ :

$$K(s) = \frac{\beta}{\alpha} = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + 1} \quad (18)$$

The main span is equipped with 25 flaps. The maximum gain of  $K(s)$  is limited to 15, to avoid excessive flow separation. The width of the flap is assumed to be 3m. The flap angle at section  $j$  ( $j=1..25$ ) depends on the deck pitch at the same section  $j$ . The leading- and trailing-edge compensator are the same (actually anti-symmetric) in order to have a configuration insensitive to wind direction. The related root locus is depicted on the left of **Figure 5**, together with the buffeting response. It is clear that the controller stabilizes the system in the whole speed range (stability objective) while reducing at the same the buffeting response (performance objective) almost at all speed (however this depends on the weights used during the optimization, namely the weight of the buffeting cost with respect to the weight related to robustness cost). It is noted that, as expected, the buffeting response of the openloop system, i.e. the bridge without controller, goes to infinity as soon as the flutter critical speed is reached, right of **Figure 5**. Further research is being carried out to design the controller while finding the minimum amount of flaps necessary for the stabilization with good robustness.

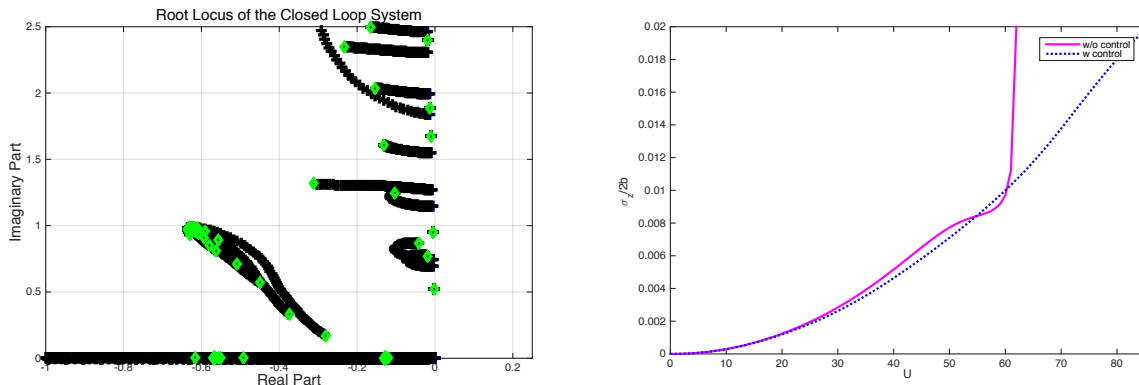


Figure 5: Root-locus (left) and buffeting response (right) of the Humber Bridge for wind velocity from 0 to 85m/s when the main span is fitted with a symmetric 2<sup>nd</sup> order compensator.

## CONCLUSION

A 3D aeroelastic model of a long-span suspension bridge equipped with leading- and trailing-edge flaps along the main span has been presented. Numerical results for the Humber Bridge has been computed and compared with those published in the literature, finding good agreement. The flutter and torsional divergence stability limits have been computed both in fully erected and partially erected conditions. A feedback controller based on the deck pitch angle has been designed and simulated. This resulted in an increase in flutter critical speed and reduction in buffeting response.

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## REFERENCES

- [1] Y. Fujino, K. Kimura, H. Tanaka, "Wind Resistant Design of Bridges in Japan", Springer, 2012.
- [2] K. Wilde, Y. Fujino, "Aerodynamic control of bridge deck flutter by active surfaces," Journal of Engineering Mechanics, vol. 124, pp.718-727, 1998.
- [3] P. Omenzetter, K. Wilde, Y. Fujino, "Study of Passive Deck-Flaps Flutter Control System on Full Bridge Model", Journal of Engineering Mechanics, 2002.
- [4] J.M.R. Graham, D.J.N. Limebeer, X. Zhao, "Aeroelastic Control of Long-Span Bridges", Journal of Applied Mechanics, Vol. 78, 2011.
- [5] G. Diana, F. Cheli, A. Zasso, A. Collina, J. Brownjohn, "Suspension Bridge Parameter Identification in Full Scale Test", Journal of wind engineering and industrial aerodynamics, pp. 165-176, 1992.
- [6] G. L. Larose, A. G. Davenport, and J. P. C. King, "Wind Effects on Long Span Bridges: Consistency of Wind Tunnel Results", Journal of Wind Engineering and Industrial Aerodynamics, 41-44 (1992) 1191-1202.
- [7] H. M. Abdel-Ghaffar, "Dynamic analyses of suspension bridge structures", California Institute of Technology, 1976
- [8] H. Max Irvine, "Studies in the statics and dynamics of simple cable systems", California Institute of Technology, 1974.
- [9] T. Theodorsen, General Theory of Aerodynamic Instability and the Mechanisms of Flutter. NACA, Report No. TR-496, 1934.
- [10] R.J. Bisplinghoff, A. Ashley, R.L. Halfman, "Aeroelasticity", Dover, 1996.
- [11] K.N. Bakis, M. Massaro, M.S. Williams, D.J.N. Limebeer, "Aeroelastic Control of Long-Span Suspension Bridges with Controllable Winglets", Engineering Structures, under review
- [12] J.M.W. Brownjohn, M. Boccione, A. Curami, M. Falco, A. Zasso, "Humber bridge full-scale measurement campaigns 1990-1991," Journal of Wind Engineering and Industrial Aerodynamics, vol.52, pp.185-218, 1994.
- [13] H. Katsuchi, N.P. Jones, R.H. Scanlan, Multimode coupled flutter and buffeting analysis of the Akashi-Kaikyo Bridge, J. Struct. Eng. 1999, 125:60-70.

- [14] Brancaleoni, F., "The construction phase and its aerodynamic issues," in *Aerodynamics of large bridges*, A. Larsen, ed., Balkema, Rotterdam, the Netherlands, 147-158, 1992.
- [15] Y. Ge, H. Tanaka, "Aerodynamic stability of long-span suspension bridges under erection," *Journal of Structural Engineering*, vol. 126, pp. 1404-1412, December 2000.
- [16] R.H. Scanlan, J.J. Tomko, "Airfoil and bridge deck flutter derivatives", *Journal of Engineering Mechanics Division*, pp. 1717-1737, 1971.
- [17] A.G. Davenport, The response of slender, line-like structures to a gusty wind, in: *Proceedings of the Institution of Civil Engineers*, Vol. 23, November 1962, pp. 389-408.
- [18] B. Etkin, *Dynamics of Atmospheric Flight*, Wiley, New York, 1972, pp. 547-548.
- [19] S. Li, M. Li, H. Liao, "The lift on an aerofoil in grid-generated turbulence", *J. Fluid Mech.* (2015), vol. 771, pp. 16-35.
- [20] G.L. Larose, "The spatial distribution of unsteady loading due to gusts on bridge decks", *Journal of Wind Engineering and Industrial Aerodynamics* 91 (2003) 1431-1443.
- [21] M. Massaro, J.M.R. Graham, "The effect of three-dimensionality on the aerodynamic admittance of thin sections in free stream turbulence", *Journal of Fluids and Structures*, 2015".
- [22] W.R. Sears, "Some Aspects of Non-Stationary Airfoil Theory and Its Practical Application", *Journal of the Aeronautical Sciences*, Vol. 8, No. 3, 1941, pp. 104-108.
- [23] H.W. Liepmann, "On the Application of Statistical Concepts to the Buffeting Problem", *Journal of the Aeronautical Sciences*, 19(12), 1952.
- [24] N.E. Busch, H.A. Panofsky, "Recent spectra of atmospheric turbulence", *Quarterly Journal of the Royal Meteorological Society*, 94: 132-148, 1968.
- [25] A.N. Kolmogorov, "The local structure of turbulence in incompressible viscous fluids for very large Reynolds number", *Dokl. Akad. Nauk SSSR*, 1941.
- [26] E. Simiu, R.H. Scanlan, "Wind Effects on Structures", 3rd Ed., John Wiley & Sons, 1996.
- [27] M.C. H Hui, Q.S. Ding, Y.L. Xu, "Buffeting Response Analysis of Stonecutters Bridge", *HKIE Transactions*, 12:2, 8-21, 2005.
- [28] D.H. Hodges, G.A. Pierce, "Introduction to structural dynamics and Aeroelasticity," *Cambridge Aerospace Series*, 2002
- [29] J.M.W. Brownjohn, M. Boccione, A. Curami, M. Falco, A. Zasso, "Humber bridge full-scale measurement campaigns 1990-1991," *Journal of Wind Engineering and Industrial Aerodynamics*, vol.52, pp.185-218, 1994.
- [30] T. Theodorsen, I.E. Garrick, "Nonstationary Flow about a Wing-Aileron-Tab Combination Including Aerodynamic Balance" *NACA Report*, TR-736, 1942.
- [31] K. Ostenfeld, A. Larsen, "Bridge Engineering and Aerodynamics", in: A. Larsen (Ed.), *Aerodynamics of Large Bridges*, A.A. Balkema, Rotterdam, Holland, pp. 3-22, 1992.
- [32] S.D. Kwon, M.S.S. Jung, S.P. Chung, "A new passive aerodynamic control method for bridge flutter", *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 48, pp 261-285, 1993.
- [33] P. Omenzetter, K. Wilde, Y. Fujino, "Study of Passive Deck-Flaps Flutter Control System on Full Bridge Model. II: Results", *Journal of Engineering Mechanics*, pp. 280-286, March 2002.

- [34] H. Aslan and U. Starossek, "Passive Control of Bridge Deck Flutter Using Tuned Mass Dampers and Control Surfaces", presented at the 7th European Conference on Structural Dynamics, Southampton, UK, 2008.
- [35] M. Green and D.J.N. Limebeer, Linear Robust Control, Dover, 2012.

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