Static aeroelastic analysis including geometric nonlinearities based on Reduced Ordered Model (ROM)

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Abstract: In this paper, a method is proposed for modelling large deflection of wing by developing efficient reduced order model (ROM). The method can be used to solve the static aeroelastic problems of wing containing geometric nonlinearities and static aeroelastic trim problems of flexible aircraft containing geometric nonlinearities. Traditional methods of aeroelastic analysis can't reflect the nonlinear characteristics of structures, its result can't satisfy the precision demand of engineering analysis. The approach for structure modelling presented here is based on a combined modal/finite element (MFE) approach that describes the stiffness nonlinearities. We apply that structure modelling method as ROM to aeroelastic analysis. The results show static aeroelastic analysis of wing and static aeroelastic trim analysis of aircraft based on structure ROM can achieve a good agreement compared to analysis based on the finite element method (FEM) and experiment result. The methods in this paper is suitable for the preliminary design and the static aeroelastic analysis of the flexible aircraft with large-aspect-ratio wing efficiently.

1 INTRODUCTION

There has been a growth of interest in the development of highly flexible unmanned aerial vehicle (UAV) aircraft. As the representative of the highly flexible UAV, high altitude long endurance (HALE) aircraft usually has the characteristics of light weight and large flexibility. The wing will produce large deformation during the flight and structural stiffness behaves in a nonlinear manner due to changing of geometric stiffness. The traditional aeroelastic analysis method of small deformation assumption can't meet the accuracy requirements of the calculation results. At the same time, significant elastic deformation of wing will bring significant changes in aerodynamics configuration and stiffness characteristics of the airplane, which makes the aeroelastic problem of large flexible aircraft. For the design requirement of HALE, Hodges, Cesnik and Patil proposed the concept of fixed-wing aircraft geometrically nonlinear aeroelastic in 1999[1-2]. Then, lots of research with diverse content of geometrically nonlinear aeroelastic problem where large flexible HALE as the main object has been carried out.[3-8] In the past of years, Zhang Jian et al.[9] have studied the effect of structural geometric nonlinearities on the static and dynamic aeroelastic characteristics of large –aspect-ratio wings. Xie Changchuan et al.[10-11] investigates the aeroelastic characteristics of a metal wing under large deformation with structure nonlinear finite element method (FEM) and generalized strips theory, flutter characteristics of the wing are also predicted by linearized method. Trim analysis is the essential parts of the aircraft design progress. Patil[1] and Zhang[9] developed a methodology by combing the nonlinear beam model and the ONERA aerodynamics model. Wang Libo et al.[12] proposed a method combied the nonlinear FEM and nonplanar vortex lattice method by iterative progress.

Nonlinear FEM is used to calculate the stiffness of model and displacement under aerodynamics in aeroelastic analysis considering geometric nonlinearities usually. Compared to FEM, reduced order model (ROM) can reduce the scale of the problem and can analyze the characteristics of large flexible aircraft geometrics nonlinearity easily. There have been a number of approaches development to use commercial finite element software to obtain the ROMs. In particular the modal/FE (MFE) approach of McEwan et al.[13] performs a static analysis for a number of prescribed static loading cases. These displacements are then curve fitted using a regression analysis in order to define nonlinear stiffness terms that can be added to the conventional modal model. J.E.Cooper et al.[14] implied the MFE approach to model the geometric nonlinearity of the Patil and Hodges HALE wing model. Having obtained the nonlinear ROM, aeroelastic analysis was performed by coupling the structural model with the rational function approximation (RFA) of doublet lattice aerodynamics using Roger method. This approach can be used to predict the static deflection, gust response and limit cycle oscillation.

This paper improves the MFE approach as ROM implied to solve aeroelastic problems, makes it more fully reflect the influence of large deformation of wing under load in the analysis of aeroelastic. The prescribed load cases with follower force and corresponding displacements from the static nonlinear FEM analysis are transformed into modal coordinates using the modal transformation of the underlying linear system where more of useful modes has been applied to participate in modelling. A regression analysis is performed to curve-fit the sets of test load and nolinear displacement maps in order to find the unknown nonlinear stiffness coefficient . When structural ROM obtained, it is introduced to solve structure displacement considering geometric nonlinearity. A nonplanar vortex lattice method[12] is used to compute the nonplanar aerodynamics. Moreover, surface spline interpolations method[15] is used for structure/ aerodynamics coupling. Finally, solve the static aeroelastic displacement by iterative method. For instance, the static aeroelastic analysis of wing including geometric nonlinearity is carried out by ROM structure solutions, nonlinear FEM solutions. Results obtained by these two methods and experiment results are also compared. Then, imply the approach to the flexible aircraft model to solve the nonlinear trim problem by iterative method combining the equilibrium equations of rigid motions of the deformed aircraft. For instance, the trim analysis of a flexible aircraft is carried out by ROM structure solutions and nonlinear FEM solutions. Results obtained by these two methods are also compared.

2 FORMULATION

2.1 Nonlinear ROM

2.1.1 Nonlinear structure equation[13]

Consider the case of an initially straight, geometrically nonlinear beam with mass proportional damping subject to forced vibration. Structural equation of motion in physical co-ordinates for forced vibration in the transverse direction is of the form

$$[\mathbf{M}]\{\ddot{\mathbf{w}}(x,t)\}+\alpha[\mathbf{M}]\{\dot{\mathbf{w}}(x,t)\}+[\mathbf{K}_L]\{\mathbf{w}(x,t)\}+[\mathbf{K}_{NL}(\mathbf{w}(x,t))]=\{\mathbf{F}(x,t)\} \tag{1}$$

Here $\{\mathbf{w}(x,t)\}$ is the transverse deflection vector, $[\mathbf{M}]$ is the mass matrix, α is the coefficient for damping, $[\mathbf{K}_L]$ is the linear stiffness matrix, $[\mathbf{K}_{NL}]=[\mathbf{K}_{NL}(\mathbf{w}(x,t))]$ is the assembled nonlinear stiffness matrix where the stiffness is dependent upon displacement, and $\{\mathbf{F}(x,t)\}$ is the external force vector.

The spatial and temporal components of the beam motion can be separated by expressing the equations of the beam motion can be separated by expressing the equations of motion in terms of modal amplitudes as

$$\{\mathbf{w}(x,t)\}=[\phi(x)]\{\mathbf{p}(t)\} \tag{2}$$

Here $\{\mathbf{p}(t)\}$ is a time-dependent vector of modal amplitudes, and $[\phi]$ is a time-independent modal matrix of the N underlying linear mode $\{\phi(x)\}_r, r=1,2,\ldots,N$, which may be obtained by solving the eigenvalue problem for free vibration:

$$[\mathbf{K}_L]\{\phi(x)\}_r = w_{Lr}^2[\mathbf{M}]\{\phi(x)\}_r, r=1,2,\ldots,N \tag{3}$$

Here w_{Lr} is the linear natural frequency of r mode. Substituting the truncated modal expansion into the system equations of motion and pre-multiplying by $[\phi]^T$, then we can get nonlinear structural equation in modal co-ordinates:

$$[\phi]^T[\mathbf{M}][\phi]\{\ddot{\mathbf{p}}(t)\}+\alpha[\phi]^T[\mathbf{M}][\phi]\{\dot{\mathbf{p}}(t)\}+[\phi]^T[\mathbf{K}_\mathrm{L}][\phi]\{\mathbf{p}(t)\}+[\phi]^T[\mathbf{K}_{NL}(p(t))][\phi]=[\phi]^T\{\mathbf{F}(t)\} \tag{4}$$

Upon completion of the modal transformation the new system equations of motion in modal space are

$$[\mathbf{A}]\{\ddot{\mathbf{p}}(t)\}+\alpha[\mathbf{A}]\{\dot{\mathbf{p}}(t)\}+[\mathbf{E}_L]\{\mathbf{p}(t)\}+[\mathbf{E}_{NL}(\mathbf{p}(t))]=\{\mathbf{f}(t)\} \tag{5}$$

Here $[\mathbf{A}]$ is the modal mass matrix, $[\mathbf{E}_L]$ is the linear modal stiffness matrix and $\{\mathbf{f}(t)\}$ is the modal force vector. It should be noted that all of the matrices in the modal equation of motion are now diagonal apart from the non-linear stiffness matrix $[\mathbf{E}_{NL}]$, which may contain cross-coupling terms and will be a function of $\{\mathbf{p}(t)\}$. The diagonalization of the linear terms occurs because of the orthogonality of the modes of the linear system.

2.1.2 Regression analysis for solving the nonlinear stiffness coefficient

Backwards to equation (5), modal mass matrix $[\mathbf{A}]$ and linear modal stiffness matrix $[\mathbf{E}_L]$ can be obtained from linear modal analysis of model, damping coefficient α , modal force vector $\{\mathbf{f}(t)\}$ is known already. Only nonlinear modal stiffness matrix $[\mathbf{E}_{NL}]$ including nonlinear stiffness coefficient is unknown.

Considering the beam in a static sense only, with velocity and acceleration terms set to zero, and all of the geometric and material properties being time invariant, then equation (5) can be simplified as

$$[\mathbf{E}_L]\mathbf{p}+[\mathbf{E}_{NL}(\mathbf{p})]=\{\mathbf{f}(t)\} \tag{6}$$

The left-hand side of equation (6) can be regarded as a stiffness restoring force, with a linear and a non-linear component. The right-hand side of equation (6) may be regarded as a statically applied load. It follows that if there exists a set of applied static loads, and corresponding displacements, then the unknown stiffness coefficient which relate the applied load to the resultant displacement may be determined using regression analysis. The set of applied loads and corresponding displacements are denoted as “static test case”, and can be solved for using MSC.Nastran finite element software package.

An ordinary polynomial approach has been used to curve fit the nonlinear force and corresponding displacement relationship. The polynomial expression of the nonlinear restoring forces, same as nonlinear stiffness coefficient is derived as the following series of up to third order:

$$\mathbf{E}_{NL}(p_r,p_s,p_t)_r=\sum_{j=0}\sum_{k=0}^{j+k+l=3}\sum_{l=0}A_r\,p_r{}^j p_s{}^k p_t{}^l,(j\times k\times l=0) \tag{7}$$

Here, $\mathbf{E}_{NL}(p_r,p_s,p_t)_r$ is the formulation of nonlinear stiffness coefficient for r mode, $\mathbf{A}_r(r,i)$ is the ith element in nonlinear structure equation corresponding to r mode, same as the nonlinear stiffness coefficient of modal polynomial combined main modal co-ordinate p_r with nonlinear stiffness coefficient under sth and tth modal co-ordinate $p_r{}^j p_s{}^k p_t{}^l$ in nonlinear equation. Nonlinear stiffness coefficient matric $\mathbf{A}_{NL}$ is what we want to obtained by regression analysis.

Consider that there are NT sets of static test load cases, complete static FEM analysis of NT sets of test loads on the model in commercial software package MSC.Nastran, then we can get NT sets of corresponding displacement. Translate the loads and displacement to modal space. The nonlinear restoring force for each of the test cases can now be fitted to find the unknown nonlinear nodal stiffness coefficients in a least squares sense. The nonlinear restoring forces for a certain mode r can now be shown in matrix form to be

$$\begin{Bmatrix} f_{r(1)}-E_{L_r}p_{r(1)} \\ f_{r(2)}-E_{L_r}p_{r(2)} \\ \vdots \\ f_{r(NT)}-E_{L_r}p_{r(NT)} \end{Bmatrix}=\begin{Bmatrix} p^2_{r(1)}p_{1(1)} & p^2_{r(1)}p_{2(1)} & \cdots & p^2_{r(1)}p_{NA(1)} & \cdots \\ p^2_{r(2)}p_{1(2)} & p^2_{r(2)}p_{2(2)} & \cdots & p^2_{r(2)}p_{NA(2)} & \cdots \\ & & \vdots & & \\ p^2_{r(NT)}p_{1(NT)} & p^2_{r(NT)}p_{2(NT)} & \cdots & p^2_{r(NT)}p_{NA(NT)} & \cdots \end{Bmatrix}\begin{Bmatrix} A_{NL}(r,1) \\ A_{NL}(r,2) \\ \vdots \\ A_{NL}(r,NA) \\ \vdots \end{Bmatrix} \tag{8}$$

Problem has been translated to standard least square problem, complete regression analysis to each order modal, we can get all of the nonlinear stiffness coefficient. Simplify equation (8), regression problem can be present in matrix short notation as:

$$\{\mathbf{q}\}_r\approx\{\hat{\mathbf{q}}\}_r=[\mathbf{D}]_r\{\mathbf{A}_{NL}\}_r \tag{9}$$

Here $\{\mathbf{q}\}_r$ is an $NT\times 1$ vector containing the mode r nonlinear stiffness restoring force for each of the data sets, $\{\mathbf{A}_{NL}\}_r$ is an $NA\times 1$ vector containing all of the unknown stiffness coefficients for each mode r. The matrix $[\mathbf{D}]_r$ is an $NT \times NA$ matrix, known as the design matrix as the fitting problem for each mode r. Nonlinear stiffness coefficients matrix $\{\mathbf{A}_{NL}\}_r$ can be obtained by solving regression formulation (8)(9).

2.1.3 Strategy for generating test load cases

Through the analysis presented before, the regression analysis is provided by the actual deformation and load testing after FEM analysis by commercial software package, so the accuracy of nonlinear stiffness coefficient directly depends on the rationality of selection of the test load case, which is related to the success of recovery of nonlinear structural equation. Selection of test load cases need to meet the following conditions:

(a).The selected cases must be able to reflect the linear and nonlinear factors of the structure

(b).The selected cases must meet the characteristics of aerodynamics in aeroelastic analysis

(c).The selected cases must be reasonable and interested in our research

(d).The selected cases must the requirements of nonlinear FEM calculation cost and complexity, the account of cases should be as fewer as possible.

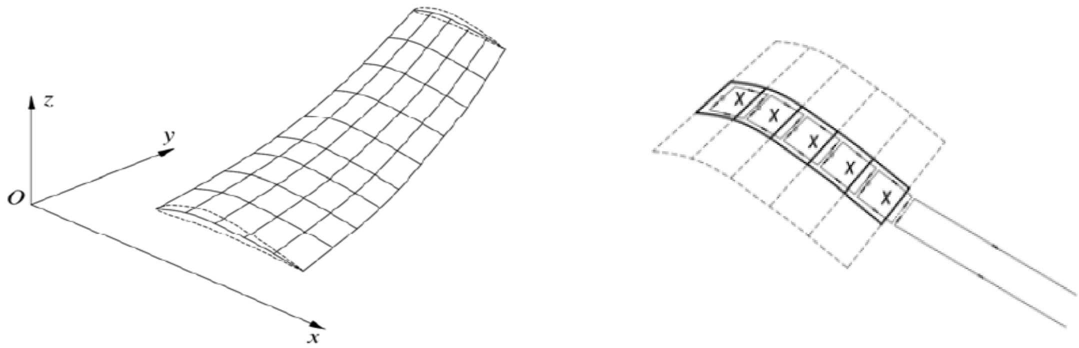
It should be noted that, in condition (d), aerodynamic force on the wing is follower force, which is more fit the actual characteristic of aerodynamic force. That is to say, take oriented load as the load test cases can't meet the requirement. In this paper, aerodynamics force under the wing's deformation combined bending modes and torsion modes is chosen as test load case, meanwhile regard bending modes and torsion modes as normal modes to realize analysis of approximate follower force load. The formulation of wing's deformation which makes aerodynamics forces should be:

$$f_{AIR}\{\mathbf{w}\} = f_{AIR}(\sum a_i\{\phi_i\}_{bend} + \sum a_j\{\phi_j\}_{torsion}) \tag{10}$$

Here $\{\phi_i\}_{bend}$ and $\{\phi_i\}_{torsion}$ are bending modes and torsion modes interested, $a_{i,j}$ are scalar modes weight factors, which can make the selected test cases reflect the nonlinear factors of the structure and interested in our research. It should be noted that deformation combined with linear modes can't reflect spanwise deformation, which is not fit actual condition and can't meet requirement of nonlinear analysis. This paper assume the beam can't be extended and shortened, solve the real spanwise deformation with geometric relationship after model wing's flapwise deformation and edgewise deformation under linear modes combination is obtained, then, make test load cases under the corrective nonlinear deformation. The correction presented above can make the test load cases fit actual condition and meet the requirement of nonlinear analysis.

2.2 Nonplanar vortex lattice method

The nonplanar aerodynamics of a deformed aircraft is computed by the nonplanar vortex lattice method. As shown in Figure 1(a), the thin-wall platform is represented by its middle camber surface, and then it is divided into panels containing vortex ring singularities. Some typical panel elements are shown in Figure 1(b). Each vortex ring consists of four segments of a vortex line, and the leading segment of the vortex ring is paced on the panel's quarter chord line. The aerodynamics of the panel acts on the midpoint of the segment. The collocation point is at the center of the three-quarter chord line, and at this point, the actual boundary condition will be implemented. The velocity induced at an arbitrary point by a typical vortex ring can be calculated by applying the Biot-Savart law to the ring’s four segments, except for the rings located at the trailing edge of the thin-wing platform.



(a) Vortex lattice model for a thin lifting surface (b) Arrangement of the vortex ring elements

Figure 1: Nonplanar vortex lattice model

Two semi-infinite training vortex lines that model the wake are shed into the flow along the x axis at each trailing edge panel, as shown in Figure 1(b). Instead of the rear segment of the vortex ring, the effect of these two semi-infinite trailing vortex lines should be considered when the velocity by a trailing edge vortex ring element is computed. The aerodynamics increments due to the deflection of control surfaces are only considered in the boundary condition, therefore, the vortex lattices that represent the control surface are not rotated physically. Then, the aerodynamics forces can be represented in special formulation.[15]

2.3 Surface spline interpolation

The surface spline is used for the coupling of aerodynamics and structure. The configuration of structure is usually considered to be embedded in a 3-D space. The undeformed configuration could be 1-D, 2-D, or 3-D, and the deformed configuration is usually 3-D.

Consider n given structural grids and the corresponding deformation vector U_S , then the deformation vector U_A of m aerodynamic grids could be interpolated.[16]

$$U_A = GU_S \tag{11}$$

Here G is the spline matrix for displacement interpolation between the aerodynamics grids and structure grids. In aeroelastic analysis, the transformation between the aerodynamics and the structural force systems requires structure equivalence rather than static equivalence. Structure equivalence means that the two force systems deflect the structure equally. When the aerodynamics forces F_A and their equivalent structure forces F_S do the same virtual work on virtual deflections, the structure equivalence of the two force system is guaranteed:

$$\delta U_A^T F_A = \delta U_S^T F_S \tag{12}$$

Here δU_A and δU_S^T are the arbitrary virtual deflections, satisfying equation (11). So:

$$F_S = G^T F_A \tag{13}$$

2.4 Static aeroelastic analysis methodology

Static aeroelasticity is the study that covers the interaction of aerodynamic and structural forces on a flexible structure with all of the geometric properties and the forces are being time invariant. This interaction results in elastic deformations of the lifting surface of the structures, hence the bending and the twist of the wing can be determined with respect to the flight condition of interest. The general form of aeroelastic static equation of motion can be represented as:

$$\{E\}\{r\} = \frac{\rho V^2}{2}[AIC_R]\{r\} + \{F_g\} \tag{14}$$

Here $\{E\}$ is stiffness matrix containing linear and nonlinear stiffness coefficient, $\{r\}$ is the convergence deformation of wing, $[AIC_R]$ is the steady aerodynamics influence coefficient matrix, $\{F_g\}$ is a vector of applied loads (e.g. gravity). Note that the rigid body motions are not included in the system of equation as for all the cases considered in this thesis, the wing structure is usually clamped on the root.

Combined nonlinear structure ROM and steady vortex lattice method to apply the static aeroelastic analysis. The interpolation of structure and aerodynamics is surface spline interpolation. In this paper, we calculate the static aeroealstic stable deformation under the given airflow velocity. Iterative process is reported in Figure 2:

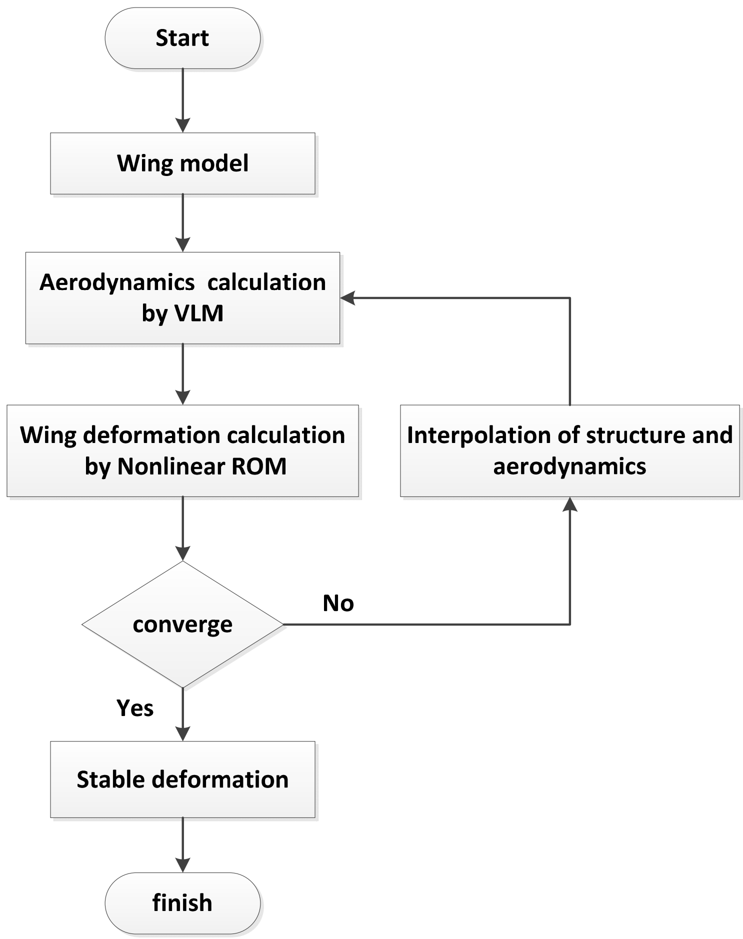


Figure 2： Iterative process of static aeroelastic analysis

2.5 Static aeroelastic trim analysis methodology

As the analysis flow chart illustrated in Figure 3, the geometric nonlinear static aeroelastic trim analysis is solved by iterative calculation of two coupling modules, which are the module for aerodynamics and rigid trim analysis and the module for structural nonlinear static analysis.

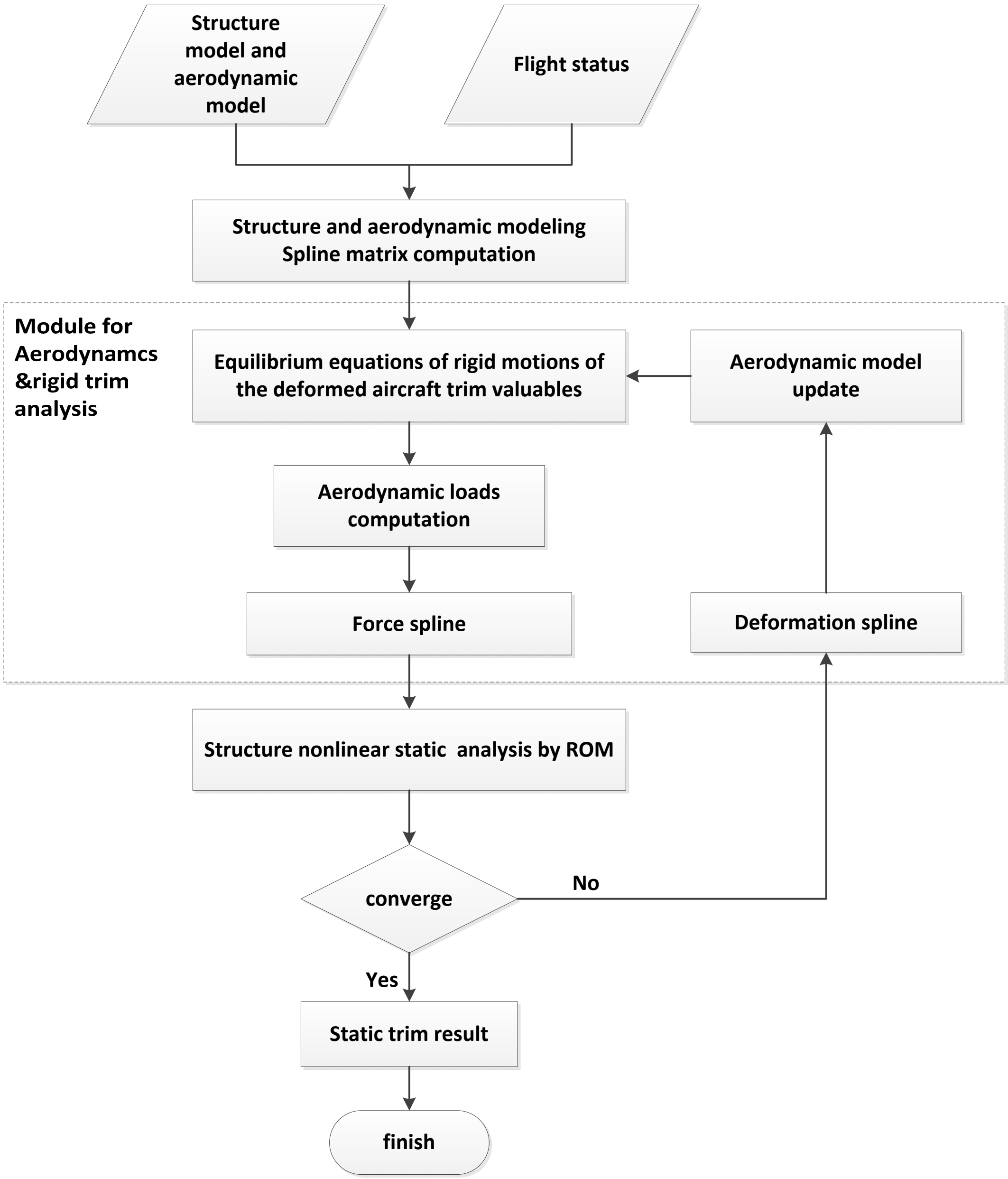


Figure 3: Iterative process of static aeroelastic trim analysis

The procedure for the geometric nonlinear statics aeroealstic trim analysis starts with appropriate aerodynamic and structure modeling and initialization, and then start iterative calculation. For each cycle of computation, the aerodynamic model is updated according to the structure deflection gained in the previous cycle. Based on the updated aerodynamics and structure models, the equilibrium equations of rigid motions of the deformed aircraft are established. The trim variables of the flexible aircraft, as the angle of attack α and control surface deflection δ_{cj}, are solved by the equilibrium equations. The trimmed aerodynamics loads are then calculated and applied to the structure, the structure deformation is solved by nonlinear ROM. Both the aerodynamics loads and inertial loads are treated as follower forces. As one cycle finishes, the deformations of specified grids will be evaluated and tested for termination. The procedure will not be ended until the termination criteria are met.

The equations of motions of the aircraft has been established in the body reference coordinate system for which the mean axes system *oxyz* is selected. The equation of motion can be represented as:

$$\begin{aligned} M\dot{V} - M\tilde{\omega}^T V &= F_{Ar} + F_{Tr} + Mg_m \\ I\dot{\omega} + \tilde{\omega} I\omega &= M_{Ar} + M_{Tr} \end{aligned} \tag{15}$$

Here M and I are the mass and inertia matrix of the deformation aircraft, V and ω are the velocity vector and angular velocity vector between the mean axes and the inertial system, F_{Ar} and M_{Ar} are the resultant force and moment of aerodynamics, F_{Tr} and M_{Tr} are the resultant force and moment of thrust and other external forces, g_m is the acceleration vector of gravity in the mean axes system.

3 NUMERICAL EXAMPLE

3.1 Model

3.1.1 Wing model

To validate the accuracy of ROM and compete static aeroelastic analysis considering geometric nonlinearity, a wing model is conducted. The wing model has large flexibility with large-aspect-ratio, the design parameters are shown in Table 1.

Item	Value
Span(mm)	487
Chord(mm)	60
Airfoil of the wing	NACA0015
Elastic axis	50% chord of the wing
Material of beam	Spring steel
Density of beam	$7.6kg / m^3$
Elastic modulus of beam	230Gpa
Section of beam	$7.03mm \times 1.14mm$ rectangular

Table 1: Design parameters of the wing model

As shown in Figure 4, the wing shape is simulated by twelve wing sections made from balsa wood and coton paper. Enough section attaches to the wing spar by the external shell. There is a wingtip store to regulate the flutter charateristics. The wingtip store is 150mm long and weight 31.5g. An aeroelastic analysis model of the wing is established, the structure FEM shown in Figure 5 uses beam elements ans lumped mass elements for the stiffness and mass simulation.

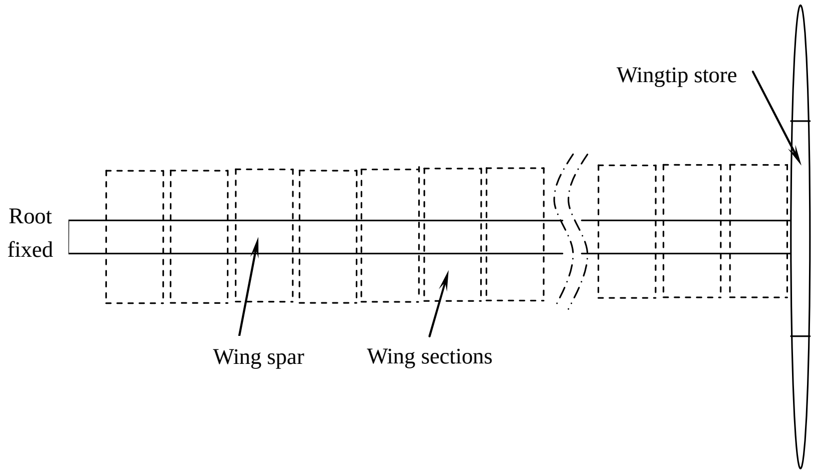


Figure 4: The layout of the flexible wing

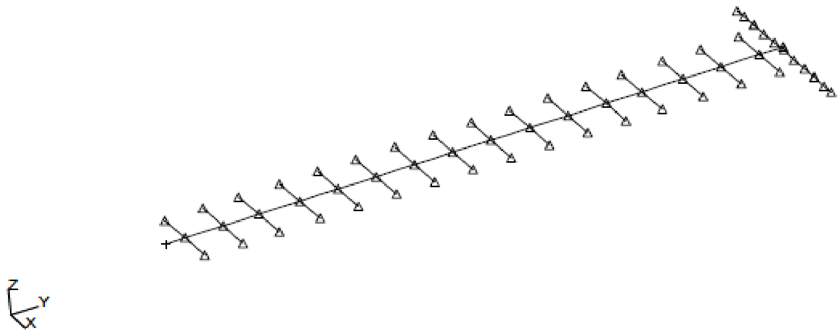


Figure 5: The structure finite element model of wing

Due to the model will exist spanwise deformation in geometric nonlinear analysis, two spanwise modes has been taken into the establishment of ROM to evaluate the accuracy. Seven modes participated in establishment of ROM in Table 2. Telescopic modes is solved by analytical method.

Mode	Description	Frequency/Hz	Modal stiffness
1	1st bending	1.7799	1. 250678E+02
2	1st edgewise bending	10.604	4. 439389E+03
3	1st torsion	13.756	7. 470369E+03
4	2nd bending	17.355	1. 189092E+04
10	2nd torsion	194.10	1. 485834E+06
	1st spanwise telescopic	244.75	2. 362461E+06
	2nd spanwise telescopic	784.51	7. 976308E+07

Table 2. Modes participated in establishment of ROM

It can be seen that, frequencies of first three modes are low, which means the flexibility of model is high.

3.1.2 Aircraft model

To complete the method for static aeroelastic trim analysis, a semispan subscale model of a flexible aircraft is conducted. A single spar with a uniform rectangular cross section is chosen for the stiffness simulation of the wing. Parameters of model is shown in Table 3.

Item	Value
Semispan of the wing(mm)	487
Chord of the wing(mm)	60
Airfoil of the wing	NACA0015
Length of the fuselage(mm)	305
Semispan of the horizontal tail(mm)	72
Chord of the horizontal tail(mm)	36
Airfoil of the horizontal tail	NACA0009
Horizontal tail volume	0.293
Rotation axis of the horizontal tail	39% chord of horizontal tail
Longitudinal location of the center of gravity	30% chord of wing
Weight of the structure(g)	85

Table 3: Design parameters of the aircraft model

The airctaft model is shown in Figure 6, the wing of aircraft is the same as wing model descrined before. There is also a wingtip store to regulate the flutter characteristics. The wingtip store is 150mm long and weight 31.5g. Compare with the flexible wing, the fuselage and horizontal tail are much stiffness, and could be treated as rigid bodies. An aeroelastic analysis model of the flexible aircraft is established, the structure FEM shown in Figure 7 uses beam elements ans lumped mass elements for the stiffness and mass simulation.

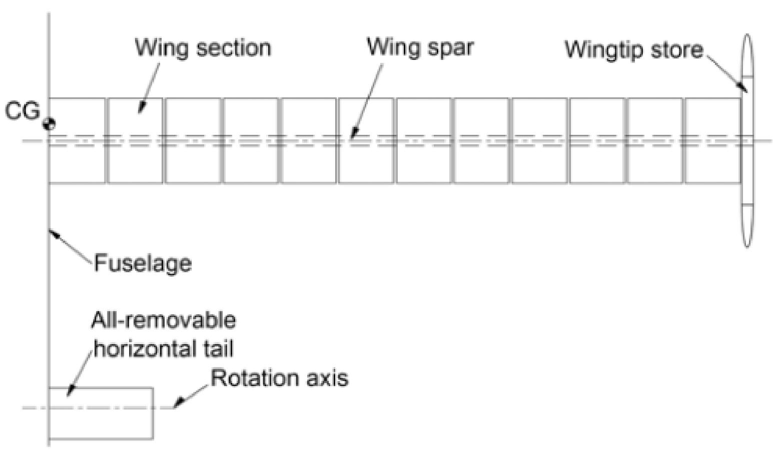


Figure 6: The layout of the flexible aircraft

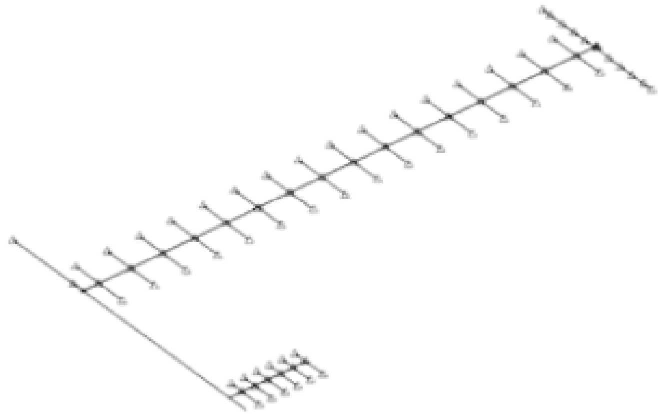


Figure 7: The structure finite element model of aircraft

3.2 Verification of nonlinear ROM

The accuracy of the estimated nonlinear ROM can be validated by contrasting the calculation results of ROM and nonlinear FEM after applying the validating load on the wing model.

Nonlinear ROM is obtained by regression analysis from test load cases and corresponding deformations, so the nonlinear structural equation must fit the result of all sets of test load cases and corresponding deformation. More important, nonlinear structure equation should fit kinds of load in different form and their deformation, having good adaptability in calculation to different form of loads, then the ROM can be applied to structural and aeroealstic analysis reasonably.

Take four sets of validation load as example, distribution of force on the wing model of those examples is shown in Figure 8. Load of case A is a distribution of actual aerodynamics force. Load of case B is a distribution of constant force. Load of case C is a distribution of linear increase force along the wing span. Load of case D is a distribution of force with two order f-

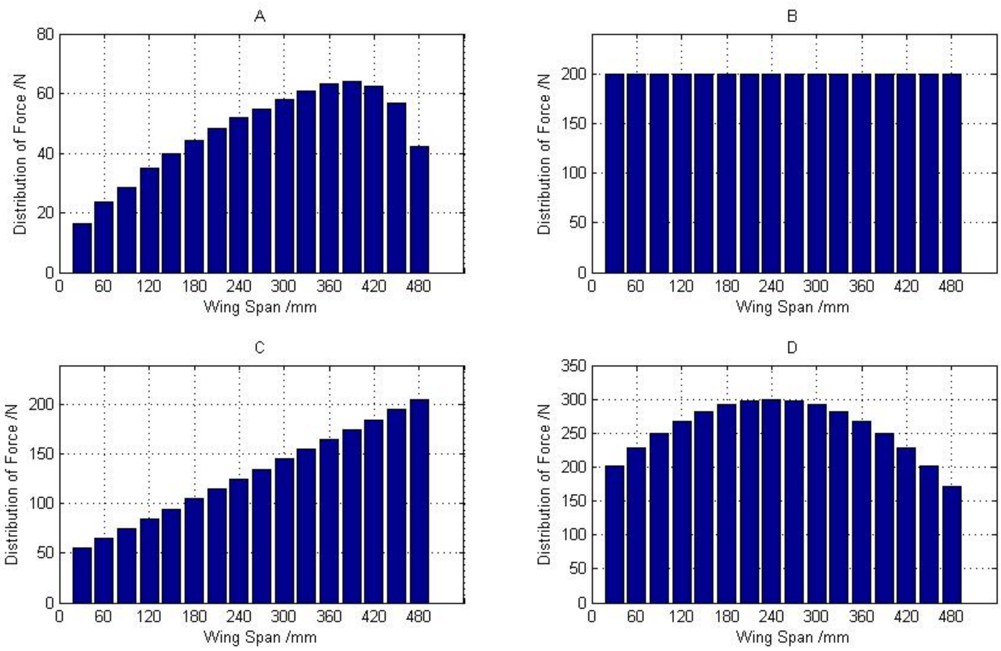


Figure 8: Distribution of force (Validating load) on the wing model

unction. Calculation results of those cases are shown in Figure 9 with comparing of ROM solutions and FEM solutions.

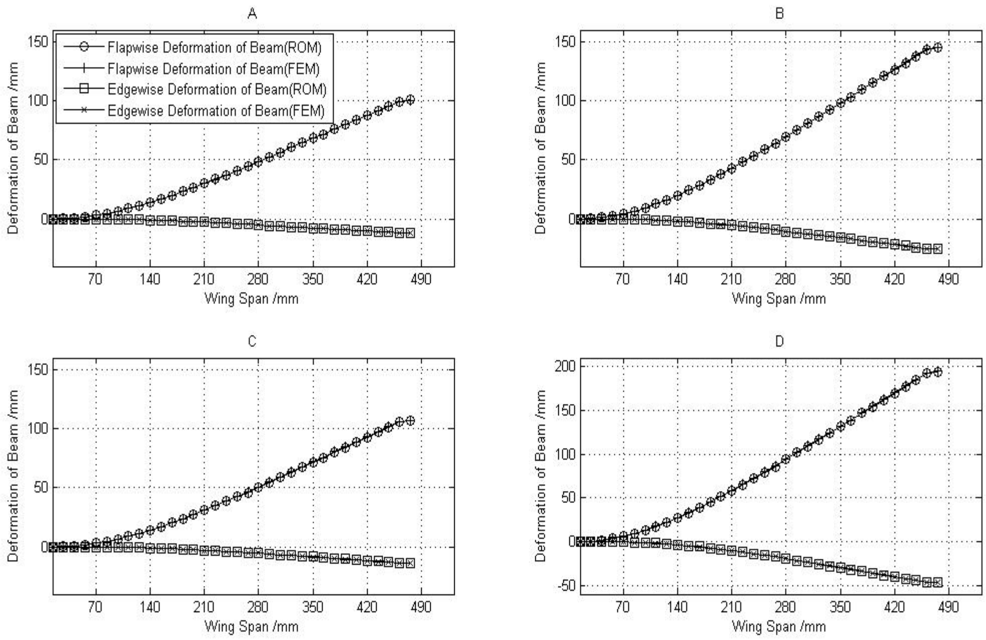


Figure 9: Calculation results of validating load

Here black line with circle represent flapwise deformation of beam in ROM solutions, black line with cross represent flapwise deformation of beam in FEM solutions. Red line with circle represent edgewise deformation of beam in ROM solutions, red line with cross represent edgewise deformation of beam in FEM solutions. ROM solutions can meet a great agreement to nonlinear FEM solutions. The structure model established is reliable.

3.3 Static aeroelastic analysis of wing model

In this paper, we calculate the static aeroealstic stable deformation under the given airflow velocity. Take airflow velocity 33m/s as example, Figure 10 shows distribution of lift, drag and cross force on the wing after deformation is stable when calculation iterative progress of static aeroelastic analysis is converged

It can be seen that end of wing is affected heavily by downwash airflow, air force nearly have ellipse distribution. Due to the clamping of wing' root and freedom of wing's tip, torsion of wing's tip is large. Lift, drag and cross force is asymmetric at tip of wing and root of wing. The velocity 33m/s in example above is large, the asymmetric is more obviously. Air force has component of cross force, due to considering of surface influence of wing's large deformation, which is different from linear aerodynamics calculation.

Calculate the static aeroelastic stable deformation of wing in a certain range of airflow velocity, Table 4 shows the comparison of vertical displacement of tip of wing between nonlinear ROM solutions and FEM solutions in a range of velocity from 20m/s to 34m/s at angle of attack of 0.0097rad.

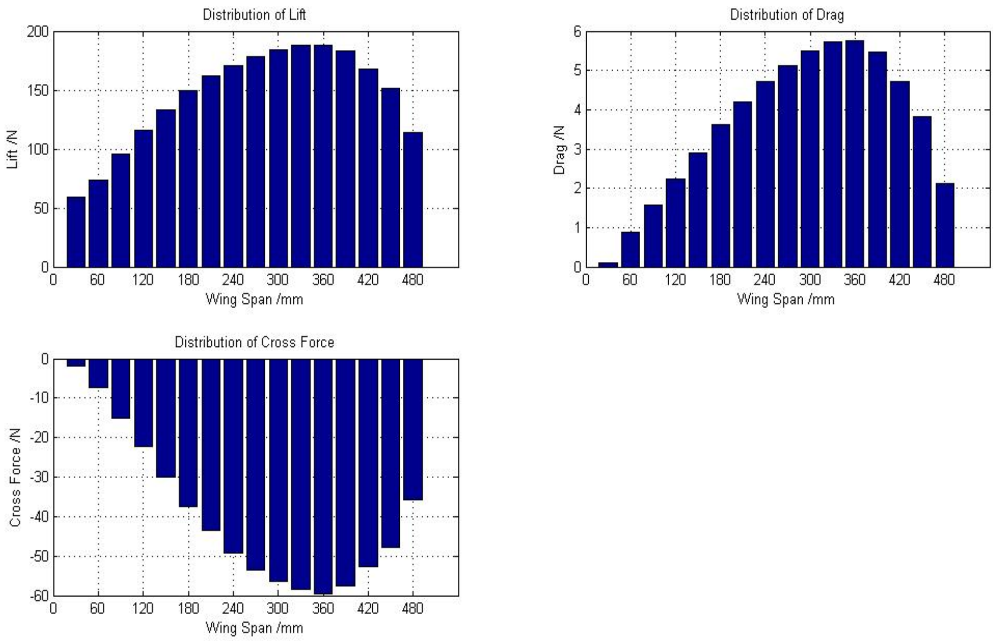


Figure 10: Distribution of aerodynamic force after calculation converged

Velocity(m/s)	ROM solutions/mm	FEM solutions/mm	Error/%
20	-59.1386	-62.5	0.71
25	-31.4230	-37.5	1.3
28	1.7816	-0.52	0.46
30	41.9373	49.23	1.5
31	72.8532	81.76	1.6
32	115.7604	124.63	1.8
33	177.7625	182.12	1.3
34	243.9941	250.54	1.3

Table 4: Comparison of ROM and FEM solutions of stable static aeroelastic deformation

Figure 11 shows the comparison of vertical displacement of tip of the wing between nonlinear ROM solutions, nonlinear FEM solutions and experiment result in a range of flight status set up before.

It can be seen that tip displacement of wing close to zero when air speed close to 28m/s when aerodynamic force equals to gravity. The tip displacement of wing change into nonlinear growth with the increase of air speed. The static aeroelastic analysis solutions based on nonlinear ROM has a good agreement to the static aeroelastic analysis solutions based on nonlinear FEM, which prove the process established is accuracy and reasonable. There are some error between experimental result and calculation result in medium speed condition, the preliminary judgment of that is there are random disturbance errors in experiment with corresponding conditions, and model has some unknown factors can’t modelling. These factors bring errors for calculation and experiment.

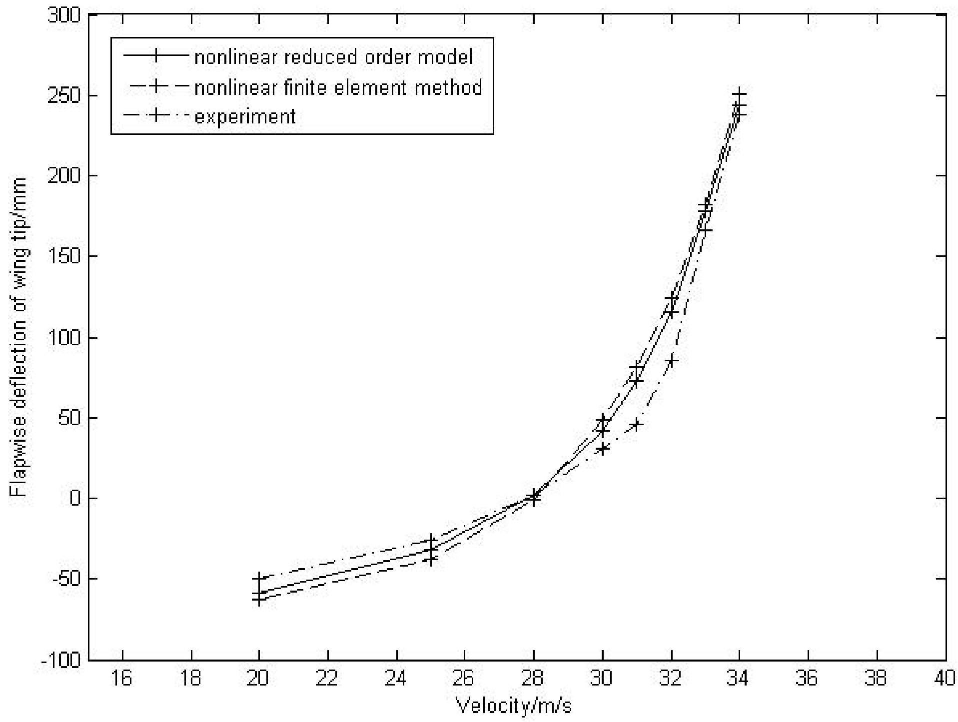


Figure 11: Comparison of ROM, FEM solutions and experiment result of stable static aeroelastic deformation

3.4 Static aeroelastic trim analysis of aircraft model

Different counterweights are added to the center of gravity of the aircraft, in order to keep its location fixed, then the longitudinal trim analysis is conducted. The variation of the trim variable with the aircraft weight is shown in Figure 12 when the aircraft is in a straight and level flight with a uniform speed of 30m/s. The trim variable are the angle of attack α and elevator deflection δ_e. The calculation result of vertical displacement variation of the tip of the wing is shown in Figure 13. FEM solutions means the static aeroelastic trim analysis is b-

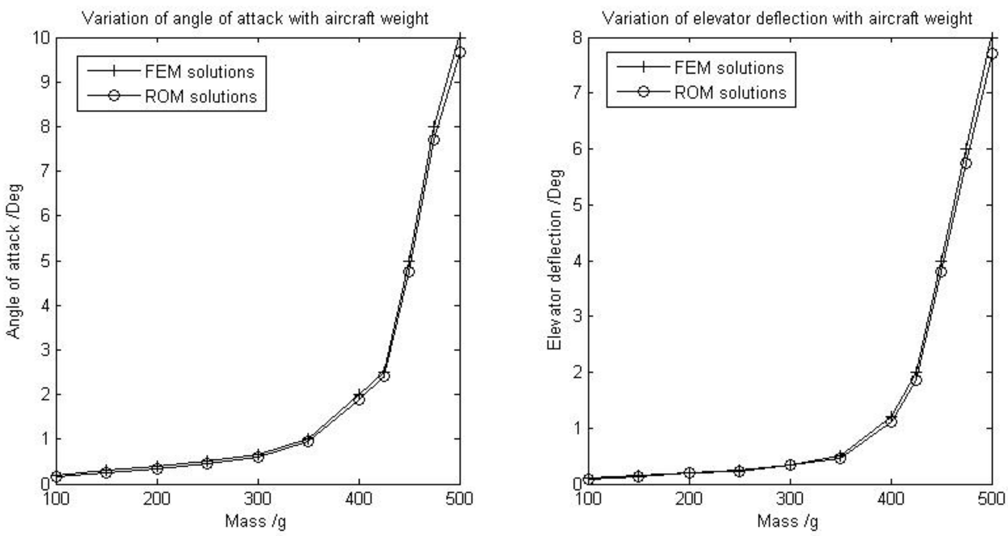


Figure 12: Variation of the trim variable with the aircraft weight

ased on structure analysis method of FEM, ROM solutions means the static aeroelastic trim analysis is based on structure analysis method of ROM.

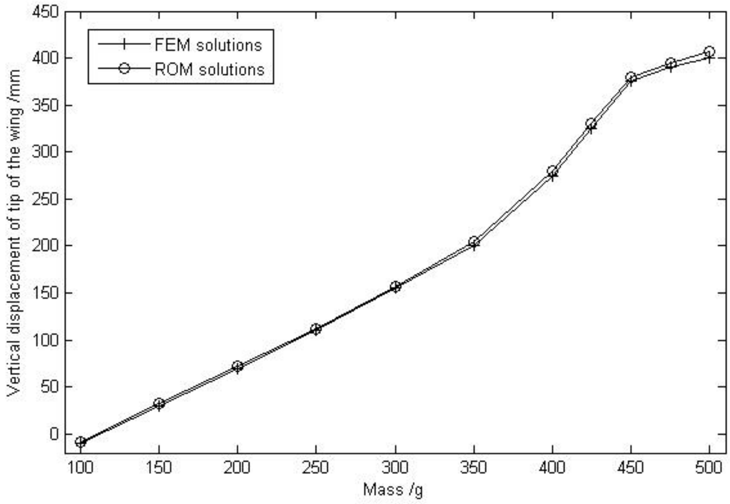


Figure 13: Vertical displacement variation of the tip of the wing with different weight

The result shows the angle of attack and elevator deflection needed for trim increase nonlinearly with the aircraft weight. When the aircraft is heavier, more than 400g, the angle of attack and elevator deflection increase rapidly and the vertical displacement at the wing tip can be larger than 55% of the semi wing span. As the nonlinear aeroelastic trim analysis shows, the structure deformation of the wing increases nonlinearly with the aircraft weight. It should be noted that, also the focus of this paper, the solutions based on ROM method can reach a good agreement to nonlinear FEM method with small error. The structure analysis method of ROM can be applied to static aeroelastic trim analysis well.

4 CONCLUSION

A method for the static aeroelastic analysis of wing and static aeroelastic trim analysis of flexible aircraft with large deformations has been presented in this paper. Main work in this paper is to improve the application of combined modal/FE method as structure ROM in aeroelastic analysis. Once the structural ROM has been defined, it is coupled with steady nonplanar vortex lattice aerodynamics method to analysis static aeroelastic behavior.

The work presented here innovatively use seven modes to recovery the large deflection of wing to reflect the characteristic of wing's large deflection, that is in order to identify the nonlinear stiffness coefficients more exactly also. Meanwhile, the method set the aerodynamic follower forces under certain deformation as test load cases in ROM which made analysis close to more real flight condition and makes result more reasonable. At last, aerodynamic influence coefficient matrix is changing for considering the changing of wing's deformation instead of a constant value based on the initial deformation. Though that makes solution more complexity, the nature of aerodynamics under the large deformation has been more considered. It can be seen from the calculated result that both of the aeroelastic analysis of wing and aeroelastic trim analysis of aircraft based on ROM can achieve a good agreement to analysis based on FEM. It is valuable to theoretical analysis and engineering application in aeroelastic with geometric nonlinearities.

Research is continuing to apply the method to dynamic aeroelastic analysis (flutter speed prediction, limit cycle oscillation behaviors and gust response prediction et al.) and flight dynamic aeroelastic stability analysis of flexible aircraft.

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