

# A HIGH-EFFICIENCY AEROELASTIC OPTIMIZATION METHOD BASED ON KRIGING MODEL AND GENETIC ALGORITHM

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**Abstract:** A high-efficiency aeroelastic optimization method based on Kriging model and genetic algorithm is presented, and the method is verified by aeroelastic optimization of composite wing. The accuracy of aeroelastic optimization based on Kriging model is analyzed and the framework of aeroelastic optimization base on Kriging model and genetic algorithm is presented. The feasibility of the method is verified by an example, and the effects of design variable number and constraint number on optimization efficiency and result are analyzed in detail.

## 1 INTRODUCTION

In modern aircraft structure design, the increasing requirement of weight loss and the great application of composite result in enhancing structural flexibility. In order to overcome the disadvantage and take adequate advantage of aeroelastic effect to increase the overall performance of aircraft, aeroelastic optimization is developed necessarily. In preliminary phase of aircraft design, aeroelastic optimization can be used as a tool to reduce weight and increase aeroelastic performance, thus enhancing the design efficiency. These effects in composite structure are especially obvious<sup>[1]</sup>.

In the past years, many methods of aeroelastic optimization were suggested and most of them were applied in actual engineering of aircraft design<sup>[2]</sup>. Isogai studied aeroelastic tailoring design of composite wing in the condition of multiple constraints with direct search method which can overcome the problem produced by the discontinuity of gradient information in gradient optimization<sup>[3]</sup>. Livne etc. suggested the concept of active controlling composite wing, which can combine aerodynamic, structure and control to consider aeroelastic tailoring<sup>[4]</sup>. Guo studied aeroelastic tailoring design of composite backswept wing using genetic algorithm<sup>[5]</sup>. Wan Zhiqiang etc. studied aeroelastic tailoring of forward-swept composite wing and high-aspect ratio composite wing using genetic/gradient-based hybrid algorithm, and the comparison of correlative design variables is also studied<sup>[6]</sup>.

These results indicated that, for aeroelastic optimization, the constraints including flutter, divergence, natural frequency, deformation, aileron effectiveness, strength and flight loads etc. are considered in the condition of satisfying weight minimization, and the design object usually needs to be analyzed repeatedly. With the development of aircraft design, structure modeling will be more complex and the number of variables and constraints will increase rapidly, both of which will result in the consumption increasing much and the efficiency

reducing much. In aeroelastic optimization design, the excessive computation consumption affects the optimization efficiency directly. Therefore, computation with a proper surrogate model is the key to reduce optimization consumption. In recent years, many surrogate models were developed. Kriging model is one of them, with its rapid computation and high prediction accuracy, applied widely.

Kriging model can predict the predictive value and error of unknown points with given points and can acquire higher optimization efficiency combined with optimization algorithm. At present, Kriging model is applied for aircraft optimization design. ShinkyuJ etc. developed a method which combined Kriging model and genetic algorithm for the aerodynamic optimization of two-dimensional airfoil profile design<sup>[7]</sup>. MashioK etc. studied multiple object optimization of multistage profile crevasse using Kriging model<sup>[8]</sup>. Nithin Kolencherry established an effective optimal method with the combination of Kriging model and genetic algorithm<sup>[9]</sup>.

However, the combination of aeroelastic and Kriging model hasn't been presented. Therefore, an effective method based on the combination of genetic algorithm and Kriging model instead of complex structural model is presented to solve the problem of excessive computation consumption.

## 2 THEORIES

### 2.1 Optimization design method

Aeroelastic optimization problem can be represented as<sup>[10]</sup>:

$$\text{Min } F(v) \quad (1)$$

$$\text{S.T. } g_j(v) \leq 0 \quad j = 1, 2, \dots, n_c \quad (2)$$

$$v_{il} \leq v_i \leq v_{iu} \quad i = 1, 2, \dots, n_d \quad (3)$$

Eq. (1) represents minimizing the objective function of  $F(v)$  which is the structural mass. Eq. (2) is the constraints including the displacement of wing tip, internal stress, aileron efficiency and flutter speed which is attained by overdamping constraint. Eq. (3) defines the upper and lower boundary of design variables which are comprised of skin thickness, section sizes of spars and section areas of stringers.

Genetic algorithm is adopted in this paper, which is an efficient global search method mimicking the process of natural selection. Genetic algorithm has many advantages, such as easy coding, good searching ability and robustness, and it has been applied to composite aeroelastic optimization<sup>[11]</sup>.

### 2.2 Kriging model

In order to reduce the analysis cost of optimization design, Kriging method is used to reduce the computation. Kriging model has been successfully applied for optimization problems, and it is suitable for approximating highly nonlinear functions and can be used to create globally valid meta-models. A wide range of correlation functions can be chosen for building the

meta-model thereby making the Kriging meta-models extremely flexible. Therefore, computation consumption can be greatly reduced.

Both the predicted value and its uncertainty are considered at the same time. This is captured by updating the Kriging model during the optimization. If the optimization algorithm is not converged, additional random designs are used to initialize and reconstruct Kriging model.

Kriging model is the combination of a polynomial model and a statistical function, and it can be written as follows<sup>[12]</sup>,

$$y(x) = F(x) + z(x) \quad (4)$$

where  $y(x)$  is the unknown function,  $F(x)$  is the known polynomial function and  $z(x)$  is the function from a stochastic process with mean zero, variance and non-zero covariance. The polynomial function  $F(x)$  approximates the design space globally and the localized deviations are created by the function  $z(x)$ . The polynomial function is taken as a constant term for this study. The covariance matrix of  $z(x)$  that represents the local deviations is given below,

$$Cov\left[z(x^i), z(x^j)\right] = \sigma^2 R([R(x^i, x^j)]) \quad (5)$$

where  $R$  is the correlation matrix, and  $R(x^i, x^j)$  is the correlation function between any two of the sample  $x^i$  and  $x^j$ . The correlation matrix considered in this paper is the common Gaussian correlation function as described below,

$$\prod_{k=1}^n \exp(-\theta_k |d_k|)^2 \quad (6)$$

where  $n$  is the number of design variables,  $\theta_k$  is the unknown correlation parameters used to fit the model and  $d_k$  is the distance between the  $k^{th}$  component of sample  $x^i$  and  $x^j$ .

### 3 FLOWCHART

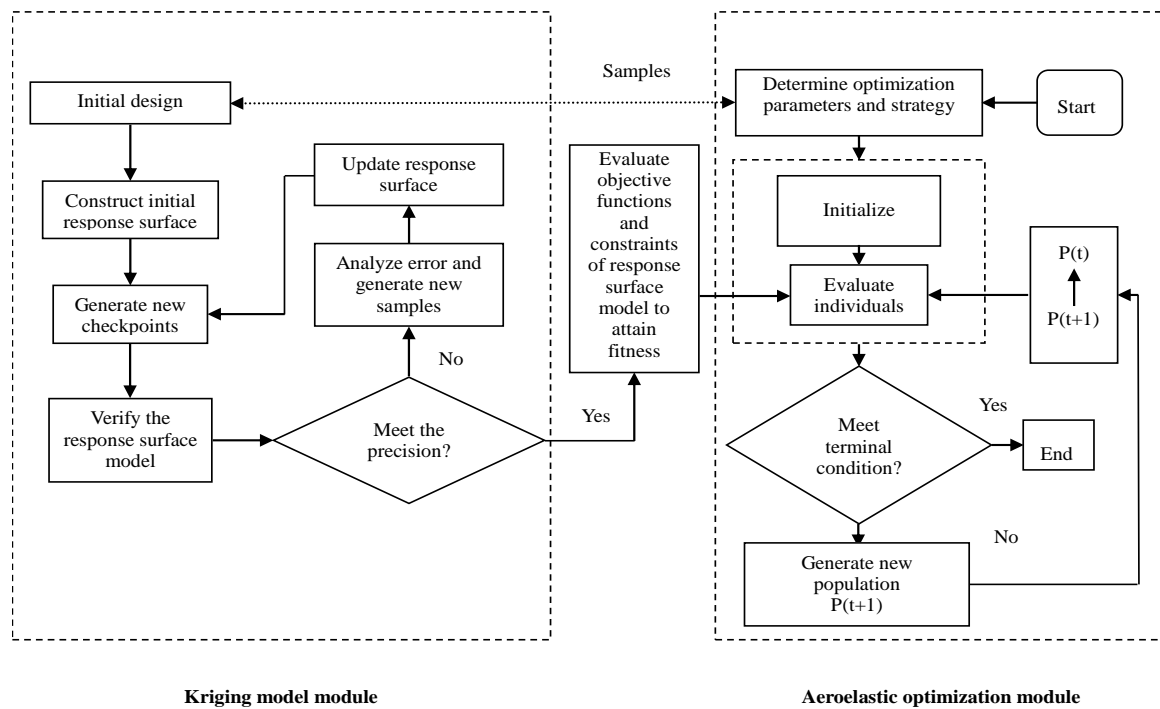


Figure 1 The flow chart of high-efficiency aeroelastic optimization method based on Kriging model and genetic algorithm

Figure 1 illustrates the flowchart of efficient aeroelastic optimization based on the combination of Kriging model and genetic algorithm. The flowchart of high-efficiency aeroelastic optimization method based on Kriging model and genetic algorithm

The essential procedures of the optimization consist of:

- 1) Determining genetic strategy and defining optimization parameters, which include population size, encoding method, convergence criterion, the method of three main operators (reproduction, crossover, and mutation) and the probability of crossover and mutation, etc.
- 2) Generating initial population, which is the sample population for Kriging model, with random function and niche technique by calling Nastran to evaluate each individual so as to attain the fitness of objective functions and constraints.
- 3) Constructing Kriging model using Kriging module with the sample population.
- 4) Constructing initial response surface with Kriging model.
- 5) Generating new checkpoints to verify response surface accuracy.
- 6) Repeating the procedure 4) to update response surface, and then repeating procedure 5) to verify the accuracy if the model doesn't meet the precision.
- 7) Completing the model successfully and continuing aeroelastic optimization module if the model meets the precision.
- 8) Calculating objective functions of current population using Kriging model to attain the fitness.
- 9) Judging the convergence from the fitness of Kriging model.

10) Operating the last population with fitness scaling, niche technique and three main operators (reproduction, crossover, and mutation) to create a better population and going back to procedure 8) if the termination criteria are not met.

11) Continuing the iteration unless meeting the termination criteria.

## 4 OPTIMIZATION OBJECT AND STRATEGY

### 4.1 Model description

The optimization object is a typical composite wing-box structure which is composed with double beams, multi-rib and stiffened skin. The skin panels and spar webs are modeled as composite shell elements. Rod elements are used to model the spar caps and stringers, while the properties are replaced by the composite shell elements using the method of displacing equal stiffness. The doublet-lattice method available in MSC/NASTRAN is used for static aeroelastic and flutter constraints calculation during the optimization.

The structural and aerodynamic models are shown in Figure 2.

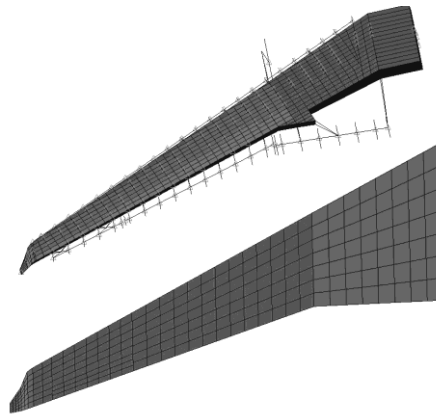


Figure 2 Aerodynamic and Structural finite element model

### 4.2 Optimization constraint

The optimization is carried out in the longitudinal load state which is a 2.5g pull-up maneuver at 11200m with a flight speed of Mach 0.78.

The objective is to minimize wing structural weight and the constraints include:

- 1) Ratio of displacement at the wing tip to the half-span length of the wing  $u_{tip} / B < 4.5\%$
- 2) Strength meeting the allowable stress and strain of composite shell.
- 3) Aileron efficiency  $\eta > 60\%$
- 4) The flutter speed  $V_F \geq 320m/s$

### 4.3 Optimization strategy

The optimization considering composite design criteria and load characteristic of large aspect ratio wing is shown as follows:

1) Symmetrical balanced lay-up is performed in wing elements, and the 0 degree fibers of the composite lay-up coincide with rear spar cap.

2) Based on the practical aerospace applications of composite materials, the composite skins have symmetrical fiber directions of 0, 90, +45, -45 degrees. Furthermore, the ply proportion of the composite shell elements is fixed, with the spar webs only consisting of the equal lay-up of  $\pm 45^\circ$ , and the thickness of lay-up of 0,  $\pm 45^\circ$  and 90 for the skins accounting 50%, 40% and 10%. The areas or thicknesses of all elements for each structural member type are divided into 12 segments and varied along the span-wise direction but are held constant in the segment. Both of web thickness and spar cap section area increase from the wing root to the engine, but decrease from the engine to the wing tip.

3) 72 design variables are considered in total, and the aeroelastic optimization method is genetic algorithm. The genetic algorithm parameters were set as follows: population size  $M = 500$ , crossover probability  $P_c = 0.7$ , mutation probability  $p_m = 0.4$  and individual number in elitist selection based on main fitness  $n_c = 30$ , and the termination criterion is the maximum number.

## 5 AEROELASTIC OPTIMIZATION DESIGN BASED ON FINITE ELEMENT MODEL

The aeroelastic optimization design based on finite element model is to attain the optimized wing structure using genetic algorithm. The comparison is shown in Table 1.

Performance indicator	Constraint conditions	Original model	Optimized model
Mass/%	—	1	90.4%
Displacement/%	$\leq 12\%$	7.82%	8.55%
Torsion Angle/ $^\circ$	$\leq 4.5$	2.33	2.39
Aileron efficiency /%	$\geq 65\%$	64.5%	61.7%
Flutter speed /(m/s)	$\geq 320$	364	356

Table 1 Performance of the optimized composite wings based on finite element model

The result indicates that structural weight can be reduced with satisfying the design constraints of stress, strain, deformation, aileron efficiency and flutter speed after aeroelastic optimization. That is to say, the structural size and stiffness distribution can be more reasonable and the weight can be reduced.

## 6 AEROELASTIC OPTIMIZATION DESIGN BASED ON KRIGING MODEL

### 6.1 Kriging modeling and analysis

Aiming at composite wing aeroelastic optimization, Kriging model is used to describe objective function and constraints. For the model in this paper, objective function is the wing mass, and many kinds of constraints are considered, including stress, strain, deformation, aileron efficiency and flutter speed etc.

Appropriate response is chosen to fit different constraints. The response value can fit the deformation and torsional angle of wing tip and aileron efficiency, while for flutter speed, stress, strain and failure constraints, the Kriging model is constructed with fitting the converted fitness.

First of all, 400 samples are selected to construct Kriging model. In order to verify predicted result accuracy, another 100 samples are selected. The absolute average value of error between the predicted value of the 100 samples and the computed value of the finite element model is shown in Table 2.

Kriging model type	Average error
Mass	0.251%
Displacement	0.849%
Torsion angle	0.886%
Aileron Efficiency	0.347%
Failure Constraints	0.106%

Table 2 Average error of the predicted value

The comparison of wing tip deformation for these 100 samples between predicted value and computed value of the finite element model is shown in Figure 3.

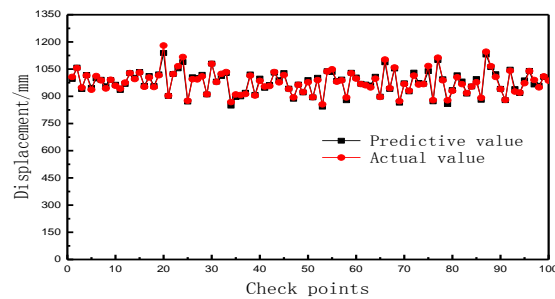


Figure 3 Comparison between predictive value and actual value of the displacement

The comparison result shows that in the condition of 72 variables, the response error is all within 1%, and the model has practical value.

## 6.2 Aeroelastic optimization based on Kriging model

500 individuals are analyzed in each population and 15 populations are computed in total. The best individual of the final population is selected as the optimized result, and comparison result between the finite element model and Kriging model is shown in Table 1.

Performance indicator	Constraint value	FEM model	Kriging model	
			Actual value	Predictive value
Mass/%	—	1	1.023	1.020
Displacement/%	$\leq 12\%$	8.55%	8.66%	8.74%
Torsion Angle/ $^{\circ}$	$\leq 4.5$	2.39	2.52	2.54
Aileron efficiency /%	$\geq 65\%$	61.7%	61.4%	61.6%
Flutter speed /(m/s)	$\geq 320$	356	358	358

Table 3 Comparison of optimized results by the finite element model and Kriging method

The comparison result shows that the optimal solution computed by Kriging model satisfies all the constraints, while the weight is a little heavier compared with the finite element model. Besides, the comparison of computation consumption of every population is shown in Figure 4.

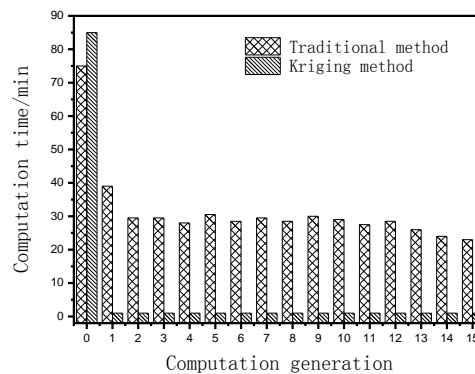


Figure 4 Comparison of computation consumption of FEM model and Kriging model

Compared with computation of other populations, the initial population based on Kriging model consumes most. The next population uses the existing results when coming across the same individual, and therefore the consumption continues to decline with the population increasing. The optimization based on Kriging model consumes most on constructing surrogate model, corresponding to initial population computation. The computation of next population based on Kriging model consumes much less, and judging from the total consumption, the computation based on Kriging model requires less and the efficiency can be improved greatly.



### 6.3 Optimization result influence factor

In order to verify the practicability of Kriging method, the influence of design variable number and the constraint number on the optimization consumption of Kriging model is studied.

#### 1) The influence of variable number

Aiming at the above example, the optimization is performed based on finite element model and Kriging model respectively with 24, 48 and 72 variables, and the comparison is shown in Table 4. (T represents finite element model, and K represents Kriging model)

Design variable number	24		48		72	
Method	T	K	T	K	T	K
Mass/%	1	1.029	1	1.031	1	1.023
Computation time/h	9.0	1.5	9.0	1.7	9.3	2.0

Table 4 Comparison in optimized results with different design variable number

Judging from the computation consumption, the influence of variable number is not obvious for finite element method, but it is obvious for Kriging model. The main reason is that the samples for constructing Kriging model is positive relevant with variables, therefore the computation consumption increases with the increasing of constraints. Judging from the objective mass, the optimal mass of Kriging model is heavier, while the mass deviation is not obvious with the variable increasing.

#### 2) The influence of constraint number

Aiming at the above example, the optimization is performed based on finite element model and Kriging model respectively with 8, 10 and 12 constraints. The comparison is shown in Table 5.

Constraint number	8		10		12	
Method	T	K	T	K	T	K
Mass/%	1	1.015	1	1.019	1	1.023
Computation time/h	8.1	2.0	8.5	2.0	9.3	2.1

Table 5 Comparison in optimized results with different constraint number

Judging from the computation consumption, the influence of constraint number for Kriging model is not obvious, but it is obvious for finite element model. The main reason is that the calculating cases for modeling Kriging model increase with the increasing of variables, which leads to consumption increasing. Judging from the objective mass, the optimal mass of Kriging model is still heavier, and the mass deviation increases a little with the design variable increasing.

## 7 CONCLUSIONS

Aiming at the great computing consumption in the aeroelastic optimization of complex structural model, a high efficiency method of aeroelastic optimization based on Kriging model instead of complex finite element model is presented.

Judging from the optimal solution, the optimal solution computed by the Kriging model satisfies all the constraints. Although the objective mass is a little heavier, it is still acceptable.

Judging from the computation efficiency, the computation consumption can be reduced much using Kriging model.

The influence of variables and constraints on computation consumption is studied. The result illustrates that, the influence of design variable number for finite element model is not obvious, but it is obvious for Kriging model, and the mass deviation is not obvious with the variable increasing, while the influence of constraint number for Kriging model is not obvious, but it is obvious for finite element model, and the mass deviation increases a little with the variable increasing.

## 8 REFERENCES

- [1] Yang Naibin, Liang Wei. Introduction to com-posite structural design for lager aircraft [M]. Beijing: Aviation Industry Press, 2009.12
- [2] Librescu L., Khdeir A. A. Aeroelastic Divergence of Swept-Forward Composite Wings Including Warping Restraint Effect[J]. AIAA Journal, 1988, V26(11): 1373-1377
- [3] Isogai K. Direct Search Method to Aeroelastic Tailoring of a Composite Wing under Multiple Constraints[J]. Journal of Aircraft, 1989, V26(12): 1076-1080
- [4] Livne E., Schmit L. A., Friedmann P. P. Towards Integrated Multidisciplinary Synthesis of Actively Controlled Fiber Composite Wings[J]. Journal of Aircraft, 1990, V27(12): 979-992
- [5] Guo S., Cheng W., Cui D. Aeroelastic Tailoring of Composite Wing Structures by Laminate Layup Optimization[J]. AIAA Journal, 2006, V44(12): 3146-3149
- [6] Wan, Z. Q. Genetic/gradient based hybrid optimization technology for composite aeroelastic structure. Beijing University of Aeronautics and Astronautics, Beijing, 2003
- [7] Shinkyu JMM, K. Y. Efficient Optimization Design Method Using Kriging Model. AIAA-2004-118
- [8] MashiroK, KentaroT, SinkyuJ, KazuomiY. Multi-Objective Aerodynamic Optimization of Elements' Setting for High-Lift Airfoil Using Kriging Model. AIAA-2006-1471.
- [9] Nithin Kolencherry, William Crossley. Multi-Fidelity Optimization Strategies Using Genetic Algorithms and Sequential Kriging Surrogates[C]. 50th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition, 2012
- [10] Liang Lu, Wan ZhiQiang, Yang Chao. Aeroelastic Optimization on Composite Skins of Large Air-craft Wings[J]. SCIENCE CHINA, 2012, 42(6): 722-728.
- [11] Tang Changhong, Wan Zhiqiang. Application of Hybrid Genetic Algorithm in Aeroelastic Multidisciplinary Design Optimization of Large Aircraft. Transactions of Nanjing University of Aeronautics&Astronautics. Jun.2013
- [12] S. Rajagopal, Ranjan Ganguli. Multidisciplinary Design Optimization of a UAV Wing using Kriging based Multi-Objective Genetic Algorithm. 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference<br>17th 4 - 7 May 2009, Palm Springs, California