MULTIOBJECTIVE OPTIMIZATION FOR SERVOELASTIC RESPONSE PREDICTIONS OF A FLY-BY-WIRE AIRCRAFT

Prabha Srinivasan[1], Ashok Joshi[2], Hemalatha E[1]

[1] Scientist, Aeronautical Development Agency
PB 1718, Vimanapura Post, Bengaluru, Karnataka, India,560017
prabha@jetmail.ada.gov.in

[2] Prof. & Head, Aerospace Engg. Dept., IIT Bombay
Powai, Mumbai, Maharashtra, India, 400076
ashokj@aero.iitb.ac.in

Keywords: servoelastic response, model updation, multi-objective optimization.

Abstract: The present study focuses on the use of optimization methods for finite element model updation, with the specific objective of achieving improved servoelastic response predictions. Test-analysis deviations of the frequency and mode shapes are generally minimized based on the frequency difference and the modal assurance criterion values. To minimize the differences between analytical and test frequency responses, multiple objective functions are set up based on the frequency domain assurance criterion. Model updation is carried out using genetic algorithms and the approach is illustrated on a generic flexible airvehicle. Fitness functions are set up to improve the test analysis correlations of direct longitudinal transfer functions arising from a symmetric wing control surface excitation, namely pitch rate response and normal acceleration response. The improvement in the analytical predictions of important modal responses associated with symmetric bending of wing and fuselage and symmetric pitching of external stores are shown for two different aircraft configurations.

1 INTRODUCTION

As compared to model updation for a structural dynamics problem, development of a robust model for servoelastic response prediction is much more challenging. Servoelasticity being a multi-disciplinary field, both structural dynamics and control dynamics have to be considered in the formulation of the responses. For a given control surface excitation, both direct and cross responses are set up and the longitudinal, lateral and directional responses have to be assessed. For a combat aircraft, different aircraft configurations arise during flight due to internal fuel and drop tank fuel consumption, release of external stores, etc. Different aircraft configurations in terms of mass variations, support conditions and combination of external stores have to be considered. For each mode, the test analysis predictions may vary for different transfer functions of sensor response to control surface excitation.

A mathematical equation relating the factors influencing the peak modal amplitudes of response of a flexible airvehicle have been identified [1]. The parameters influencing the peak amplitude of modal response are the stiffness, mass and modal damping, accelerometer or rate gyro locations, coupling inertia and stiffness and the actuator stiffness and damping. A sensitivity study carried out has shown the extent of influence of these parameters on the servoelastic response. The study also indicated that tuning a combination of the above parameters to optimized values would result in an improved model for servoelastic response predictions. In the present work, a multiobjective optimization approach using genetic algorithms has been developed for model updation. The multi-objective optimization approach for modal analysis of a cantilever beam problem from literature [2], with a cost function for minimization comprising frequency and modal assurance criterion (MAC) has been studied. The method is extended for model updation of a real life response problem for a flexible fly by wire aircraft, by carrying out optimization using a cost function based on the frequency domain assurance criterion (FDAC).

2 MODEL UPDATION METHODS

A considerable amount of literature exists covering analytical model improvement methods in structural dynamics. The studies are aimed at achieving an improved test analysis correlation and comprehensive reviews are given in [3-5]. Two methodologies in use are model refinement and model updating. The former involves changing the physical principle in modeling like using non-linear methods, microscale modeling, etc. whereas model updating uses mathematical means to match predictions with physical observations [6]. Model updation methods which can in general be classified into modal domain methods and frequency response based methods. The methods can be further classified into direct and indirect methods. Direct methods involve model updation by direct (mass, stiffness, damping) matrix updation [7]. They have the limitation that corrections may result in loss of connectivity and physical significance. Indirect or iterative methods involve tuning of material or geometric parameters of the structure [8-9]. Here model parameters are identified and updated. The solutions are not unique but the physical significance is retained. It is generally assumed for model updation that the test modes and frequencies are accurate and the analytical model is good enough so that only small changes have to be effected to achieve a good correlation with test results. But limitations exist in industrial application of above methods due to the size and complexity of real-life problems. In the aerospace domain, an aircraft finite element model could easily have over a lakh of degrees of freedom. To cover both symmetric and asymmetric condition, for instance, where a store is released from the left or right wing, the full model has to be analysed. Due to the use of different composites and zones, different materials and physical properties as necessitated by design, the number of variables that can be tuned is considerable. The selection of appropriate influencing parameters and regions to tune or finding an optimal combination of parameters for updation or assigning appropriate weightage to different parameters is generally problem specific and presents a big challenge.

The current study therefore focuses on the use of optimization methods for finite element model updation, which requires the setting up of an objective function or a set of objective functions for minimization or maximization. Optimization procedures can be classified into classical, evolutionary and multiobjective optimization methods.

In classical optimization, an optimum solution is achieved based on deterministic procedures [10]. An initial value has to be specified and the algorithm computes the search direction and identifies a single solution in each iteration. Classical optimization methods can be further classified into direct methods and gradient based methods. In a direct search, only the objective function and the specified constraints are used. In the gradient based method, the search procedure uses the objective function and specified constraints along with the first/second derivative information of the objective function to guide the search towards the optimum. Therefore, gradient based methods are generally more efficient and converge faster as compared to direct methods.

Classical optimization techniques however, have some limitations. The initial solution dictates the convergence to an optimal solution or only a local optimum may be detected. The algorithms are generally problem specific and a parallel mode of working may be difficult to achieve. Evolutionary algorithms may alleviate some of the above difficulties and are increasingly replacing classical methods for solving practical optimization problems.

Evolutionary algorithms are different in the sense that they adopt search and optimization procedures based on principles of nature [11]. The basic difference as compared to classical optimization is that the procedures are stochastic and more than one optimum solution is detected. This is useful in a complex real world problem, especially where multiple conflicting objectives exist. The optimization procedure adopted in the present work for application to finite element model updating uses genetic algorithms (GA). Other currently popular optimization methods in the literature are the particle swarm optimization (PSO) and the simulated annealing (SA) technique. The salient features of GA and its implementation are summarized below.

2.1 Genetic Algorithms

The concept of Genetic algorithms was first introduced by John Holland of the University of Michigan, Ann Arbor, USA in 1965 [12]. Natural genetics and natural selection, Darwin's theory of evolution and 'survival of the fittest', form the basis of genetic algorithms. In GA terminology, a solution vector $x \in X$ is called a chromosome. Chromosomes are made of discrete units called genes. Each gene controls one or more features of the chromosome. A collection of chromosomes is called a population. For starting the optimization, a random population is chosen as the initial set. Each solution in the population is assigned a fitness value. An optimality check is carried out and if required, the next generation of population evolves by the process of reproduction, crossover and mutation. In each subsequent generation, fitter and fitter population is produced until a feasible optimal solution is identified. The flowchart in Fig.1 [11] illustrates the working of the genetic algorithm.

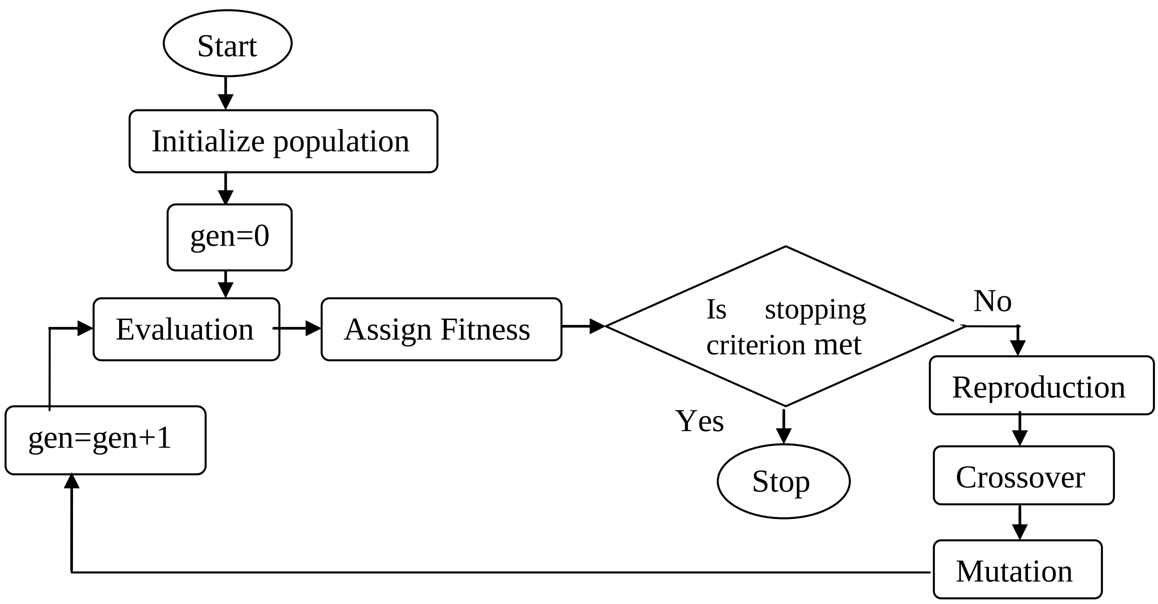

Figure 1 : Flowchart showing working of GA

The reproduction operator makes multiple copies of good solutions and uses these to replace bad chromosomes in a population set. It does not create any new solutions. Some common methods adopted for reproduction are tournament selection, proportionate selection and ranking selection. On the other hand, creation of new solutions is achieved by both crossover and mutation operators. In the crossover operation, two chromosomes with better fitness values are picked from the parent population set and are combined together to create two new chromosomes. Single point or multi point crossovers can be employed. The mutation operator makes random changes to a single chromosome from the parent population set. It is responsible for introducing diversity into the population. If bad chromosomes are created, these are expected to be eliminated by the reproduction operator in subsequent generations. Since GA works on a population, a set of solutions can be expected in any iteration.

2.2 Multi-objective optimization

The GA can be used either for single objective optimization or for multi objective optimization, which is more representative of a general real-life problem. A multi-objective optimization problem (MOOP) deals with the minimization or maximization of more than one objective function. Each of these may have constraints which the feasible solution must satisfy. The general form of a MOOP is given as :

$$\text{Optimize} \quad f_m(x) \qquad m=1,2,\dots,M$$

subject to

$$g_j(x) \geq 0 \qquad j=1,2,\dots,J$$

$$h_k(x) = 0 \qquad k=1,2,\dots,K$$

$$x_i(L) \leq x_i \leq x_i(U) \qquad i=1,2,\dots,N$$

where, $x=\{x_1 \ x_2 \ \dots \ x_r\}^T$ is the vector of r decision variables, lying in a decision variable space with lower and upper bounds $x_i(L)$ and $x_i(U)$. Functions $g_j(x)$ and $h_k(x)$ define the inequality constraint and the equality constraint. Additionally, in a multi-objective optimization, the set of objective functions constitute the objective space Z i.e., $f(x) =\{z_1 \ \ z_2 \ \dots \ z_M\}^T = Z$.

A feasible solution lies within the decision variable space and satisfies all defined constraints. The set of all feasible solutions is the feasible region S. Since the multiple objectives may generally conflict with each other, the solutions are divided into a dominant set and a non-dominant set. A solution $x^{(1)}$ is said to dominate another solution $x^{(2)}$ if (a) and (b) below are true :

(a) $x^{(1)}$ is no worse than $x^{(2)}$ in all objectives

(b) solution $x^{(1)}$ is better than $x^{(2)}$ in at least one objective

In the context of mutli-objective optimization, all non-dominant solutions are called Pareto optimal solutions. The curve joining all Pareto optimal solutions which exist in the search space is termed the Pareto-optimal front (Fig.2).

Two approaches are generally followed in multiple-objective optimization. One is to combine the individual objective functions into a single function using suitable weighting functions. However, in practice, it is generally rather difficult to select the weights as even slight changes in the assigned weights may influence the results considerably. The second approach is to determine a Pareto optimal solution set. Here the user can make a trade-off among the various conflicting requirements, which would be very useful in real-life problems.

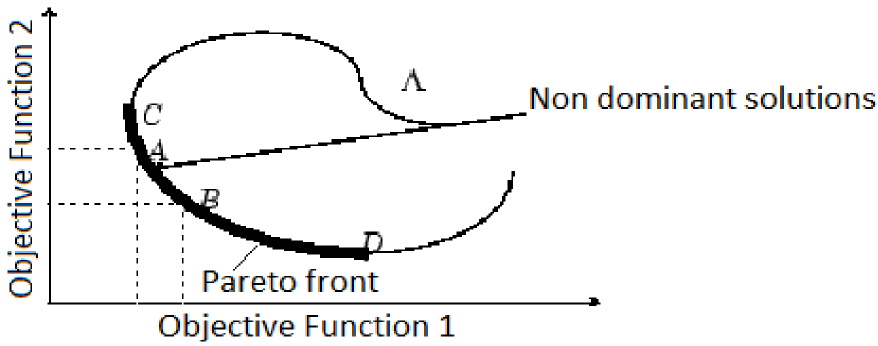

Figure 2 : Pareto optimal front

3 SERVOELASTIC RESPONSE COMPUTATIONS

An analytical model has been developed to compute the longitudinal, lateral and directional responses for symmetric and antisymmetric wing control surface excitation and for rudder excitation of a flexible airvehicle. In the present work, the normal acceleration response (n_z) and the pitch rate response (q) to unit symmetric excitation of the control surface (de) are studied. The normal acceleration response is computed as :

$$n_z = \ddot{T}_z - l_x \ddot{R}_y + l_y \ddot{R}_x + \sum_{ne} \varphi_i \ddot{e}_i \tag{1}$$

And the pitch rate response is given by :

$$q = \dot{R}_y + \sum_{ne} \varphi'_i \dot{e}_i \tag{2}$$

Here, T_z, R_x, R_y are the aircraft translation/ rotations about CG along global axis. l_x ,l_y are distance of acclerometer from aircraft C.G. The single and double dot superscripts indicate the velocity and acceleration terms. *ne* is the number of flexible modes considered for the analysis and φ_i, φ'_i are the modal amplitude and slope at sensor location for the e_i^{th} modal response.

3.1 Rigid and elastic components of response

As seen from Eq.(1) and (2), servoelastic responses comprise both rigid and elastic components. It is the feedback of the elastic response that could lead to structure control coupling instabilities and is to be attenuated. The rigid, elastic and total servoelastic response with respect to normal acceleration and pitch rate are plotted in Fig.3 and 4 for aircraft with outboard wing store and with all wing stations loaded respectively. It is seen that at lower frequencies upto 6 Hz, the rigid modes are the major contributors to the SE response. The levels of response picked up at the higher frequencies corresponding to elastic modes of the structure are significant. Therefore the estimation of frequencies and gain amplitudes of servoelastic response for different aircraft configurations becomes important.

The responses in Fig.3 and 4 can be explained with respect to the actuator frequency response function. In the frequency range where the rigid actuator gain is almost constant, the aircraft response due to control surface actuation is linear. Beyond this, the response is governed by the rate at which the actuator gain falls per decade. Therefore, for the present case, it is seen that the n_z gain amplitude is almost constant and the q gain amplitude falls at around 20 dB per decade.

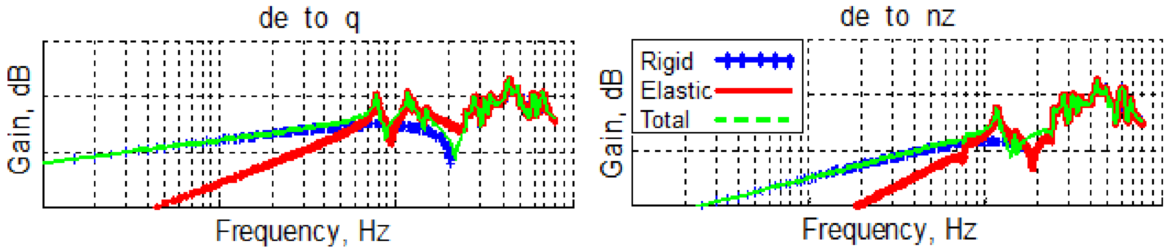

Figure 3: Rigid, Elastic and Total SE response, with OB wing store (a) *de-q* (b) *de-nz*

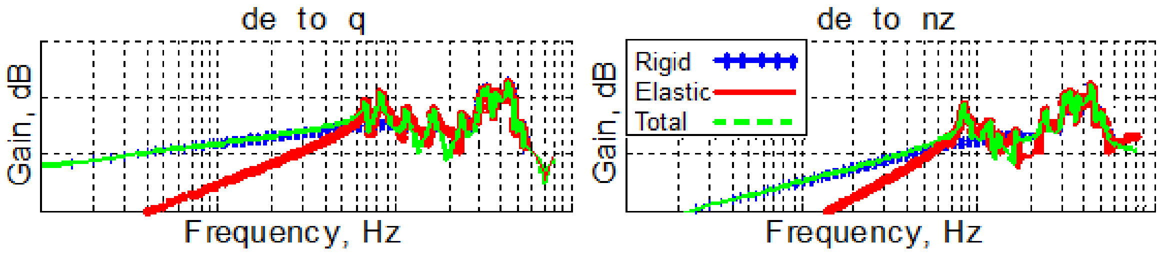

Figure 4 : Rigid, Elastic and Total SE response, with wing stores (a) *de-q* (b) *de-nz*

3.2 Gain amplitude of response

From the equation of equilibrium for servoelastic response and basic structural dynamics it can be shown as in [1] that the amplitude of normal acceleration at the i^{th} mode is given by :

$$(n_{zo})_i = \frac{\varphi_i[\omega_i^4 (M_{q\delta})_i - \omega_i^2 (K_{q\delta})_i]\delta_o}{g_i (K_{qq})_i} \tag{3}$$

Similarly, the amplitude of pitch rate response at the i^{th} mode is given by :

$$(q_o)_i = \frac{\varphi_i'[\omega_i^3 (M_{q\delta})_i - \omega_i (K_{q\delta})_i]\delta_o}{g_i (K_{qq})_i} \tag{4}$$

where, ω_i is the resonance frequency in the i^{th} elastic mode, δ_o is the control surface rotation at that frequency for unit harmonic command input. (g_i), $(K_{qq})_i$ are the generalized damping and stiffness and $(M_{q\delta})_i$, $(K_{q\delta})_i$ are the inertia coupling and stiffness coupling terms in the i^{th} elastic mode. From Eq.(3) and (4), it is seen that the longitudinal responses are a direct function of the natural frequency, accelerometer and rate gyro locations and the coupling inertia and stiffness terms. The response levels vary inversely with the structural stiffness, modal damping, actuator stiffness and damping. Further, the more critical among these influencing parameters as identified though a sensitivity study are the mass, stiffness, modal damping, sensor location, coupling inertia and actuator damping and stiffness.

4 TEST ANALYSIS CORRELATION

Testing is a part of the design and certification process and it plays an important role in confirming the design adequacy and assists in certification. Multiple test platforms and procedures are adopted to cater to all the requirements of an integrated aircraft system. Tests are conducted at every stage in the development of aircraft, starting from material characterization, component and system level to full scale aircraft. Ground Vibration Tests (GVT) and Structural Coupling Tests (SCT) are among the first tests to be conducted on a flight standard (in terms of stiffness and mass distribution, FCS hardware and software configuration) version of a developmental combat aircraft.

A schematic of the generic flexible fly-by-wire airvehicle used for the present study, with the actuator and sensor placement is shown in Fig.5. The aircraft has five control surfaces, inboard and outboard elevons on each wing and the rudder. Structural coupling tests were carried out for important aircraft configurations for assessment of servoelastic responses. The inertial forces to excite the aircraft were generated by applying stepped sine signals to the control surfaces in a frequency range chosen to cover all important fundamental structural modes of the aircraft. The amplitude of the sinusoidal signals was selected so as to ensure that the structural response is approximately linear.

The analytical computations were carried out using Eq.(1) and (2). Both analytical and experimental results were expressed as transfer functions of sensor response to input control surface symmetric excitation (*de-q* and *de-nz)*. The initial test analysis correlation of the longitudinal servoelastic responses are shown in Fig.6.

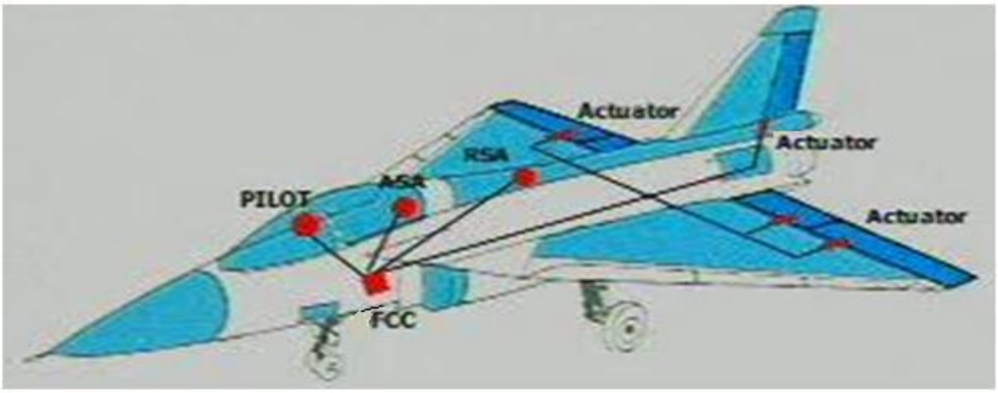

Figure 5 : Schematic of Aircraft model with actuators and sensors

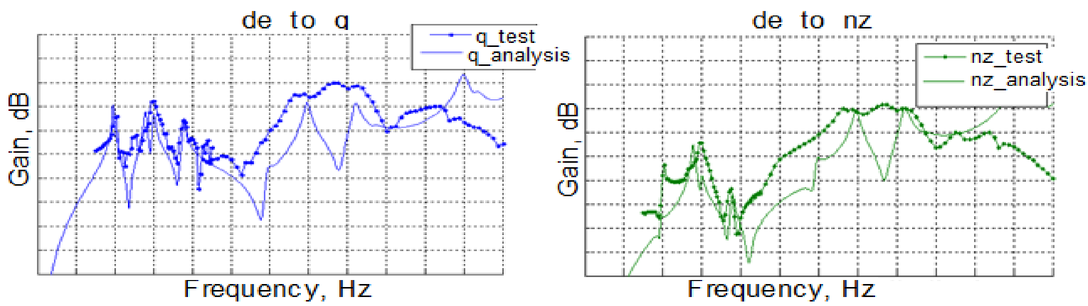

Figure 6 : Test analysis correlation of longitudinal response (a)*de-q* (b)*de-nz*

5 SETTING UP FITNESS FUNCTIONS FOR MINIMIZATION

Optimization using multiobjective GA is carried out for a cantilever plate and the methodology is extended for a generic flexible aircraft model. The fitness functions for the finite element model updation in respect of natural frequency and mode shapes for the cantilever plate are set up as in [2]. For the aircraft model, the objective is to arrive at an improved test analysis correlation of servoelastic responses in a required range of frequency.

Fitness functions are set up based on the fundamental frequencies and the frequency response functions (FRF).

5.1 Frequency and Mode shape updation : Cantilever Plate

Multiobjective optimization using GA is implemented as in [2] for updation of modal analysis results. A steel cantilever plate of 95 cm x 30 cm, with a thickness of 4 mm is considered for the study (Fig.7a). A crack is introduced at the root end of the plate (Fig.7b). The objective of the study is to reproduce the reduced frequencies due to the crack and the associated mode shapes by updating the model of the original defect free plate. On the basis of sensitivity of the objective function with respect to an updation parameter, the model is divided into three super elements (along the length) or six super elements (along length and width). The problem has been reproduced in the current work and the results obtained vary slightly from [2]. This can be attributed to the fact that optimization results depend on the finite element discretization from which initial predictions are generated, use of GA, initial values and bounds assumed for influencing parameters, etc.

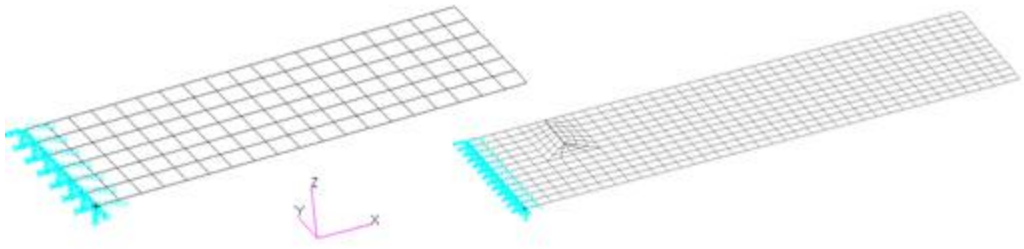

Fig.7 (a) Cantilever plate (b) Cantilever plate with a crack

The first four modes of the cantilever plate were considered and minimization of following objective functions based on test analysis frequency differences and MAC was carried out using multiobjective GA :

$$\sum_{i=1}^{4}(fx_i - fa_i)^2, (1 - MAC_{ii}) \text{ for } \quad i = 1 \quad \text{to} \quad 4 \tag{5}$$

with $$MAC_{ij} = \frac{\left|\{\phi_{Ai}\}^T \{\phi_{Xj}\}\right|^2}{\left(\{\phi_{Ai}\}^T \{\phi_{Ai}\}\right)\left(\{\phi_{Xj}\}^T \{\phi_{Xj}\}\right)} \tag{6}$$

where, ϕ_{Ai} and ϕ_{Xj} are the i^{th} analytical modal vector and j^{th} test modal vector [13].

Table 1 shows the test values, initial analysis values along with the improvement achieved by carrying out the analysis using updated models containing three and six super elements.

	Test	Initial Analysis		Analysis with 3 superelements		Analysis with 6 superelements	
Mode	Freq, Hz	Freq,Hz	MAC	Freq,Hz	MAC	Freq,Hz	MAC
1	3.58	3.79	0.9910	3.47	0.9721	3.53	0.939
2	22.53	23.63	0.3172	22.47	0.6751	22.15	0.901
3	23.73	23.87	0.2693	23.74	0.7038	23.51	0.935
4	65.08	66.18	0.9044	65.10	0.871	65.16	0.81

Table 1: Test analysis comparison : Initial and updated values of frequencies and MAC

As optimization is carried out with multiple objective functions, with each iteration comprising different population sets, many feasible solutions may be identified. For six different solution sets from the pareto front, the fitness function values are plotted in Fig.9(a) and 9(b) for model with three super elements and six super elements respectively. It is seen that dividing the model into six sets of superelements gives better results in terms of simultaneous updation of frequencies and mode shapes. As seen in Fig.9(b) for model with six superelements, while solution sets 6 and 3 are the preferable options, the former leads to better updation for mode shapes whereas the latter results in better frequency updation.

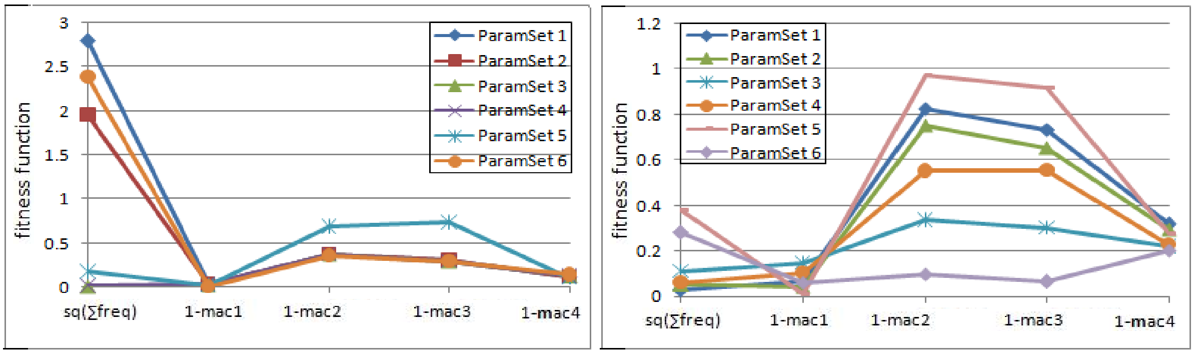

Fig.9 Values of Fitness functions for different pareto optimal parameter sets :
Model with (a) 3 superelements (b) 6 superelements

5.2 Frequency Response updation : Aircraft Model

Six rigid body and fifty four elastic modes were used along with control surface rotation and actuator degrees of freedom for estimation of servoelastic responses. Test analysis deviations, in terms of frequencies and gain amplitudes are within 15% as seen in Fig.6. The initial analytical model represents the basic dynamic characteristics of the aircraft and model updation can be taken up for an improved prediction of servoelastic responses. The test analysis correlation of three important modes of longitudinal response, wing symmetric bending (WSB), outboard store symmetric pitch (SSP) and fuselage vertical bending (FVB) (Fig.6 and 7), were studied for a baseline configuration of aircraft with outboard wing stores present (Config.I). FDAC was computed for the three associated frequency ranges. FDAC correlates analytical and experimental FRF [14] and is evaluated like MAC :

$$FDAC(\omega^A,\omega^X)=\frac{\left|\{H(\omega)^A\}.\{H(\omega)^X\}^T\right|^2}{\left(\{H(\omega)^A\}.\{H(\omega)^A\}^T\right).\left(\{H(\omega)^X\}.\{H(\omega)^X\}^T\right)} \tag{7}$$

where, $H(\omega)^A$ and $H(\omega)^X$ are the analytical and test FRF respectively.

Multiobjective optimization using GA was carried out using three objective functions for minimization :

$$(1\text{-}FDAC_1),\ (1\text{-}FDAC_2) \text{ and } (1\text{-}FDAC_3) \tag{8}$$

The three initial FDAC values were computed in a certain frequency range associated with WSB, SSP and FVB (shown in Fig.10) as 0.85, 0.7 and 0.72 and need to be enhanced to a value ≥ 0.9.

The initial test-analysis correlation of mode shapes is shown in Fig.11. The MAC values were computed for all the three modes and found to be ≥ 0.6. Therefore, the initial mode shapes are satisfactory and it is assumed that the mode shapes do not need corrections.

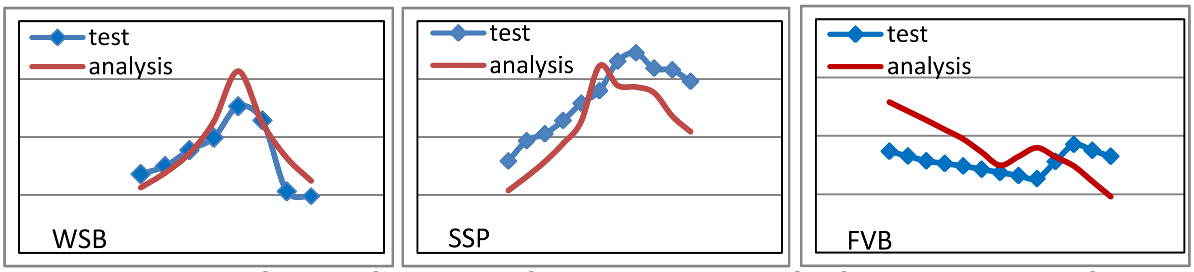

Fig.10 Test analysis correlation : Servoelastic response associated with WSB, SSP, FVB modes

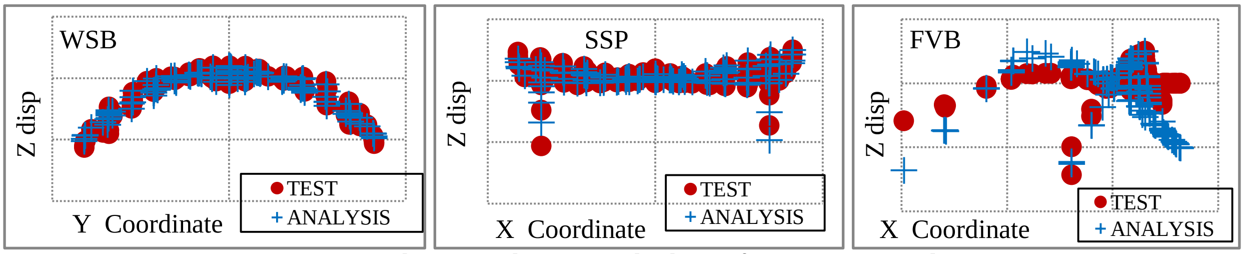

Fig.11 Test analysis correlation : mode shapes for WSB, SSP and FLB

To enhance the servoelastic response predictions, the variables or the parameter set considered for updation were stiffness, modal damping, actuator stiffness and damping and the inertia coupling terms. Bounds of ±10% were imposed for each parameter. This ensures that the implemented updation is meaningful and still representative of the original structure.

A second optimization was carried out, considering additionally the frequency differences for the WSB, SSP and FVB modes. A total of six fitness functions were considered for the multiobjective optimization :

$$(\textit{analysis frequency} - \textit{test frequency})_i^2 \text{ and } (1\text{-}FDAC_i), \qquad \text{for } i = 1 \text{ to } 3 \tag{9}$$

While similar results can be achieved by both sets of fitness functions (8) and (9), the latter approach with a set of six fitness functions was beneficial since additional weightage to the modal frequencies of interest was ensured and the number of iterations could be reduced. The resulting updated test analysis correlations for WSB, SSP and FVB are shown in Fig.12.

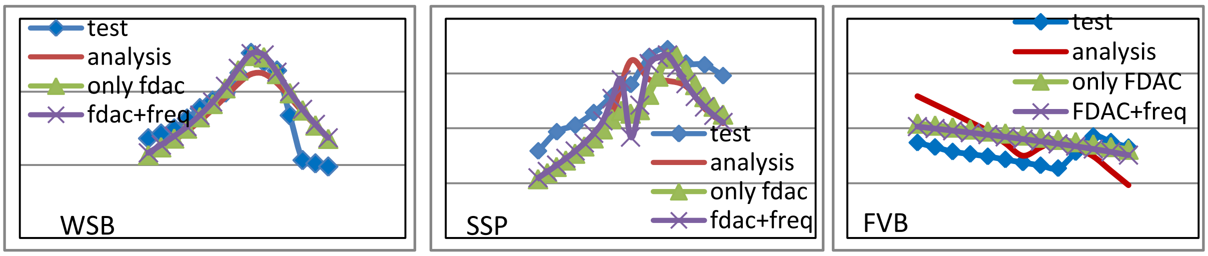

Fig.12 Test analysis correlation for WSB, SSP and FVB : model updation with FDAC and FDAC & frequency

The above optimization approach was applied for another aircraft configuration, namely with all wing mounted stores present (Config.II). The region of store attachments, wing tip and front fuselage regions were the different zones taken for computing the optimal parameter correction factors. Here, the servoelastic responses corresponding to the other store modes also had to be considered and the fitness functions for optimization were taken as :

$$(\textit{Analysis frequency} - \textit{Test frequency})_i \text{ and } (\textit{1-FDAC}_{\,i}) \text{ , for } i = 1 \text{ to } 5 \qquad (10)$$

The five important longitudinal modes considered in (10) above were wing symmetric bending and fuselage vertical bending along with the symmetric pitching of inboard (ISP), midboard (MSP) and outboard (OSP) wing mounted external stores. The initial and updated results for servoelastic responses associated with the wing and fuselage modes and for the store pitching modes are shown in Fig.13 and Fig.14 respectively. Further work is being carried out for better identification of critical zones for parameter updation, which would result in a more improved test analysis correlation.

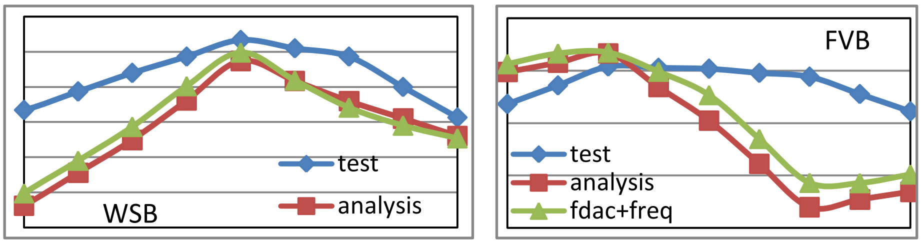

Fig.13 Test analysis correlation for WSB and FVB (Config.II) : model updation with FDAC / Frequency

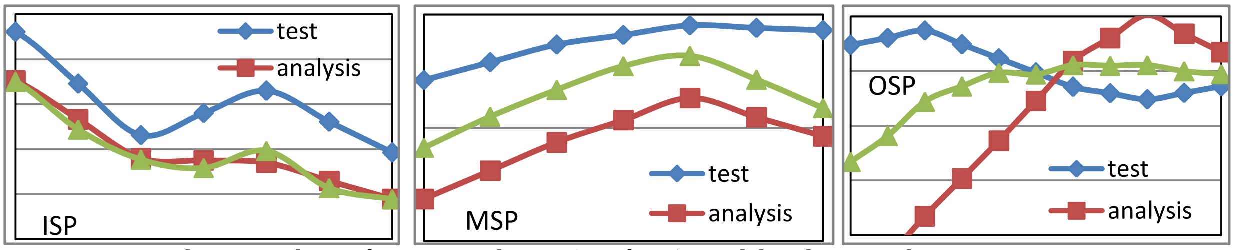

Fig.14 Test analysis correlation for WSB and FVB (Config.II): model updation with FDAC / Frequency

6 CONCLUSIONS

The objective of developing an approach to arrive at an improved test analysis correlation of servoelastic responses of an aircraft was achieved using multi-objective optimization and genetic algorithms. Fitness functions were based on the differences in the test-analysis predictions of fundamental frequencies and the frequency response functions. The parameters to be updated included actuator stiffness, damping and inertia coupling terms in addition to the mass, stiffness and modal damping generally considered. The procedure can be adopted for preliminary analytical estimation of servoelastic responses with different store combinations or for studying the effects of minor changes in the airframe structure.

7 ACKNOWLEDGMENTS

The first author thanks the Technology Director (AirFrame) and Program Group Director and Director (CA) for their permission to publish this paper.

8 REFERENCES

[1] Ashok Joshi, Prabha Srinivasan, Indira Narayanaswamy and Hemalatha E., Identification of critical parameters for improved servoelastic response predictions of a flexible aircraft, *18th NASAS*, VNIT, Nagpur, India, December 2014.

[2] G.H. Kim and Y.S. Park, Finite Element Model Updating Using Multiobjective Optimization Technique, *Proceedings of the 19th International Modal Analysis Conference,* November 2005, pp. 348-354.

[3] J.E.Mottershead and M.I.Friswell, Model updating in structural dynamics: A Survey, *Journal of Sound and Vibration*, 167(2), 1993, pp. 347–375.

[4] Pilkey, D. and Inman, D., "A Survey of Damping Matrix Identification", *Proceedings of the 16th International Modal Analysis Conference*, February 1998, pp. 104-110.

[5] M.S.Ewins, Comparison of popular finite element model updating techniques, *AIAA paper* 92-2135-CP, pp.550-555

[6] Ying Xiong, Wei Chen and Kwok Leung Tsu, A better understanding of model updating strategies in validating engineering models, *49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Schaumburg, April 2008.

[7] Berman A., and Nagy, E.J., Improvement of a large analytical model using test data, *AMA Journal,* No.21, No.8, August 1983, pp.1168-1173.

[8] Hsin-Hsen Huang and Roy R.Craig, "Model updating based on sensitivities of system matrices", AIAA-93-1651-CP, pp.3041-3049.

[9] Pilkey, D. and Inman, D., "An iterative approach to viscous damping matrix identification", *Proceedings of the 15th International Modal Analysis Conference*, February 1997, pp. 1152-1159.

[10] S.S.Rao, Engineering Optimization, Theory and Practice, 3rd Ed., Publ. New Age International, 2010.

[11] Kalyanmoy Deb, Multi-Objective Optimization using Evolutionary Algorithms, Publ. John Wiley & Sons, 2010.

[12] John Holland, Genetic Algorithms, *Scientific American*, July 1992, pp.66-72.

[13] D.J.Ewins, Model validation: Correlation for updating, *Sadhana*, Vol.25, Part 3, June 2000, pp.221-234.

[14] D.Fotsch and D.J.Ewins, Application of MAC in the frequency domain, *IMAC-XVIII*, International Modal Analysis Conference, 2000.

9 COPYRIGHT STATEMENT