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RELIABLE & ROBUST OPTIMIZATION OF A LANDING GEAR SYSTEM

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Abstract: Optimization strategies are widely used in the design process across all engineering fields. The consideration of uncertainties increases the difficulties in determining an optimum nominal design, and this is particularly true for systems whose numerical model is computationally expensive to simulate. Robust design optimization (RDO) and reliability-based design optimization (RBDO) are the two techniques commonly adopted to deal with the optimization of performance of systems under uncertainty. In the presence of a very computationally expensive numerical model, nonlinear behaviour and multiobjective problems, the traditional RDO and RBDO are not always suitable because of two main problems: the prohibitive computational cost and the neglect of higher-order moments common for the RDO and RBDO techniques. Thus it is advantageous to develop a method that can assure reliability and robustness in the designs of complex engineering structures. In this paper, novel optimization techniques based upon an evolutionary approach are presented and demonstrated considering the design of landing gear systems to eliminate 'shimmy', an undesirable limit cycle oscillation.

1 INTRODUCTION

Engineering analyses aim to validate and to evaluate a system already designed or to actually define the design that fulfils some fixed requirement. Optimization processes are considered in the definition and improvement of such a design. Depending on the goal of the optimization, the optimization technique is selected; reliable or robust optimum can be of interest. On one hand the robustness consists of the *minimization of the variance* in the determined optimum, on the other hand reliability is achieved based on whether the *minimization of occurrence* of limit-state or constraints violations is assured. Difficulties during the optimization process arise due to the complexity of the description of locii of interest in the analysis and the presence of uncertainties (always present in all branches of physics and which limits the extent of analysis to be done due to the computational burden). These difficulties are particularly true for systems whose numerical model is computationally expensive to be simulated. In the optimization process, uncertainty in the system has not always been considered and the process is developed via a deterministic analysis ([1, 2]) or sometimes introducing a safety factor ([3]). The consideration of uncertainties in the development and improvement of optimization processes has recently become of significant interest in research and in industry. In fact, industrial organizations themselves are aware that a deterministic approach, with the application of a safety factor, involves either over or under designing each system system.

Techniques that are commonly adopted to optimize a system under uncertainties ([4]) are Robust Design Optimization (RDO) ([5]) and Reliability-Based Design Optimization (RBDO) ([6]). In RDO, the mean of the response of interest is optimized by minimizing its variance. In the RBDO, a cost function is minimized and the uncertainty considered, introducing specific risk and target reliability constraints that require tail statistics. In the presence of a very computationally expensive numerical model, nonlinear behaviour and multi-objective problems, the traditional RDO and RBDO are not always suitable because of two main issues: the prohibitive computational cost and the neglect of higher-order moments common for the RDO and RBDO techniques. Alternative optimization strategies have been proposed to overcome these problems, for instance aggressive design procedures in the field of turbomachinery. The main idea behind aggressive design techniques is to exploit the existence of a desired target; a desired nominal response of interest and its statistical properties are defined and the optimization is then performed such that the sought target-desired performance is matched as closely as possible ([7]). The difficulty in applying such an approach is that, depending on the problem, it is not assured that the desired target is matched 'closely' enough or that the computational effort is actually reduced.
The complexity of a problem can be reduced thanks to the adoption of specific techniques. The first step that should be accomplished to reduce the complexity is to identify the quantities that are of interest for the analyzed problem; the label QoI is here used as abbreviation for these entities. The factors that most influence the system can be isolated using sensitivity analysis techniques ([8]). Then, surrogate models and reduced order models (ROMs) can be constructed in terms of the identified factors and QoI. Finally, using the ROMs and suitable strategies to perform optimization and to propagate uncertainties, the optimum design candidates need to be critically assessed in terms of the defined objective functions and requirements, considering the most influential factors as uncertainties or design variables and applying suitable optimization strategies.

In this paper, the analysis is focused on a landing gear design, in which the goal of the optimization is to decrease the probability of occurrence of shimmy during ground manoeuvres. Two optimization techniques have been developed and are presented here, validating them using two landing gear systems: an analytic and multi-body landing gear system. Both the optimization techniques aim to limit the number of evaluations of the objective function without involving approximations in the computation, guarantee a minimization of the probability of failure, and avoid gradient calculations. The first to be presented (Iterative Distribution Evolutionary Algorithm - I.D.E.A.) is a reliability based method, while the second (Robust and Reliable Evolutionary Algorithm - R.R.E.A.) can be tuned in order to achieve a more reliable or robust optimum. The R.R.E.A. calculate the statistical quantities, keeping the computational cost low thanks to the use of Univariate Reduced Quadrature ([9]). The objective of this study is to demonstrate an improved approach to the design optimisation of complex systems in a computationally efficient way.

2 MATHEMATICAL MODEL

The developed techniques aim to decrease the probability of occurrence of shimmy phenomena in landing gear systems during ground manoeuvre [10, 11, 12]. Shimmy results from the nonlinear interaction between the follower forces acting on the tyre and the modes of vibration, resulting in Limit Cycle Oscillations (LCOs). Two landing systems have been considered for the validation of the developed techniques and bifurcation analysis has been adopted for determining boundaries of stable dynamic regimes [13], investigating limit cycle occurrence. The landing gear models, an overview of bifurcation analyses techniques and an explanation on how to perform bifurcation analysis for the case of interest is presented in this section.

2.1 Landing Gear Model

Both analytic and multi-body landing gear models are considered. The analytic model has been implemented first as it has been used for previous studies using Bifurcation analysis ([14]). Then, numerical models are introduced, due to the possibilities in describing the system of interest in more detail. In fact, they are adopted in industry for static and dynamic analyses in order to limit the assumptions adopted to describe the dynamics of the landing gear model. Therefore, implementing the analyzed techniques on a system represented by a multi-body modeling framework demonstrates their applicability not only in an academic environment but also potentially in industry.

Analytic Model. The analytic model is the one presented by Howcroft ([14]) representing a dual-wheel landing gear in which free-play and wheel gyroscopic effects are omitted. The deflection of the landing gear structure is modeled in terms of three degrees of freedom and an additional DoF is introduced for the tyre dynamics. There are seven states, since the differential equations for the first three DoFs are of second order while the last is of first order. The degrees of freedom are:

1. torsional, ψ, describing the rotation of the wheel/axle assembly about the local axis z;
2. in-plane, δ, expressing the bending of the oleo piston in the side-stay plane. This DoF is approximated as a rotation about a point at a distance L_δ from the axle;
3. out-of-plane, β, describing the rotation of the landing gear about the two attachment points;
4. lateral tyre displacement, λ, which is represented adopting the straight tangent model ([15]). The tyre model is the one used in ([14]) and does not consider the longitudinal side slip dynamics.

The adopted analytical landing gear model has been presented in [10] where uncertainty quantification in terms of shimmy phenomena was performed, thus the details of the system equations characterizing the model are omitted here.

Multi-body Model. The multi-body landing gear system is also dual-wheel and is modelled in LMS Virtual.Lab Motion [1]. Specific names are used here to refer to the different rigid body-parts that comprise the landing gear system. The landing gear has 4 main parts and five revolute joints: the wheels; the axle to allow the wheels to rotate (the wheels rotate around the axle

[1] Virtual.Lab Motion, url = http://www.plm.automation.siemens.com, date accessed June, 2015

through revolute joints). The wheel axle is connected to the bottom of the main structure of the landing gear (the 'trail') through a revolute joint that allows rotation around the 'y-axis' (Δ). The trail body is then connected to the main structure of the landing gear through another revolute joint, which allows the rotation around the 'z-axis' (ψ). The main structure is decomposed into three rigid parts, the one connected to the trail body is Main 3. The fuselage is connected to Main 1 through the revolute joint that allows the rotation around the 'x-axis' (δ). The Main body is divided in three parts in order to introduce a nonlinear shock absorber between the Main 3 and Main 2 and an additional revolute joint between Main 2 and the Main 1. This revolute joint allows a rotation around the 'y-axis' (β). The shock absorber is modelled via a vector force acting along the vertical axis connecting the central point of the lower and upper face of the Main 2 and Main 3 structures. The vector force simulates the presence of a nonlinear shock absorber in terms of stiffness (F_{shock_s}), damping (F_{shock_d}) and friction (F_{shock_f}) forces.
The model has 9 degrees of freedom, considering all the adopted constraints. The dynamics of each degree of freedom is described by a second order differential equation, so each degree of freedom is fully described by the use of two independent variables, the state and its derivative. The model and values adopted for the nonlinear shock absorber refer to the modeling presented in [16]. Regarding the adopted tyre models, a Pacejka-based model ([15]) has been adopted: this differs from the one used for the analytic landing gear model by introducing the longitudinal side slip dynamics as well as the lateral.

2.2 Bifurcation analysis

The term bifurcation was first introduced by Poincaré to describe the 'splitting' of equilibrium solutions in a family of differential equations. J. Guckenheimer provides the following definition [13]:

Definition Given a system of differential equations [2]

$$\dot{\mathbf{x}} = \mathbf{f}_{\mathbf{p}}(\mathbf{x}); \qquad \mathbf{x} \in \mathbb{R}^{\mathrm{n}}; \qquad \mathbf{p} \in \mathbb{R}^{\mathrm{k}} \tag{1}$$

depending on the k-dimensional parameter $\mathbf{p}$, then a value $\mathbf{p}_0$ of equation (1) for which the flow of (1) is not structurally stable is a *bifurcation value* of $\mathbf{p}$.

Common analysed bifurcations are those related to equilibrium solutions obtained varying the k-dimensional parameter $\mathbf{p}$. In particular, solutions are given by smooth functions of $\mathbf{p}$ and the graph representation of each of these solutions will be called *branches* of equilibria of (1). An equilibrium $(\mathbf{x}_0, \mathbf{p}_0)$ is called a point of bifurcation if at such a point the Jacobian matrix presents an eigenvalue with zero real part; in fact in this situation the equilibrium point $(\mathbf{x_0}, \mathbf{p}_0)$ may belong to several branches of equilibria. The implementation of bifurcation analysis entails the solution of all the steady states of the system in the parameter range of interest, along with a determination of their stability. Changes in local stability as a parameter varies are then assessed using bifurcation theory to infer the mechanisms governing more global behaviour. The results obtained can be graphically visualized and the plots are called bifurcation diagrams. Various methods can be adopted to perform bifurcation analysis, identifying the equilibrium branches for bifurcation diagrams and bifurcation points, possibly also evaluating the *periodical solutions*

[2] $\mathbf{x}$ is the state vector of the system.

in more than one parameter. Continuation has been here adopted since it gives the possibility of analyzing both equilibrium and periodical solutions and of performing bifurcation analysis directly using multi-body systems.

2.3 Bifurcation analysis and landing gear shimmy phenomena

Having considered *shimmy* in landing gear as the case study, the selected bifurcation parameters are the forward velocity V and the vertical load acting on the main structure of the landing gear. In fact, the operational parameter space for investigating the shimmy phenomena is the one identified by the variation of vertical load and forward velocity. The analysis has been performed using AUTO as the continuation and bifurcation software ([17]). Since the developed methodologies are implemented in Matlab, the Matlab interface with AUTO (the Dynamical System Toolbox [18]) has been adopted. The Dynamical System Toolbox[18] integrates AUTO into Matlab via mex functions to perform bifurcation analysis of dynamical systems for which an analytic description is available or that are modelled in software able to interface with Matlab. If an analytic system is considered, then the equations of motion can be expressed in a Matlab function that is evaluated during the continuation analysis ([17]). Figure 1 presents the exchange of information occurring between AUTO implementation in Matlab and the analytic system described using a Matlab function. If a multi-body model is adopted then it is necessary to couple multi-body software to the code that is able to perform bifurcation analysis, i.e. the Dynamical System Toolbox in this application[18]. Regarding the multi-body software, LMS Virtual.Lab Motion[3] is selected. This was interfaced with Matlab using a library available in VLM itself so that all the states of the model are available in the Matlab environment. Figure 2 presents the flow chart of the general process adopted for the coupling. To analyse the multi-body landing gear systems, independent states need to be selected and the decision as to which states are used is not unique; the way in which the independent states are chosen influences the results given by the continuation analysis and their correctness. Thus, particular attention needs to be paid to this matter [11].

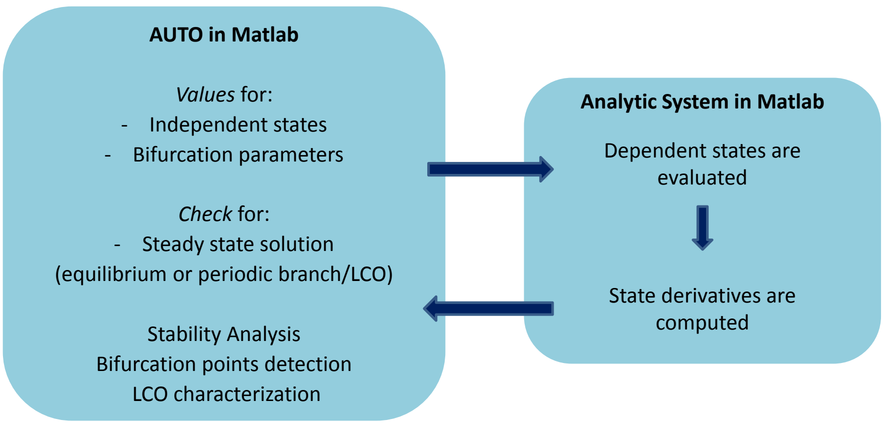


Figure 1: Flow chart describing the exchange of information needed to perform the bifurcation analysis of an analytic system using the Dynamical System Toolbox.

3 OPTIMIZATION TECHNIQUES

An optimization problem is formulated as follows. A vector of values $x_1, x_2, ..., x_N$ for an input factor vector $\mathbf{x}$ is sought in order to minimize the defined objective function $f(\mathbf{x})$. In the

[3]Virtual.Lab Motion, url = http://www.plm.automation.siemens.com, date accessed June, 2015

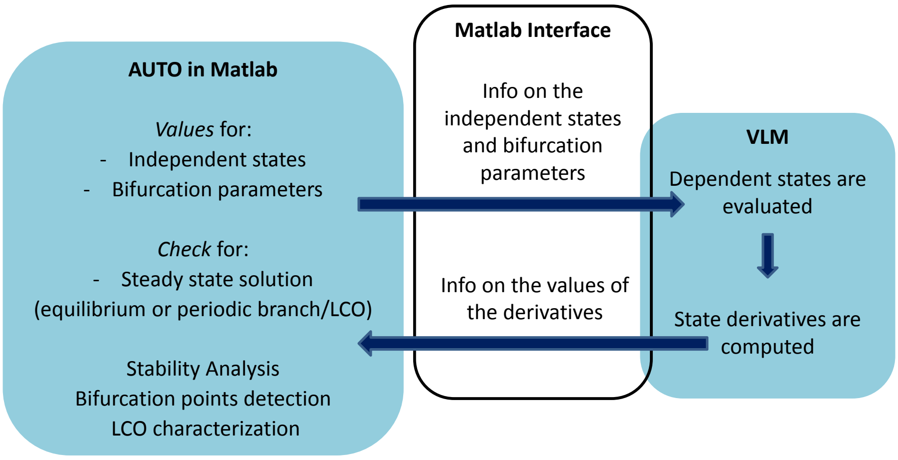


Figure 2: Flow chart describing the exchange of information needed to perform the bifurcation analysis coupling AUTO with VLM using the Dynamical System Toolbox and the Matlab interface available in VLM to 'handle' the states of the multi-body model.

proposed optimization strategies, the nominal values of the selected design factors are regarded as uncertain. However, there is no limitation in the design/uncertain factors that can be selected. The strategies are conceived in order to solve problems that are very time demanding and for which it is difficult (and expensive) to determine derivatives and to identify and define the optimum set of parameters. The achieved reduction in computation time is both in terms of number of cases to be analyzed directly through experiments, or runs of numerical models, and in eliminating the need to compute gradients. The completeness of the analysis is fulfilled thanks to the inclusion of methodologies to perform SA and propagate uncertainties in the system already considered by the authors for different applications, dealing with correlated aircraft loads ([19], [20], [21], [22], [23]), flutter ([24]) and shimmy occurrence ([12]). The application here has the aim of optimizing the landing gear design, decreasing the probability of occurrence of shimmy during ground manoeuvre. Figure 3 is a two-parameter bifurcation diagram, showing an example of what can occur due to uncertainty in the system.

The continuous blue and red lines are the lower and upper confidence bounds for the loci of Hopf bifurcation (Shimmy onset) points determined using a singular value decomposition (SVD) or high-order SVD (HOSVD) based method ([12]). The dashed red line is the operational trend. This is the variation of vertical force with V in typical take-off and landing scenarios. We do not consider uncertainty or variability in the trend and it can be regarded as representing a worst-case condition. For all the values of the forward velocity V between the first and second intersecting points, LCO (shimmy) can occur due to the uncertainty in the system. In the considered optimization, the operational trend shown in figure 3 is the limit state function (i.e. the variation of the vertical load on the landing gear as the forward velocity changes during a static manoeuvre on the ground of an illustrative aircraft in equilibrium conditions.) The stated variation can be determined employing equilibrium equations, giving the following expressions:

$$F_n = \frac{B_m}{B}(W - L) \qquad L = \frac{1}{2}\rho V^2 S C_L \tag{2}$$

where B_m is the track of the main assembly, B is the distance between the nose and the axis of the main assembly, W is the weight of the aircraft and L is the lift.

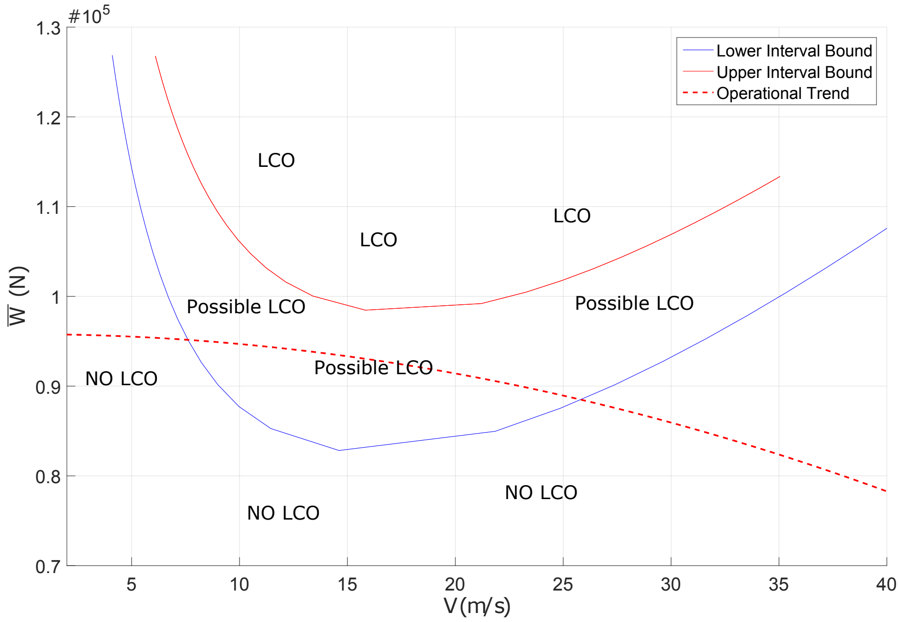


Figure 3: Example of lower and upper confidence bounds for the loci of Hopf bifurcation points and operational trend.

Given the task of selecting design parameters to avoid shimmy, the two developed techniques are:

- Iterative Distribution Evolutionary Algorithm - I.D.E.A., which has the aim of decreasing the probability of occurrence of shimmy during ground manouevres. In particular, this has been addressed here by making the probability of intersection between the loci of interest (loci of Hopf bifurcation) and the limit state function (the operational trend) as low as possible.
- Reliable & Robust Evolutionary Algorithm - R.R.E.A., which minimizes the selected objective function $f(\mathbf{x}) = |\mu_d + S\sigma_d|$, where S is a scale factor whose value affects the robustness/reliability of the optimum, μ_d and σ_d are the mean and deviation of the output of interest, which is $d(\mathbf{x})$. $d(\mathbf{x})$ is the distance $\sqrt{(\overline{W}_{operational} - \overline{W}_{loci})^2 + (V_{operational} - V_{loci})^2}$ [4] of the point on the locus of Hopf bifurcation that is at the maximum positive distance from the point on the same direction of interest but on the limit state function. The direction of interest for which the stated distance is maximum, is called critical. The selected objective function is chosen such that the mean is negative and the minimization of the mean is actually giving a maximization of the distance of the locus of Hopf bifurcation point from the operational trend. The adopted objective function is such that the optimization looks for the parameter values that give the tangency assuming that the tail ends at $4\sigma_d$ ($S = 4$).

The second optimization process (R.R.E.A.) can be used for systems that allow the analysis of interest to be completed at any conditions; the Univariate Reduced Quadrature ([9]) has been adopted to compute statistical quantities of the output of interest, a method that requires $2n + 1$ evaluation of the responses of interest, significantly less computational expense than MCS. This

[4] $\overline{W}$ stands for the generic vertical load, the subscripts *operational* and *loci* are adopted to indicate if the considered point is on the operational trend or on the locus of Hopf bifurcation

reduction is important if bifurcation analysis is adopted. In fact, it would be unfeasible to determine statistical quantities without the use of techniques allowing the evaluation of significantly fewer bifurcation diagrams than that required by a technique such as MCS. This limitation is due to the computational burden required by the bifurcation analysis applied to landing gear systems. Unfortunately, for some conditions, the code that couples AUTO to Virtual.Lab Motion to compute the bifurcation diagrams for the multi-body models fails because the continuation does not converge. This behaviour is probably due to the reaching of conditions, during the continuation analysis, that are beyond physical constraints, but mathematically acceptable. Thus, so far the second optimization strategy has been applied only to the analytic landing gear system.

The Iterative Distribution Evolutionary Algorithm and the Reliable & Robust Evolutionary Algorithm consist of three phases and are now discussed step by step. The first phase is common to both the techniques, thus it is presented once for both the methods (subsection 3.1). Then, the remaining phases featuring the Iterative Distribution Evolutionary Algorithm and Reliable & Robust Evolutionary Algorithm are presented.

3.1 Range of Factor Variation for the Optimization Process

The starting phase for both the developed iterative procedures is to identify the range of design parameter variation to start the optimization process. This set of values can be given either by the set of values for the parameters $\mathbf{F}^*$ for which the relative locus of interest LoI is tangent to the defined limit-state function g or for which the distance along the direction of interest d for which the probability of failure is highest is a minimum, given a starting set for the parameters $\mathbf{F}$. This starting phase consists in two steps, preparation and data collection, which are explained in what follows.

First step: Preparation. The objectives of the optimization are established including possible acceptable tolerances. In particular, the limit state function g that delimits the failure region has to be defined. Moreover, in the presence of a system with a lot of parameters, like in a landing gear system, sensitivity analysis needs to be performed to detect the most influential ones for the considered objectives in the optimization process. Sobol indices ([8]) are adopted. Having identified the parameters to be considered during the optimization process, the maximum negative and positive range of variation $P_{max_{ineg,pos}}$ for the ith considered parameter ($i = 1...N$) are defined. In case a symmetric variation of the parameter of interest is adopted, then $P_{max_{ineg}}$ is equal to $P_{max_{ipos}}$ and the maximum percentages is P_{max_i}.

Second step: Data Collection. The quantities of interest (QoIs), those that describe the locus of interest and limit-state function, are evaluated for a suitable number of points in the parameter space by directly running the numerical model or doing experiments. These are needed to train surrogate models adopted in the SVD based methodology ([19, 22, 10, 20]). Using the SVD/metamodelling based methodology can reduce by 95% the time consumption to investigate the parameter space to look for the set of nominal values $\mathbf{F}^*$ for which the stated tangency occurs. Knowing $\mathbf{F}^*$, the interval $(x_{j,low}, x_{j,upp})$ of interest for each jth design parameter are selected such that the point $\mathbf{F}^*$ is internal to the optimum uncertain interval of variation identified at the end of the I.D.E.A. process and that is around the optimum nominal value $\overline{\mathbf{F}}_{opt}$. If $\mathbf{F}^*$ is selected considering the tangent condition, it is worth noticing that the analyses are performed numerically, thus the tangency can be defined as the state for which the locus of interest LoI is the nearest one to the defined limit function g a long the direction of interest d previously identified. Mathematically, this can be expressed as $\mathbf{F}^* := \mathbf{F}|(dist(g - LoI(\mathbf{F}^*)_d) = \min(dist(g - LoI(\mathbf{F})_d)$.

1. Determine the upper and lower bounds for the optimum nominal value. Given that the point $\mathbf{F}^*$ needs to be internal to the final optimum uncertain interval of variation, the stated bounds are shown in eqs. (3) and (4).

$$\overline{\mathbf{F}}_{opt_{upp}}(1 - P_{max_{i_{neg}}}) = F_i^* \tag{3}$$

$$\overline{\mathbf{F}}_{opt_{low}}(1 + P_{max_{i_{pos}}}) = F_i^* \tag{4}$$

2. Define the maximum possible interval of variation for the lower or upper nominal value $\overline{\mathbf{F}}_{opt}$

$$[\overline{\mathbf{F}}_{opt_{low}}(1 - P_{max_{i_{low}}}), \overline{\mathbf{F}}_{opt_{upp}}(1 + P_{max_{i_{upp}}})] \tag{5}$$

3. Substitute the lower and upper bounds of eqs. (3) and (4) in 5: the sought expression is obtained (eq. (6)).

$$[F_i^* \cdot \frac{1 - P_{max_{i_{neg}}}}{1 + P_{max_{i_{pos}}}}, F_i^* \cdot \frac{1 + P_{max_{i_{pos}}}}{1 - P_{max_{i_{neg}}}}] \tag{6}$$

The range of variation defined in eq. 6 is the same for both the optimization strategies, but, as shown, the definition comes from considerations related to the I.D.E.A. technique. The user can decide to consider more than one set $\mathbf{F}^*$ to continue in the optimization process and eventually pick the best set according to other requirements, such as robustness. In the following subsections the other steps needed to complete the proposed optimization processes are presented.

3.2 Iterative Distribution Evolutionary Algorithm

The Iterative Distribution Evolutionary Algorithm has the capability of minimizing the probability of failure (a *reliable optimizer*) and providing an understanding of the acceptable range of uncertainties. The Iterative Distribution Evolutionary Algorithm (I.D.E.A.) consists of three steps: the first two have been already presented in subsection 3.1 and Figure 4 presents the flow chart describing the last step, which is the iterative one.

Third step: Iterative Process. The third step is the iterative part that has evolutionary characteristics. A general evolutionary algorithm has three main steps: generation, mutation and selection ([25]). Each generation consists of separate selection and mutation steps performed iteratively. In the I.D.E.A. the generation is the region-hypercube of interest identified for each set $\mathbf{F}^*$ the user wants to use; this hypercube has as many dimensions as the number of design factors and each value of the design factors is delimited by the defined interval (eq. (6)). Moreover, for each generation a full factorial design is considered to define sampling points in which to evaluate the objective function. This consists of generating a well structured sampling plane that has the aim to not exclude any values for a specific parameter that could match desired requirements for some precise values of other parameters. For each generation and each point in the full factorial sampling plane, the QoI need to be evaluated. Thanks to the surrogate models already trained, a saving in time can be achieved. The surrogate models are used to evaluate the

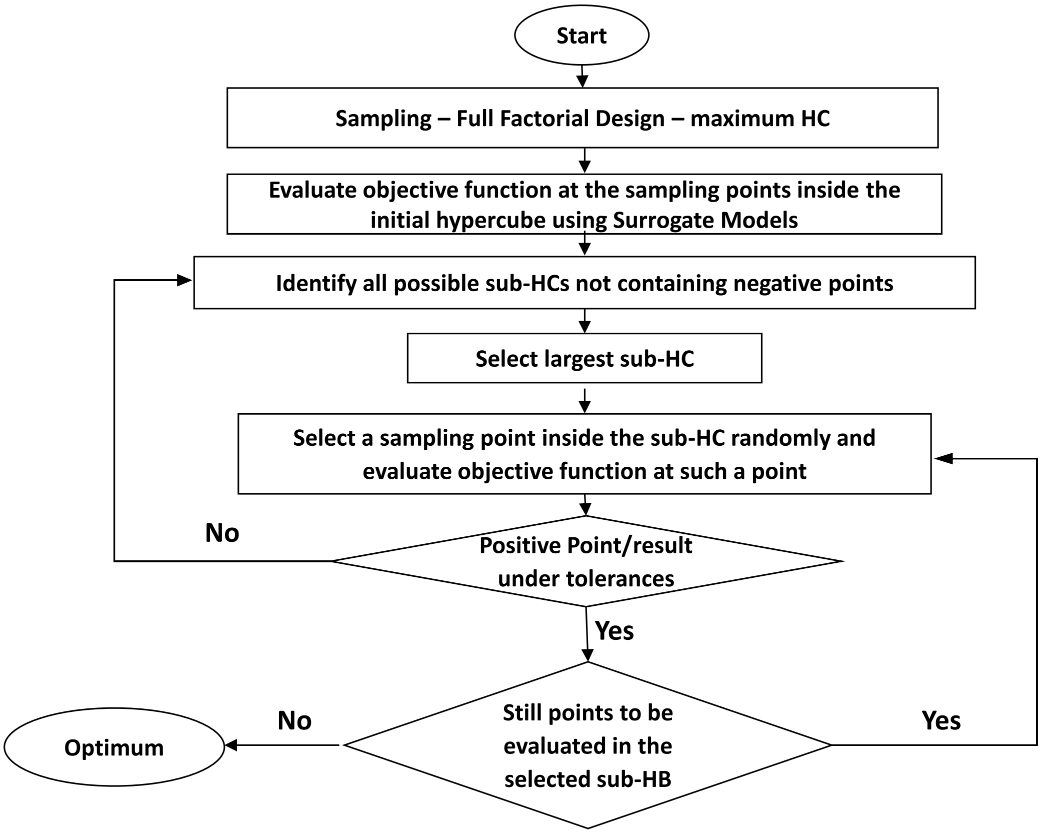


Figure 4: Flow chart describing the iterative phase characterizing the Iterative Distribution Evolutionary Algorithm; HC is an abbreviation of hypercubes.

QoI at the points that are in the range considered in the first phase. Depending on the obtained QoI the relative points are divided into two groups: positive and negative. The positive points are those for which the loci identified by the QoI are not in the failure region or if they are, the defined tolerance Tol_ϵ is fulfilled such that

$$dist(g - LoI(\mathbf{F}_G)_d) - dist(g - LoI(\mathbf{F}^*)_d) <= Tol_\epsilon \cdot dist(g - LoI(\mathbf{F}^*)_d) \qquad (7)$$

where G is used here to emphasize belonging to a particular generation, $\mathbf{F}_G$ is the set of values of the parameters at the generation G, $\mathbf{F}^*_G$ is the set of values of the parameters at the generation G. All the other points are negative and always present since the optimization process is considered for problems that do not have an acceptable probability of failure.

The mutation consists of a subdivision of the hypercube along particular directions such that all the new hypercubes contain the point $\mathbf{F}^*$. The directions for the subdivisions are identified by the negative set of points, and in particular by the values assumed by the parameters at such a point. The directions are identified by varying all the parameters at the negative point but one. The number of the directions for each negative point is equal to the dimension of the hypercube, i.e. the number of design parameters.

The selection step consists of sorting the hypercube in a descending order in terms of the volume and evaluating the QoI at the points not in the range considered in phase one. This selection can only be done by directly running the numerical model or using experimental results. Finally, the hypercube that does not have a tolerance greater than Tol_p of points for which the loci of interest are intersecting the limit state function is selected, having accepted also the tolerance shown in eq. (7). Tol_p is defined as the percentage of the number of negative points acceptable in the hypercube with respect to the total number of points belonging to the considered hypercube. The optimum set of values is given by the mean point in the hypercube.

3.3 Reliable & Robust Evolutionary Algorithm

The Reliable & Robust Evolutionary Algorithm has the capability of minimizing the objective function of interest $f(\mathbf{x})$ defined in terms of the mean μ_d and standard deviation σ_d, ($f(\mathbf{x}) = |\mu_d + S\sigma_d|$). Looking at the selected objective function, the expectation is that the higher the value of the coefficient S, the more the optimization tries to minimize the other statistic quantity σ_d. However, this can also be untrue if, for instance, the coefficient S is increased and the required minimization can then be fulfilled by decreasing the mean without the need to reduce the variance with respect to the one related to higher value of S. Comparing the two optimization methods, R.R.E.A. can be used only if the input statistical quantities are known; in the lack of such information the I.D.E.A. process must be used instead. In fact, the R.R.E.A. method use some formulas for approximating the output statistical quantities. This formulae need the first four input statistical quantities to be known, i.e. the mean μ_x, the variance σ_x^2, the skewness γ_x and the kurtosis Γ_x. The R.R.E.A. method consists of three steps; the first two have been already presented in 3.1, thus the last one is discussed here.

Third step: Optimization. The last step of the algorithm is the core of the optimization. In order to determine the set of values of the input factors that minimizes the selected objective function a technique to approximate the statistical quantities has been used and an evolutionary algorithm has been considered. The approximated statistical quantities have been determined using the Univariate Reduced Quadrature Technique (URQ) ([9]). The URQ belongs to the numerical approximation techniques to compute the integrals that define the stated statistical quantities such as Taylor-based Moment Propagation, Gaussian Quadrature, Monte Carlo Simulations (MCS) and Statistic Expansion. The URQ technique provides formulas consisting of sums of functional values that try to match the highest possible number of terms of the mean and variance expressions based on a third order Taylor-series expansion. The URQ has the advantage of having one less level of differentiation if compared to the methods based on first derivatives, thus it is advantageous with respect to the gradient-based optimization techniques. URQ method has a higher accuracy than those characterizing the linearization method, but similar computational cost. This has been shown by [9] using the fourth-order Taylor-series expansion. Both the URQ mean and variance accuracy is $O(œ_x^2)$ for symmetrical distributed uncertainties in the input and in case of asymmetrical distribution the accuracy is $O(œ_x^3)$. The use of such a technique requires the first four moments of the input uncertain parameters to be known, i.e. the mean μ_x, the variance σ_x^2, the skewness γ_x and the kurtosis Γ_x. Further details on the proposed formula by URQ are given in [9]. URQ is attractive for a computationally expensive system since it requires $2n + 1$ [5] evaluations of the objective function to evaluate the output statistical quantities; however, if for some of the $2n + 1$ conditions the analysis does not converge for physical or software constraints then the application of URQ techniques has some limitations. The R.R.E.A. can be used for systems that allow to complete the analysis of interest at any conditions. As previously stated, the optimization is based on the self-adaptive differential evolutionary paradigms. All the versions of the Differential Evolutionary algorithm are composed of three main steps that need to be followed after having generated the first pop-

[5] n is the dimension of the input factor $\mathbf{x}$

ulation; these steps are mutation, crossover and selection. After having identified the objective function to be considered during the optimization process, some variables need to be initialized before starting the optimization. These variables are the initial and minimum population size NP, the scale factor F and crossover variable CR, the constant adopted to perform self-adaptation (τ_F, τ_{CR}), the maximum number of function evaluations $N_{max_{Feval}}$, the maximum number of generations G_{max}, the value to reach if known and relative acceptable error (VTR, ϵ) and the counter variables for the number of function evaluation and population reduction. Moreover, the bounds $(\mathbf{x}_{low}, \mathbf{x}_{upp})$ for the considered input factors are determined during the second phase (section 3.1). After initializing the stated variables, operations are followed to reach the global optimum. The first population of input factors as well as the first set of scale factor F and crossover variable CR are generated according to the assumed variation of the considered factors and in the interval of defined variation, i.e. $[\mathbf{x}_{min}, \mathbf{x}_{max}]$,$[F_{low}, F_{upp}]$ and $[CR_{low}, CR_{upp}]$. Then there are the operations of crossover, mutation and selection. Three additional steps have been introduced by the author in the differential evolutionary algorithm presented by [25]. The flow diagram in Figure 5 presents the adopted code and the stated additional steps are:

- reduction in the dimension of the population. This step is considered when the 'if-conditions' are fulfilled. Labeling $N_{max_{Feval}}$, p_{max}, NP and $count_G$ as the maximum acceptable number of function evaluations, maximum number of population size reduction, population size and number of evaluated generations, the mathematical expression of such a condition is
$$count_G > \frac{N_{max_{Feval}}}{p_{max} \cdot NP} \tag{8}$$
The meaning is that considering the worst case scenario, the number of populations evaluated for all the possible reduced generation needs to be less than the maximum number of function evaluations. The condition presented in equation (8) on one hand is conservative since it considers that actually all the possible p_{max} reductions will be done and it does not take into account that the current population NP number can be reduced; on the other hand it does not take into account that NP could have been greater in previous steps.
- stratagem to avoid local minimum. This step has been introduced in order to avoid that a local optimum, determined after a certain number of evaluated generations $count_G$, can 'monopolise' the optimization process. To this end, a random variation of the identified best individual is generated.
- sorting the individuals at the end of the algorithm depending on the relative values of the objective function. At the end of the optimization process, it has been considered necessary to sort the set of obtained values for the quantity of interest and relative input factors in order to keep the best ones at the beginning of the ordered set of quantities. Such a sorting is significant if then a reduction of the size of the population is considered and some individuals of the population in the previous generation are excluded from the iterative process.

The selected stopping criterion is in terms of the maximum number of generations G_{max} and/or the error with respect to the desired optimum value of the objective function $VTR \pm \epsilon$, if known or given. In order to use the presented evolutionary algorithm some constants need to be fixed, thus the goodness of the results depends also on the set of considered values. The constants are $\tau_F, \tau_{CR}, p_{max}, NP_{min}, NP_{init}, N_{max_{Feval}}, VTR, \epsilon, G_{max}, F_{upp}, F_{low}, CR_{upp}, CR_{low}$ and are defined depending on the analyzed problem.

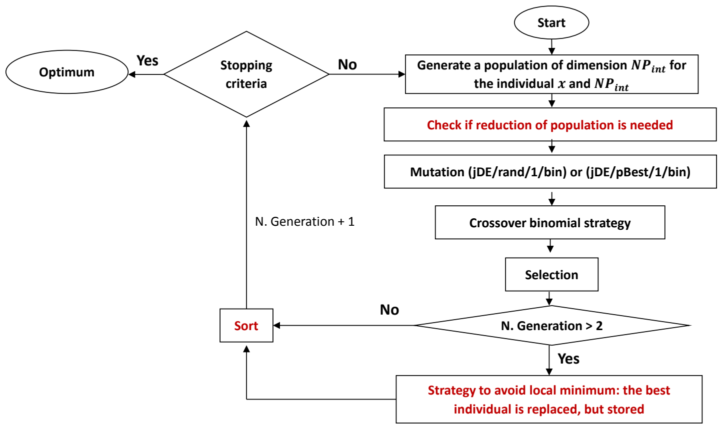


Figure 5: Flow chart describing the iterative phase characterizing the Reliable and Robust Evolutionary Algorithm.

4 RESULTS

In this section the results for the considered application are shown together with the values adopted for the constants to be fixed in the optimization techniques. The common element for both the application is related to the interval of variation, the adopted tolerances adopted and all the coefficients that need to be defined in the Evolutionary Algorithm.

- For all the analyzed cases, the percentage variation $P_{max_{i_{low}}}$ is fixed equal to $P_{max_{i_{upp}}}$. In what follows, the percentage variation is called P_{max} and is fixed equal to 3.5%.
- The stated tolerances Tol_ϵ and Tol_p are fixed equal to 0.01 and 0.05, respectively (section 3.2).
- Regarding the adopted coefficients in the Evolutionary Algorithm, the constants presented in section 3.3 are specified here.
  - $N_{max_{Feval}}$ is fixed to a value sufficiently high so as not to prematurely stop the iterative optimization process,
  - τ_F and τ_{CR} have been fixed equal to 0.01
  - ϵ and VTR are equal to 10^{-5}
  - N is 3 as it is equal to the dimension of the uncertain factors
  - $F_{upp}, F_{low}, CR_{upp}$ and CR_{low} have been fixed equal to $1, 0.2, 1$ and 0.05 respectively.
  - NP_{min} and NP_{init} are equal to 3 and 10.
  - G_{max} is fixed equal to 10
  - p_{max} changes as the dimension of the input factor varies:

$$p_{max} = 30 \cdot log(2 \cdot D) \tag{9}$$

The values adopted for τ_F and τ_{CR}, ϵ and VTR, $F_{upp}, F_{low}, CR_{upp}$ and CR_{low} and p_{max} are kept equal to those identified as good choice during the validation performed minimizing functions that are usually used to test optimization algorithms.
The values adopted for the minimum and initial population size (NP_{min} and NP_{init}) are to assure a combination of generations at an acceptable computational time. Same considerations have been done to select the value for the maximum number of generation G_{max}. However, more investigation into the time computational burden and accuracy of the solution can be done, if of interest, changing the selected values for the parameters.
min_x, max_x are the last coefficients to be set and they are different depending on the analysed landing gear and range of variation for the parameters, they are then given in the respective sections 4.1 and 4.2.

4.1 Analytic Model

For both techniques, first of all the values adopted to define the limit function trend are specified (Table 1). Then, the variables and the percentage variation need to be defined. The variables for the analytic model are selected as the one that influence the most the problem of interest ([10]): the stiffness (k_1 and k_4) and damping (d_1) of revolute joints which have the rotation about the global y-axis as free. Adopting the surrogate models trained using the bifurcation diagrams already evaluated to perform the Uncertainty Quantification ([10, 26]) exploiting the latin hypercube sampling (LHS) plane, the set of values $\mathbf{F}_G^*$ and the interval (eq. 6) have been determined for each considered case. From this point, different approaches need to be followed depending on the adopted strategy. First the I.D.E.A. and then the R.R.E.A. strategies and results are presented.

parameter	value	parameter	value
B	11.04 m	B_m	$Bm = B - xcg$
x_{cg}	8.6m	W	$8 \cdot 10^4$ kg
ρ	1.225kg/m^3	$S \cdot C_L$	128.1m^2

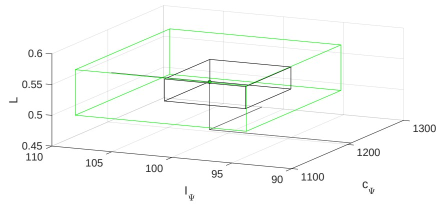
Table 1: Values adopted to describe the limit function for the optimization of the analytic landing gear design.

I.D.E.A. Considering the I.D.E.A. strategy, the Evolutionary phase starts and Fig. 6a-7a provide the results obtained for one generation each, where the green point stands for $\mathbf{F}_G^*$. The hypercubes in the parameter space (in this case a cube) are determined and sorted in an ascending order. Using a full factorial design, the hypercubes are filled with points (red in Fig. 6b) and the QoI (F_n and V) are evaluated for the points inside the initial range of variation for which the surrogate models are trained. The dimension of such a full factorial design is 36. Negative points, i.e. those for which the locus of Hopf bifurcation points intersect with the operational trend under the considered tolerance, are determined and the hypercube is further subdivided as explained in section 3.2. At this step, AUTO and the numerical model is run for all the other sampling points inside the hypercubes that can be further considered due to containing the point $\mathbf{F}_G^*$ and no negative points. Finally, the hypercubes are sorted again and the one with the greatest volume is picked out for each considered generation and the optimum sets are determined. A post-processing of the data can be considered and the SVD/HOSVD based method applied to eventually propagate the uncertainty for the determined optimum sets (the black points in Fig. 7a). The comparison of the initial confidence bounds and the optimum ones are shown in Fig. 8. Finally, for the sake of completeness Fig. 7b shows a comparison of PDF obtained for the initial set of nominal values and the optimum one, considering the same percentage of variation. Table 2 provides the initial set of nominal/percentage values and the optimum set and relative acceptable uncertainties.

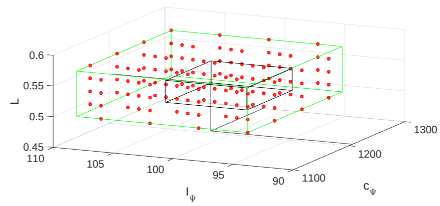
	$[c_\psi, I_psi, L]$
Starting Point	$[1200, 100, 0.53]$
Optimum	$[1189.80, 91.72, 0.525]$
Initial Uncertainty Range ($\%$)	$[3.5, 3.5, 3.5]$
Acceptable Uncertainty ($\%$)	$[4.56, 4.77, 4.56]$

Table 2: Results obtained applying I.D.E.A. to the analytic landing gear model considering two different sets of uncertianties.

The results show a very encouraging achievement for the proposed novel optimization algorithm (I.D.E.A). The lower confidence bound of the loci of Hopf bifurcation moved, decreasing the

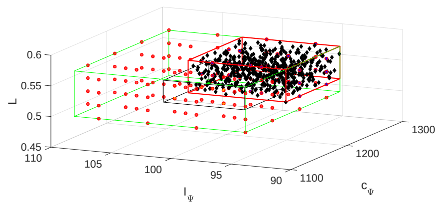


(a) I.D.E.A. - one generation step. The green point is that for which the locus of interest is tangent to the limit-state function.

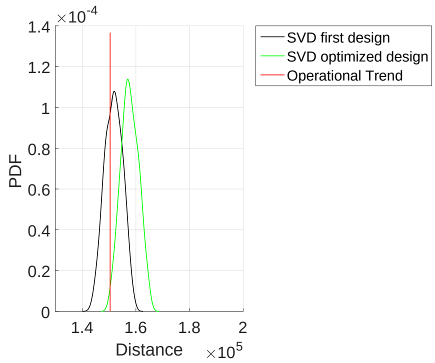


(b) I.D.E.A. - Full factorial design.

Figure 6



(a) I.D.E.A. - Optional postprocessing analysis. The loci are evaluated using the SVD/HOSVD based method at the black points.



(b) Comparison of PDF obtained for the initial set of nominal values and the optimum one, considering 3.5% variation.

Figure 7

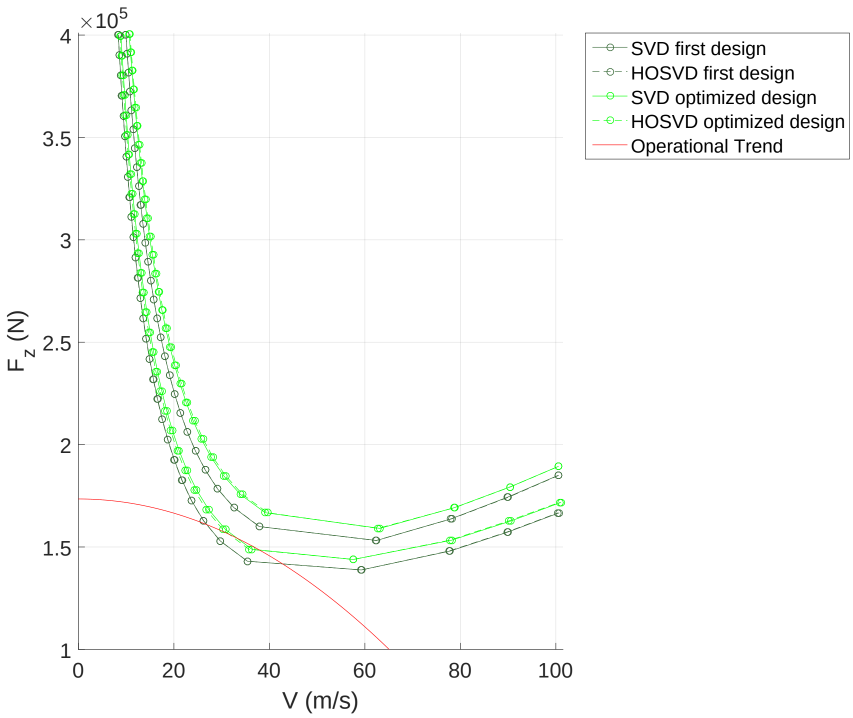


Figure 8: Comparison of the initial confidence bounds and the optimum ones for the analytic landing gear model.

probability of failure and assuring a reliable structure. Moreover, a high level of robustness is also retained, which is shown by the width of the PDF along each direction of interest starting from the points on the discretised locus of Hopf bifurcation that are between the intersections with the operational trend. Finally, thanks to the introduced iterative procedure, there is no requirement to perform bifurcation analyses and then give statistical properties to the results, propagating the uncertainty, thus even using surrogate models spending at least 1 hour for each analysed set of parameters. In the presented method the uncertainty is considered throughout the optimization procedure and there is no need to propagate them separately for each set of design parameters.

R.R.E.A.. If the R.R.E.A. technique is adopted, after having selected the range of variation for the input parameters selected in the optimization process, the enhanced evolutionary algorithm presented in section 3.3 can be applied. This second optimization process has been applied to the analyitic landing gear problem. As stated at the beginning of the present section, this decision is due to the requirement of having valid bifurcation analysis for all the considered test cases in order to use the URQ formulae; this cannot be assured using the multi-body landing gear model directly through the code coupling AUTO and Virtual.Lab Motion. In fact, for some conditions the continuation does not converge and this is due to conditions that are mathematically acceptable but physically not reproducible in Virtual.Lab Motion. Having therefore chosen the analytic landing gear model as the one to be used for the validation, the range of variation considered for the nominal values of the three more influential parameters is presented in Table 3. Moreover, since the URQ is adopted then the first four statistical moments of the input factors need to be specified. The first statistical moment, the mean, is fixed to the values characterizing each individual in the populations used in the evolutionary algorithm. The other statistical quantities are determined assuming a continuous uniform distribution for the design factors, i.e.

$$\sigma^2 = \sqrt{(max(x_{iG}) - min(x_{iG}))^2/12}; \qquad \gamma = 0; \qquad \Gamma = -6/5 + 3; \tag{10}$$

where i and G are the indexes adopted for the individual of the population and the generation at which the population belongs, respectively.

Parameter	Label	Minimum	Maximum
inertia of ψ DoF	I_ψ	105.01	91.29
damping coefficient of ψ DoF	c_ψ	1262.13	1097.18
tyre relaxation length	L	0.566	0.492

Table 3: Parameters and the range of values adopted in the optimization using R.R.E.A. technique.

Investigation has been carried out considering $f(\mathbf{x}) = |\mu_d + S\sigma_d|$ as the objective function. Different values for the coefficient S have been adopted and 4 is the value for which the tangency occurs. MCS analysis with 100 sampling points is used for the validation of the approximated statistical quantities. Table 4 shows the obtained results, also for the nominal case. The results related to the validation reveal that sometimes the discrepancy between URQ and MCS approximations can reach a percentage error of 28%. The error can be due to one or the other approximation, the only conclusion that can be made is that there is a lack of coherence between the two approximation methods. Looking at the mean and deviation, and making a comparison with the one related to the nominal conditions, it is apparent that minimizing the objective function, and so the distance of the point on the PDF along the critical direction of interest from the limit state function (the operational trend), the results are reliable keeping the same robustness.

6

case	$\mu_{f_{MCS}}$	$\mu_{f_{UQR}}$	$\lvert\epsilon_\mu\rvert$	$\sigma^2_{f_{MCS}}$	$\sigma^2_{f_{UQR}}$	$\lvert\epsilon_\sigma\rvert$
Nominal	-860.35	-623.58	0.83	3433.5	3404.9	28
	-14305.73	-13901.27	1.23	3491.83	3448.87	2.83

Table 4: Results obtained applying R.R.E.A. to the analytic landing gear model for three different objective functions.

Figure 9 presents the interval bounds and Figures 10 the probability distribution along the direction of interest for each case.

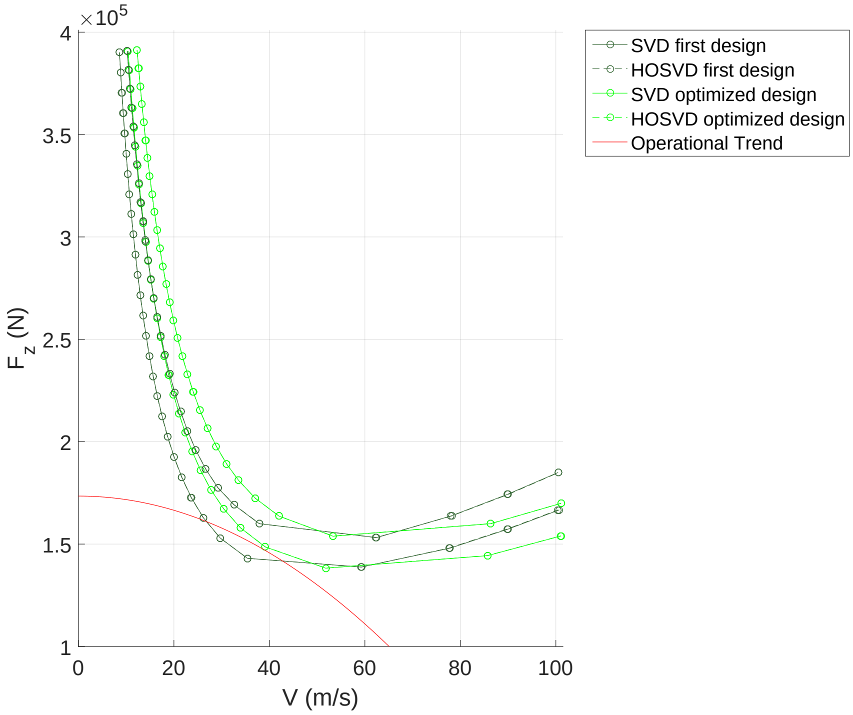
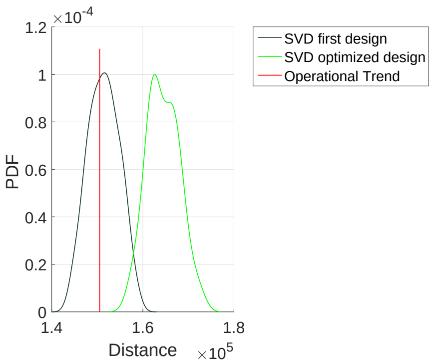
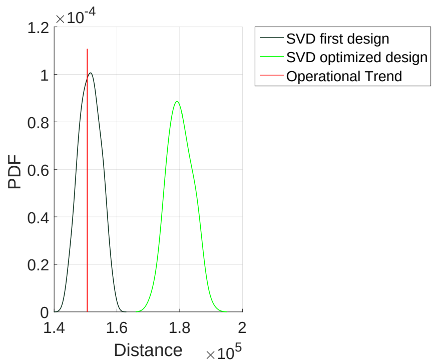


Figure 9: Comparison of the initial confidence bounds and the optimum ones for the multi-body landing gear model.



(a) $S = 4$



(b) $S = 8$

Figure 10: Probabitility Distribution Function along the direction of interest.

[6]The reliability is assured by the selected objective function and the robustness is linked to the obtained variance.

Considering the results presented here it is apparent that these is significant potential for using the enhanced differential evolutionary algorithm together with the Univariate Reduced Quadrature for complex analyses that assure acceptable results to be achieved whatever condition in the acceptable range of parameter variation is considered.

4.2 Multi-body Model

As for the analytic landing gear model, the coefficients used to define the limit function trend are first specified (Table 5).

parameter	value	parameter	value
B	11.04 m	B_m	$Bm = B - xcg$
x_{cg}	9.5m	W	70000 kg
ρ	1.225kg/m^3	$S \cdot C_L$	128.1306m^2

Table 5: Parameter values adopted to describe the limit function for the optimization of the multi-body landing gear design.

The optimization of the multi-body landing gear model using I.D.E.A. strategy follows exactly the same steps as for the analytic landing gear model adopting for this application a full factorial design with a dimension of 91. For the sake of conciseness the description of the different steps in the stated phase are not repeated and can be found in section 4.1. The comparison of the initial confidence bounds and the optimum ones are shown in Figure 11. A comparison of PDF obtained for the initial set of nominal values and the optimum one is shown in Figure 12. Table 6 provides the initial set of nominal/percentage values and the optimum set and relative acceptable uncertainties.

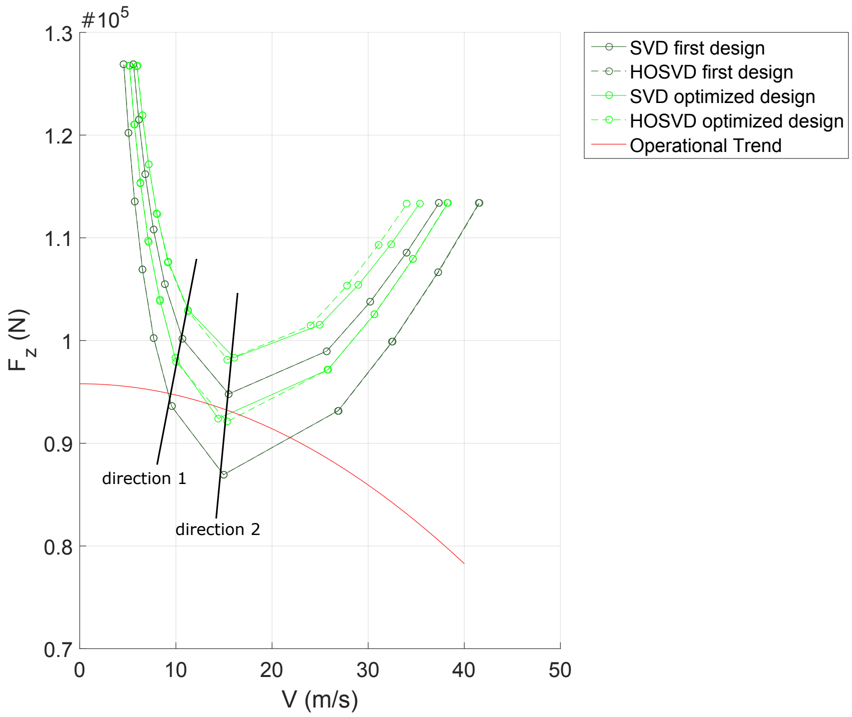


Figure 11: Comparison of the initial confidence bounds and the optimum ones. Two directions of interest are shown.

The application done for the multi body landing gear model is very encouraging for the pro-

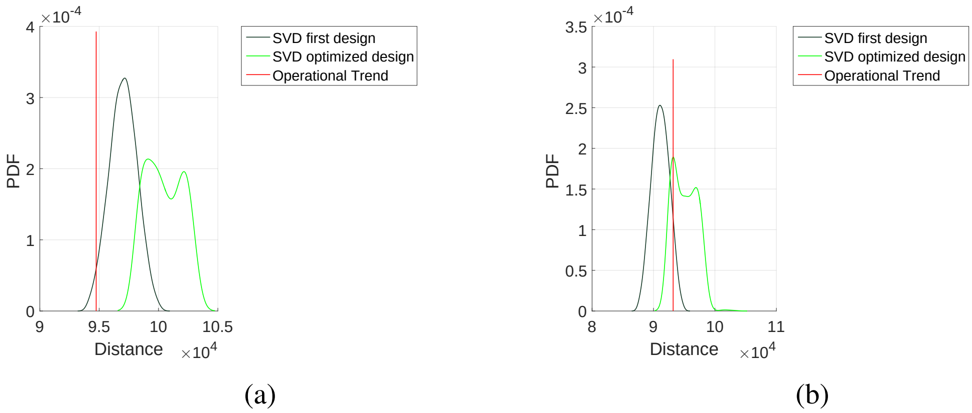


Figure 12: Comparison of PDF obtained for the initial set of nominal values and the optimum one a long the two directions of interest shown in figure 11. (a) direction 1; (b) direction 2.

	$[c_\psi, I_\psi, L]$
Starting Point	$[1.5 \cdot 10^6, 750000, 400]$
Optimum	$[1.5257 \cdot 10^6, 794059, 411]$
Initial Uncertainty Range (%)	$[3.5, 3.5, 3.5]$
Acceptable Uncertainty (%)	$[4.55, 4.55, 7]$

Table 6: Results obtained applying I.D.E.A. to the multi-body landing gear model.

posed novel optimization algorithm (I.D.E.A). The most critical probability of failure is decreased assuring an increment in the structure reliability. The examples underline that an increase of robustness is not guaranteed, since it is not the goal of the I.D.E.A. algorithm.

5 CONCLUSIONS

Two optimization strategies have been developed in order to reduce the computational time required by existing techniques. The reduction in computational time is significant especially if complex and nonlinear dynamics are dealt with. The techniques are based on evolutionary algorithm concepts and are the Iterative Distribution Evolutionary Algorithm (I.D.E.A.) and the Reliable & Robust Evolutionary Algorithm (R.R.E.A.); I.D.E.A. does not require statistical information for the input quantities and is a reliable based method, R.R.E.A. requires information about the statistical quantities of the design factors and adopts an approximate formulation for calculating the statistic quantities of the output. In order to validate the proposed techniques, they have been applied to identify an optimum set of structural design factors for a landing gear system such that the onset of shimmy is avoided. The adopted landing gears systems are analytical and multi-body ones. Both the techniques have been successfully applied to the analytic system and just the I.D.E.A. strategy to the multi-body system, since apparently physical limitation for having the continuation analysis converging whatever is the analysed configuration. Still, it has been of significant importance the possibilities of actually performing bifurcation analysis and optimization of a multi-body system embodying the strategy in an automatic environment. The results are extremely encouraging since the developed strategies have allowed the probability of shimmy occurrence to be minimized with a reduced computational time.

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