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A STATIC AEROTHERMOELASTIC RESPONSE ANALYSIS METHOD WITH CONSIDERING STIFFNESS UNCERTAINTY

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Abstract: This paper provides a method to consider the stiffness uncertainty in aerothermoelastic analyses for hypersonic vehicles. A framework is established to analyze the aerothermoelastic response of hypersonic vehicles. To compare the analysis results of this framework with that of references, the framework proves to be feasible. Then, under this framework, the interval method is used to consider the stiffness uncertainty. With considering uncertain stiffness the aerothermoelastic displacement limits of the metal plate are obtained by interval parameter perturbation method. And some meaningful conclusions are obtained.

1 INTRODUCTION

The hypersonic vehicles are the vehicles that operate at a Mach number range of 5 to 15. They can fly in and across the atmosphere with a hypersonic velocity. The hypersonic vehicles have the advantages of fast speed, long voyage, flexibility and quick reaction. They can adapt to the high-technology war and satisfy the requirement of high speed transportation in the future. Therefore, the research about hypersonic vehicles is a hotspot in recent years.[1]

The aerodynamic shapes of most hypersonic vehicles are slender body, lifting body or waverider configuration. In order to lose weight, the fuselage, airfoil and control surface always have large elasticity.[2] Therefore, it significant to consider the aeroelastic effects of hypersonic vehicles. Furthermore, hypersonic vehicles need to fly with hypersonic velocity in the atmosphere, this causes drastic aerodynamic heating effect on the surface of the vehicles. Then, structural stiffness and aeroelastic response are influenced by the aerodynamic heating. That means, under the action of aerodynamic force and aerodynamic heating, the hypersonic vehicles face the aerodynamic-thermal-structural coupling problem.[3] So, during design stage of this kind of air vehicle, the aerodynamic heating effect on the aeroelastic response must be seriously considered.

In addition, because the existing test methods are not efficient enough, the experimental results of hypersonic problems and the real situation always differ to some extent.[4] In other words, uncertainty exists. The uncertainty exists in the calculation parameters, such as the parameters of the structural materials, the distribution of aerodynamic pressure, and others. The uncertainty of these parameters complicates the aerothermoelastic problems which is already complex. The uncertainty problem in the field of aeroelasticity has gained considerable attention in recent years.[5]

Thus, the main objectives of this paper are:

1. Develop a method to consider the stiffness uncertainty in static aerothermoelastic analysis.
2. Study the effects of stiffness uncertainty on aerothermoelastic resopnse.

2 FRAMEWORK DESCRIPTION AND CALCULATION METHOD INTRODUCTION

2.1 Framework Description

An aerothermoelastic analysis framework with considering stiffness is established, it is shown in Fig.1.[6]

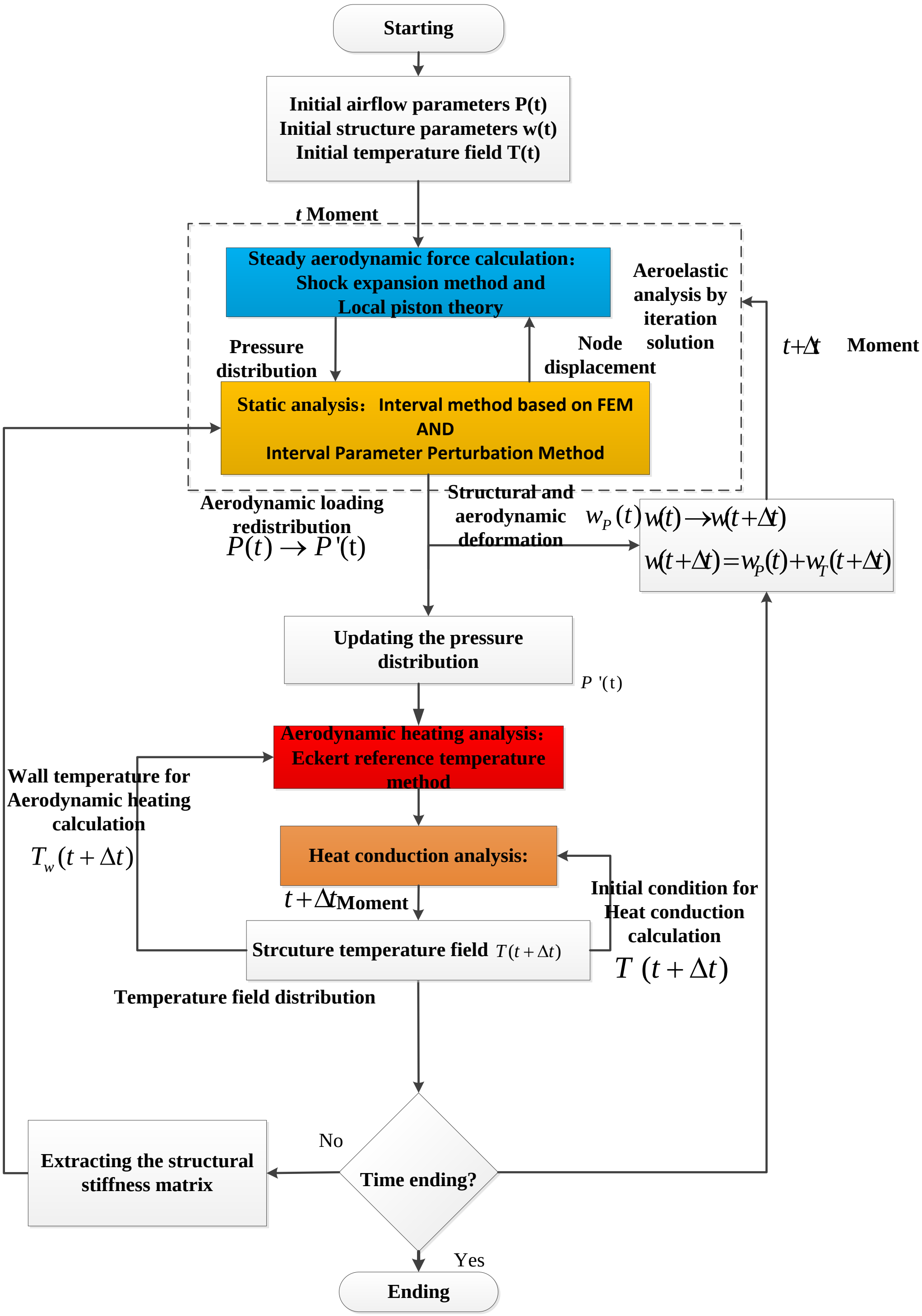


Figure 1: The aerothermoelastic analysis framework with considering stiffness

The steps of the analysis are illuminated as follows.

1. At the beginning of the analysis, the initial parameters of air flow, the initial structure parameters and the initial temperature field are given. The steady aerodynamic force is calculated by shock expansion method and local piston theory. This steady aerodynamic force

is loaded on the structure finite element model and the deformation is obtained by interval method based on FEM. Then the deformation is utilized in the aerodynamic calculation mode. Keeping solving iteratively until convergence, the static aeroelastic response of the structure at moment t is obtained.

2. Aerodynamic heat is calculated by using the aerodynamic force distribution and Eckert reference temperature method. The aerodynamic force is redistributed.

3. The finite element method is used to analyze heat conduction, and the temperature field of the structure at the next time step is obtained. At the same time, the structure temperature field, which is the initial condition of heat conduction calculation, and the updated stiffness matrix are obtained.

4. Judge whether the time is over. If so, the calculation ends, otherwise the stiffness matrix in aeroelastic analysis and the structure deformation will be updated. The process return to the interval parameter perturbation method aeroelastic calculation, the aeroelastic response at the moment is obtained, and the loop continues.

2.2 Calculation Method for Each Discipline

In this paper, the FEM is used to calculate the structural deformation and heat transfer, the shock expansion method and local piston theory are used to calculate the aerodynamic force, and the Eckert reference temperature method is used to calculate the aerodynamic heat flux. More detail is introduced in reference [6].

2.3 Interval Parameter Perturbation Method

In the step1 of the framework, the interval parameter perturbation method is used to consider the stiffness uncertainty during calculating the structure deformation. This method is introduced as following.[7]

The uncertain parameters is expressed as interval parameters,

$$\boldsymbol{a}^{\boldsymbol{I}} = [\underline{\boldsymbol{a}}, \bar{\boldsymbol{a}}] = [\boldsymbol{a}^{c} - \Delta\boldsymbol{a}, \boldsymbol{a}^{c} + \Delta\boldsymbol{a}] = \boldsymbol{a}^{c} + [-\Delta\boldsymbol{a}, +\Delta\boldsymbol{a}] = \boldsymbol{a}^{c} + \Delta\boldsymbol{a}^{\boldsymbol{I}} \tag{1}$$

$$\Delta\boldsymbol{a}^{\boldsymbol{I}} = [-\Delta\boldsymbol{a}, +\Delta\boldsymbol{a}]$$

The governing equation of finite element method with uncertain interval parameters is:

$$\boldsymbol{K}(\boldsymbol{a}^{\boldsymbol{I}})\boldsymbol{u}^{\boldsymbol{I}} = \boldsymbol{f}(\boldsymbol{a}^{\boldsymbol{I}}) \tag{2}$$

Where,

$$\boldsymbol{u}^{\boldsymbol{I}} = [\underline{\boldsymbol{u}}, \bar{\boldsymbol{u}}] = (\mathrm{u}_{\boldsymbol{i}}^{\boldsymbol{I}})_{\boldsymbol{n}} \tag{3}$$

$$\underline{\boldsymbol{u}} = \min_{\boldsymbol{a}\in\boldsymbol{a}^{\boldsymbol{I}}} \{(\boldsymbol{K}(\boldsymbol{a}))^{-1} \boldsymbol{f}(\boldsymbol{a})\}$$

$$\bar{\boldsymbol{u}} = \max_{\boldsymbol{a}\in\boldsymbol{a}^{\boldsymbol{I}}} \{(\boldsymbol{K}(\boldsymbol{a}))^{-1} \boldsymbol{f}(\boldsymbol{a})\}$$

For structural stiffness matrix $\boldsymbol{K}$ and load vector $\boldsymbol{f}$,

$$\boldsymbol{K} = \boldsymbol{K}_1 + \boldsymbol{K}_2 + \boldsymbol{K}_3 + \boldsymbol{K}_4 + \dots\boldsymbol{K}_{\mathrm{m}} = \sum_{\mathrm{i=1}}^{\mathrm{m}} \boldsymbol{K}_{\mathrm{i}} = \mathrm{a}_1\boldsymbol{K}_1 + \mathrm{a}_2\boldsymbol{K}_2 + \dots\mathrm{a}_{\mathrm{m}}\boldsymbol{K}_{\mathrm{m}} = \mathrm{a}_{\mathrm{i}}\sum_{\mathrm{i=1}}^{\mathrm{m}} \boldsymbol{K}_{\mathrm{i}} \tag{4}$$

$$\boldsymbol{f} = \boldsymbol{f}_1 + \boldsymbol{f}_2 + \boldsymbol{f}_3 + \boldsymbol{f}_4 + \dots\boldsymbol{f}_{\mathrm{m}} = \sum_{\mathrm{i=1}}^{\mathrm{m}} \boldsymbol{f}_{\mathrm{i}} = \mathrm{a}_1\boldsymbol{f}_1 + \mathrm{a}_2\boldsymbol{f}_2 + \dots\mathrm{a}_{\mathrm{m}}\boldsymbol{f}_{\mathrm{m}} = \mathrm{a}_{\mathrm{i}}\sum_{\mathrm{i=1}}^{\mathrm{m}} \boldsymbol{f}_{\mathrm{i}} \tag{5}$$

Then, for structural stiffness matrix with interval parameters $\boldsymbol{K}(\boldsymbol{a}^{\boldsymbol{I}})$ and load vector with interval parameters $\boldsymbol{f}(\boldsymbol{a}^{\boldsymbol{I}})$, based on natural interval expansion in interval Mathematics,

$$\boldsymbol{K}(\boldsymbol{a}^{\boldsymbol{I}}) = \sum_{\mathrm{i=1}}^{\mathrm{m}} \alpha_{\mathrm{i}}^{\boldsymbol{I}} \boldsymbol{K}_{\mathrm{i}} \tag{6}$$

$$\boldsymbol{f}(\boldsymbol{a}^{\boldsymbol{I}}) = \sum_{\mathrm{i=1}}^{\mathrm{m}} \alpha_{\mathrm{i}}^{\boldsymbol{I}} \boldsymbol{f}_{\mathrm{i}} \tag{7}$$

Considering equation (1), the above two equations can be wrote as,

$$\boldsymbol{K}(\boldsymbol{a}^{\boldsymbol{I}}) = \sum_{\mathrm{i=1}}^{\mathrm{m}} \alpha_{\mathrm{i}}^{\boldsymbol{c}} \boldsymbol{K}_{\mathrm{i}} + \sum_{\mathrm{i=1}}^{\mathrm{m}} \Delta\alpha_{\mathrm{i}}^{\boldsymbol{I}} \boldsymbol{K}_{\mathrm{i}} \tag{8}$$

(9)

$$\boldsymbol{f}(\boldsymbol{a}^{\boldsymbol{I}}) = \sum_{\mathrm{i}=1}^{\mathrm{m}} \alpha_{\mathrm{i}}^{\mathrm{c}} \boldsymbol{f}_{\mathrm{i}} + \sum_{\mathrm{i}=1}^{\mathrm{m}} \Delta\alpha_{\mathrm{i}}^{\mathrm{I}} \boldsymbol{f}_{\mathrm{i}} \tag{10}$$

Let

$$\begin{aligned} \boldsymbol{K}^{c} &= \sum_{\mathrm{i}=1}^{\mathrm{m}} \alpha_{\mathrm{i}}^{c} \boldsymbol{K}_{\mathrm{i}}, & \boldsymbol{f}^{c} &= \sum_{\mathrm{i}=1}^{\mathrm{m}} \alpha_{\mathrm{i}}^{\mathrm{c}} \boldsymbol{f}_{\mathrm{i}} \\ \delta\boldsymbol{K}(\boldsymbol{a}^{\boldsymbol{I}}) &= \sum_{\mathrm{i}=1}^{\mathrm{m}} \Delta\alpha_{\mathrm{i}}^{\boldsymbol{I}} \boldsymbol{K}_{\mathrm{i}}, & \delta\boldsymbol{f}(\boldsymbol{a}^{\boldsymbol{I}}) &= \sum_{\mathrm{i}=1}^{\mathrm{m}} \Delta\alpha_{\mathrm{i}}^{\mathrm{I}} \boldsymbol{f}_{\mathrm{i}} \end{aligned} \tag{11}$$

The equation (2) can be wrote as,

$$(\boldsymbol{K}^{c} + \delta\boldsymbol{K}(\boldsymbol{a}^{\boldsymbol{I}}))\,\boldsymbol{u} = \boldsymbol{f}^{c} + \delta\boldsymbol{f}(\boldsymbol{a}^{\boldsymbol{I}}) \tag{12}$$

Think of $\boldsymbol{K}^{c}$ and $\boldsymbol{f}^{c}$ as the standard value of the structure system. Then, Think of $\boldsymbol{K}^{c} + \delta\boldsymbol{K}(\boldsymbol{a}^{\boldsymbol{I}})$ and $\boldsymbol{f}^{c} + \delta\boldsymbol{f}(\boldsymbol{a}^{\boldsymbol{I}})$ as the perturbed value of the structure system. The static displacement problem of structure with considering interval parameters can be turn into perturbed static displacement problem of structure.

The first order approximation perturbation of uncertain value of structural static displacement is

$$\Delta\boldsymbol{u}_{\alpha}^{\boldsymbol{I}} = \sum_{\boldsymbol{j}=1}^{\mathrm{m}} \Delta\alpha_{\mathrm{j}}^{\boldsymbol{I}} ((\boldsymbol{K}^{c})^{-1} \boldsymbol{f}_{\mathrm{j}} - (\boldsymbol{K}^{c})^{-1} \boldsymbol{K}_{\mathrm{j}} \boldsymbol{u}^{c}) \tag{13}$$

Where, $\Delta\boldsymbol{a}^{\boldsymbol{I}} = [-\Delta\boldsymbol{a}, +\Delta\boldsymbol{a}]$

Let

$$\Phi = (\varphi_{\mathrm{i}}), \varphi_{\mathrm{i}} = (\boldsymbol{K}^{c})^{-1} \boldsymbol{f}_{\mathrm{j}} - (\boldsymbol{K}^{c})^{-1} \boldsymbol{K}_{\mathrm{j}} \boldsymbol{u}^{c} \qquad \mathrm{i,j}=1,2,...,\mathrm{m} \tag{14}$$

In this equation, Φ is a M-dimensional matrix

Substituting equation (14) into equation (13),

$$\Delta\boldsymbol{u}_{\boldsymbol{a}}^{\boldsymbol{I}} = \Phi\Delta\boldsymbol{a}^{\boldsymbol{I}} = \Phi[-\Delta\boldsymbol{a}, +\Delta\boldsymbol{a}] = [-|\Phi|\Delta\boldsymbol{a}, |\Phi|\Delta\boldsymbol{a}] \tag{15}$$

Then, the first order approximation perturbation value of structural static displacement is

$$\boldsymbol{u}^{\boldsymbol{I}} = [\underline{\boldsymbol{u}}, \overline{\boldsymbol{u}}] = \boldsymbol{u}^{c} + \Delta\boldsymbol{u}_{\alpha}^{\boldsymbol{I}} = \boldsymbol{u}^{c} + [-|\Phi|\Delta\boldsymbol{a}, |\Phi|\Delta\boldsymbol{a}] = [\boldsymbol{u}^{c} - |\Phi|\Delta\boldsymbol{a}, \boldsymbol{u}^{c} + |\Phi|\Delta\boldsymbol{a}] \tag{16}$$

Because of the necessary and sufficient conditions for equal intervals

$$\begin{aligned} \underline{\boldsymbol{u}} &= \boldsymbol{u}^{c} - |\Phi|\Delta\boldsymbol{a} \\ \overline{\boldsymbol{u}} &= \boldsymbol{u}^{c} + |\Phi|\Delta\boldsymbol{a} \end{aligned} \tag{17}$$

3 CALCULATION MODEL DESCRIPTION

A titanium alloy plate whose span and a chord length is both 500mm was used as the model, its thickness is 10mm. In order to simulate the full-motion wing, the 7 nodes in the middle of the wing root are clamped to restrain their movement and rotational degrees of freedom. The meshing of the model is shown in Fig. 2. Nodes are subdivided along the spanwise chord. The node number of the front of the root is 1, and it increases along the chord. There are 400 meshes and 441 nodes.

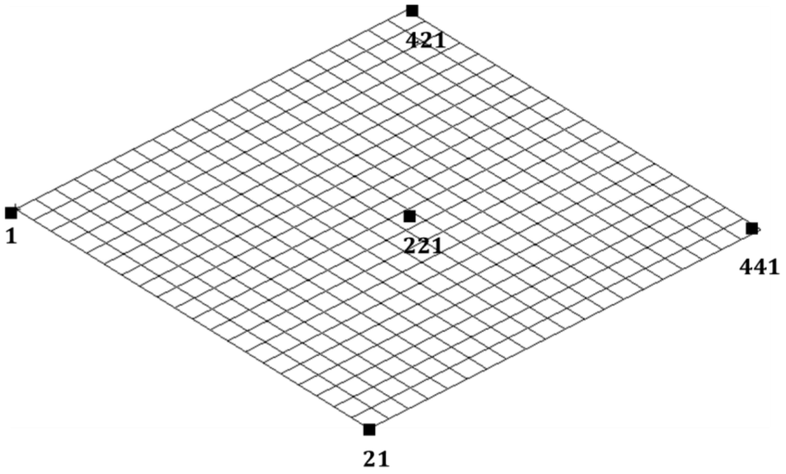


Figure 2: The calculation model and the meshing of the model

The material of airfoil is TC4 titanium alloy, the relationship between the material parameters (elastic modulus, Poisson's ratio, etc.) and temperature is shown in Table 1 and Figure 3.

Density（kg/m^3）	4450
Young's modulus（Pa）	$E=(132.7-0.0791T)\times e09$
Poisson's ratio	0.34
Thermal expansion coefficient（1/℃）	$\alpha=(1+0.000389T)\times 4.7e-6$
Specific heat capacity（$J/(kg*K)$）	520
Thermal conductivity（$W/(m*K)$）	22

Table 1: The relationship between material parameters and temperature

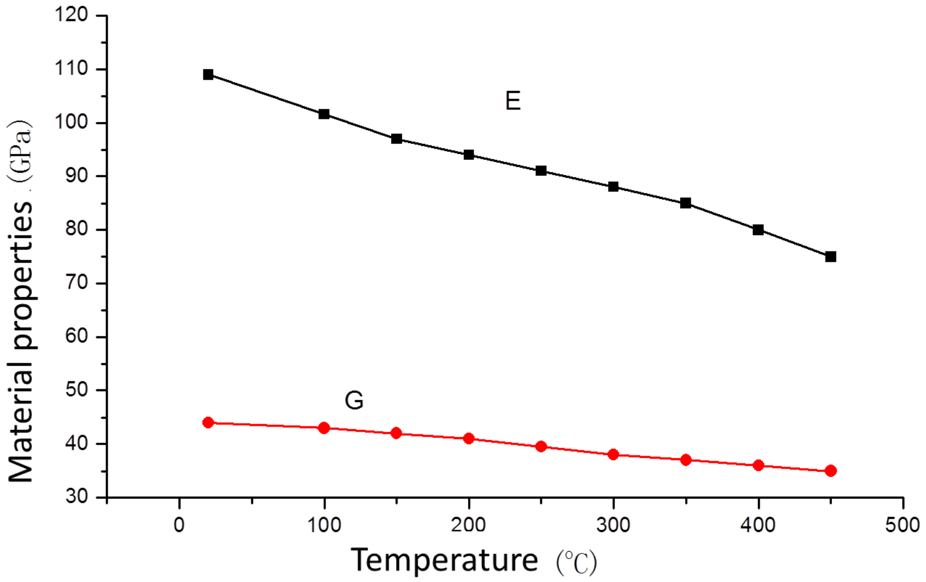


Figure 3: The relationship between Young 's modulus and temperature

Flight height is 25 km, the height of the standard atmospheric parameters is shown in Table 2.

Height（m）	25000
Temperature（K）	221.552

Pressure（Pa）	2549.2
Density（kg/ m^3 ）	0.04008
Viscosity coefficient（ $kg/(m*s)$ ）	1.4484e-005

Table 2: Standard atmospheric parameters at 25km height

Flight status is cruise. The total calculation time is 30 seconds and the step is 2 seconds.

4 CALCULATION RESULT

For the case that the angle of attack is 3 ° and the Mach number of the forward flow is 5,7,9,11 ,13. When the uncertainty is not considered, the results of upper surface pressure coefficient calculated by different methods are shown in Table.3.

Ma0	5	7	9	11	13
ZONAIR	-0.03103	-0.02482	-0.02150	-0.01944	-0.01800
shock expansion method	-0.01830	-0.01216	-0.00890	-0.00680	-0.00540
third order piston theory	-0.01790	-0.01210	-0.00890	-0.00690	-0.00550

Table 3 Upper surface pressure coefficient calculated by different methods

The displacement of the model without considering uncertainty is shown in Fig.4.

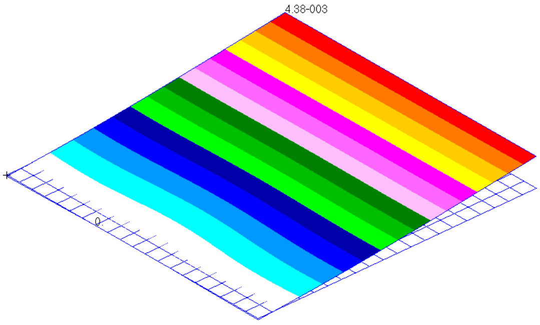


Figure 4: The model displacement without considering uncertainty

The node displacement without considering uncertainty calculated by different method is shown in Table.4.

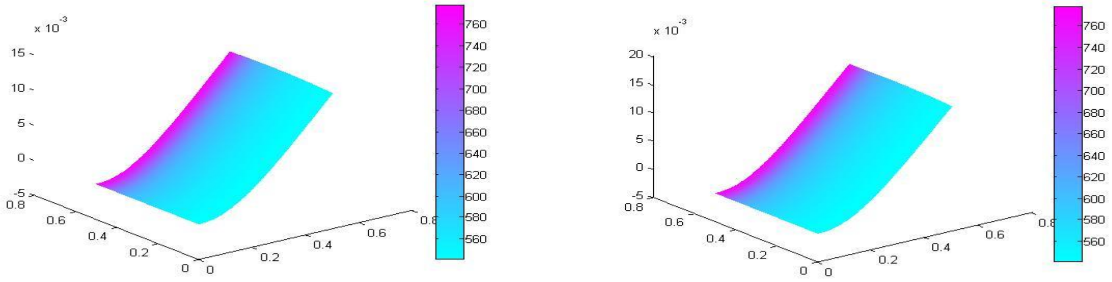
Node Number	Method in this paper（mm）	NASTRAN（mm）
1	-0.1747	-0.1803
21	-0.1511	-0.1606
221	1.6907	1.7183
421	4.3306	4.3804
441	4.286	4.3397

Table 4: The comparison of displacement calculated by different method

When considering the stiffness uncertainty, the uncertainty is assumed to be 10%.

That means the Young 's modulus of material of the entire plate will be expressed as $\mathrm{E}=[(132.7-0.0791T)\times e09\times 0.9,(132.7-0.0791T)\times e09\times 1.1]$,then the structural static displacement can be obtain by using the method presented in this paper.

Figure 5 shows the comparison of the displacements. Figure 5A gives the standard displacement, and Figure 5B gives the maximum displacement while considering stiffness uncertainty. The displacement in Figure 5B is 12% larger than that in Figure 5A.



A

B

Figure 5: The comparison of structural static displacements

More results is shown in Table 5.

Uncertainty	Maximum Displacement	Minimum Displacement
0	100%	100%
5%	105.75%	94.81%
10%	112.12%	89.96%

Table 5: Structural static displacements when the entire plate has the same Young 's modulus

Then, the plate is divided into 9 parts, as shown in Figure 6.

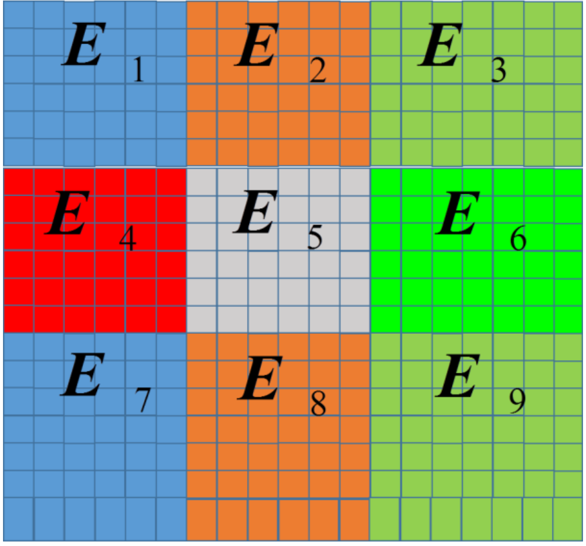


Figure 6: 9 parts of the plate

The material of each parts has its own Young 's modulus $E_i \quad i=1,2,...,9$, $E_1 = E_2 =$, . . . , $= E_9 = [(132.7 - 0.0791T)\times e09 \times 0.9, (132.7 - 0.0791T)\times e09 \times 1.1]$,each E_i is independent of each other.

Then the displacement is calculated by the method again, the results are shown in Table 6.

Uncertainty	Maximum Displacement	Minimum Displacement
0	100%	100%
5%	103.72%	96.44%
10%	108.13%	92.02%

Table 6: Structural static displacements when each E_i is independent of each other

Comparing Table 5 and Table 6, it can be found that the interval of displacement become smaller when the plate be divided into 9 parts. This conclusion coincides with the results in reference [8].

5 CONCLUSION

This paper describes the significance of hypersonic thermoelasticity and points out that the uncertainty problem exists in this problem. Then the aerothermoelastic analysis method which can consider the uncertainty of stiffness is established. And the method was verified by simulating a titanium alloy plate. By analyzing the calculation results, the following conclusions are drawn

First, the stiffness uncertainty seriously impacts the aerothermoelastic response. Second, the interval parameter perturbation method can consider the stiffness uncertainty. Third, the independence and distribution of the uncertain parameters have a decisive effect on the aerothermoelastic response.

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