

## MODELING OF FORCED VIBRATIONS OF THE AIRPLANE WITH THE ENGINE IMBALANCE AT GROUND RESONANCE TESTS

Regina V. Leonteva<sup>1</sup>, Mikhail A. Pronin<sup>2</sup>, Vsevolod I. Smyslov<sup>3</sup>

<sup>1</sup> Aeroelasticity Department, TsAGI  
1, Zhukovsky str., Zhukovsky, Moscow Region, Russia, 140180  
e-mail: [regina.leontjeva@gmail.com](mailto:regina.leontjeva@gmail.com)

<sup>2</sup> Aeroelasticity Department, TsAGI  
1, Zhukovsky str., Zhukovsky, Moscow Region, Russia, 140180  
e-mail: [pronin\\_m@pochta.ru](mailto:pronin_m@pochta.ru)

<sup>3</sup> Aeroelasticity Department, TsAGI  
1, Zhukovsky str., Zhukovsky, Moscow Region, Russia, 140180  
e-mail: [smysl@mail.ru](mailto:smysl@mail.ru)

**Keywords:** blade off, engine imbalance, wind milling, forced vibrations, simulation, ground vibration tests.

**Abstract:** Theoretical basics and the analysis of ground simulations' features of airplane forced vibrations after engine blade off, technical means and the experimental procedure are given. Examples of simulation predictions of airplane vibrations with the engine imbalance obtained during standard ground vibration tests (GVT) are given.

### 1 INTRODUCTION

For the analysis of flight safety of the airplane at engine blade off, appropriate computational researches have to be conducted. Their reliability is provided by the correction of dynamic design models according to ground experimental data.

Despite the existence of today's complex software systems, for example, FEM-based, the reliability of results is insufficient even for the problem of determining airplane natural vibration frequencies and mode shapes (the problem of eigenvalues). Generally, the airplane design model is linear, without regard to actual damping characteristics and other design features. This causes the generally accepted need of carrying out GVTs of the airplane.

The above is especially related to the problem of forced vibrations (the problem "with the right-hand side"), which results depend on imperfections of the computational structural model.

Therefore there is the actual need for the detailed consideration and the development of the experimental procedure to obtain data to increase the reliability of the analysis.

Engine blade loss in flight causes at first moment the huge vibration force in the order of hundreds of thousand Newtons. The subsequent engine shutdown reduces the rotary speed up to some stationary value, determined by the dynamic pressure.

Airplane forced vibrations considered in this paper are related to wind milling – the steady flight condition with the engine off, which low-pressure rotor is rotating under the action of the dynamic pressure.

## 2 STUDY OBJECTIVE

The purpose of this study is, on the one hand, the analysis of ground simulations' features of airplane vibrations with the engine imbalance, the development of the experimental procedure and necessary software and hardware.

On the other hand, no less important problem is to approbate the possibility of simulating the imbalance during GVT (normally, mandatory for any airplane type), this is a necessary condition for the practical implementation of the method.

For the first time the idea of modeling the imbalance and the need to conduct the experiment simultaneously with airplane standard GVTs were proposed in TsAGI paper in the 1990s [1]. Later, a set of tests on different elastic models to debug the procedure of the simulation and the test equipment was carried out.

The first ground tests of the airplane with the engine imbalance were conducted in ONERA [2]. In TsAGI the first experiments with the imbalance modeling were conducted during standard GVTs in 2011 and 2012 [3].

In [3-5] predicted research data and the general idea of the imbalance simulation are given, the continuation and further development of indicated studies is presented below.

## 3 THE BASIS OF THE THEORY

During ground tests of the airplane, engine rotors do not rotate, and force actions from the imbalance are modeled only on one of the engines, conventionally "damaged". The correction of the design model of the structure is needed to carry out for this option, based on results of ground tests. The corresponding vibration equation can be written as:

$$\mathbf{C}\ddot{\mathbf{q}} + \mathbf{H}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}^I(t) + \mathbf{M}^G(t, \dot{\mathbf{q}})/l, \quad (1)$$

where  $\mathbf{C}$ ,  $\mathbf{H}$ ,  $\mathbf{K}$  are matrices of inertia, damping and stiffness of the structure, respectively;  $\mathbf{q}$ ,  $\mathbf{F}^I$ ,  $\mathbf{M}^G$  are vectors of generalized coordinates, the inertial force and the gyroscopic moment,  $l$  is the characteristic dimension. Impacts of the damaged engine are carried out on the right-hand side of Eq. (1), in general, they need to be modeled.

Projections of the inertial force -  $F_Y$  and  $F_Z$  - on fixed axes, vertical and horizontal, can be represented as:

$$F_Y = m [\Omega^2 r \cos(\Omega t) + \ddot{y}], \quad F_Z = m [\Omega^2 r \sin(\Omega t + \pi/2) + \ddot{z}], \quad \Omega = \psi(t), \quad (2)$$

where  $\Omega$  is the angular rotational speed of the rotor,  $m$  is the mass of the lost blade,  $r$  is the distance from the rotor axis to the mass center of the blade,  $\ddot{y}$ ,  $\ddot{z}$  are linear accelerations of the rotor in the section of the lost blade.

In the wind milling mode, the angular velocity is constant at the flight condition, the engine forced oscillations occur at the angular frequency  $\Omega$  :

$$\ddot{y} = \Omega^2 y \cos(\Omega t + \varphi_1), \quad \ddot{z} = \Omega^2 z \sin(\Omega t + \varphi_1), \quad (3)$$

where  $\varphi_1$  is the phase shift.

The gyroscopic moment arises at engine angular oscillations, according to the approximate theory of the gyroscope its projections relative to same axes are written in the form:

$$M_y = (I_x \Omega) \dot{\theta}, \quad M_z = (I_x \Omega) \dot{\psi}, \quad (4)$$

where  $I_x$  is the axial inertia moment of the rotor,  $\theta$ ,  $\psi$  are pitch and yaw angles.

At some flight conditions, magnitudes of angular oscillations, as well as magnitudes of accelerations, can be relatively large. But, in ground tests, they can be neglected, they are limited to displacements at excitation points, amplitudes of engine vibrations are two orders smaller than the magnitude of  $r$ , the amplitude of angular vibrations is of the order of 0.01 rad. For harmonic oscillations the following relations are valid (in rad):  $|\theta| \ll 1$ ,  $|\psi| \ll 1$ , therefore inequalities  $\Omega \gg |\dot{\theta}|$ ,  $\Omega \gg |\dot{\psi}|$  are satisfied, which are the initial in the approximate gyroscope theory.

#### 4 SIMULATING THE FORCE ACTIONS

From the description of the blade loss phenomenon, it follows, in the general, it is of interest as the transient process, with a decreasing number of turns, and the steady process, i.e. wind milling.

Firstly, the force action of a large level, imitating the initial period of the blade loss, is practically excluded in ground tests of the airplane. Secondly, the duration of measurements with the imbalance simulation should be relatively small in relation to the total time of GVTs.

It follows from (2), the simulation of the spatial rotating force is possible by modeling its two projections onto fixed axes (generally, not necessarily at 90° angle), i.e. two forces that are fixed in space.

It follows from (4), the simulation of gyroscopic moments with respect to two axes is possible, with allowance for (2), by simulating two pairs of forces that are stationary in space. It is enough that each pair of forces is in the same plane (in general, this is not necessary).

So, the complete simulation of force actions of the damaged engine, simultaneously inertial force and gyroscopic moment, may be implemented with the help of two pairs of fixed forces, each pair of forces may be applied in one of two different planes.

Finally, from the condition for carrying out the simulation during GVT, it follows that the orientation and maximum values of forces should be the same, so as not to complicate the experiment and not to increase its duration, if possible.

## 5 TECHNICAL MEANS

In the GVT process, as in electromechanical modeling (EMM) of force actions [6], precision modal exciters and power amplifiers are used, which allow modelling control signals in the form of forces applied to the airplane structure, and with sufficient certainty.

$$F = EU, \quad (5)$$

where  $E$  is the calibration coefficient of the exciter/amplifier pair.

In addition, hardware and software that generate control signals and collect data are needed. The management of the experiment and the data collection due to GVT are provided by the modern specialized equipment with software systems such as P-WIN MODAL (Prodera, France), LMS Test.Lab (Siemens, Germany), PULSE (Brüel&Kjær, Belgium). The same tools provide tests with simulation of the engine imbalance.

Measurements of oscillations are provided by accelerometers that, in the operating range of the airplane natural frequencies, convert accelerations into electrical signals:

$$e = D\ddot{y}, \quad (6)$$

where  $D$  is the calibration coefficient of the accelerometer.

Limits of frequency variations practically overlap the required range of the corresponding engine rotating speed. The total number of accelerometers due to GVT is quite large, from several dozens to several hundreds; it fully corresponds to purposes of simulating the imbalance.

In addition to the above, it may be necessary to generate additional control signals, which are not provided for by the specialized equipment for GVTs, specified above. Such a formation is completely analogous to electromechanical modeling of force effects (EMM), in which sensor signals are converted into mechanical forces without time lags. For this purpose, high-speed, real-time modules are used, for example, in LabVIEW environment (National Instruments, USA).

Examples of error estimates in the force simulation due to the imbalance are given below, the errors in modeling the force actions due to the nonlinearity of characteristics in this case are relatively small, about 1-2%, corresponding data are given, in particular, at exciters design specifications [7]. An example of relatively detailed error analysis when modal testing of the airplane is given in [8], one can assume the total mean square error in the range of 3-5%, this value may be considered satisfactory.

## 6 FEATURES OF PROCEDURE

Since due to GVT the excitation points of engine vibrations are calculated for vertical or horizontal forces, simulating the rotating force in the case of the imbalance is carried out by same forces, at the  $90^\circ$  angle, in the plane along the normal to the rotor axis, preferably in the plane of the motor fan.

A second pair of exciters is needed for simulating gyroscopic moments, oriented parallel to the one that simulates the inertial force.

Since the doubling of equipment and the preparation of measurements substantially increase the laboriousness of the experiment and its duration, in this study it is decided to confine itself to simulating only the inertial force in the process of wind milling.

In tests, the constant amplitude excitation is given. Since rotations during wind milling are determined by the value of the dynamic pressure, values of the frequency and the excitation force are set for its selected value (or for each pair of corresponding values of Mach number  $M$ , flight height  $H$ ). Frequency response functions (FRF) of the structure are measured in a standard way: step by step frequency change with necessary excitation levels. Programmatically the frequency and the duration of the pause for the time of oscillations are changed.

The FRF of interest is assigned, and the result at each discrete value has the form of two components: in-phase with excitation,  $\text{Re}U$ , and quadrature,  $\text{Im}U$ , shifted by a quarter periods:

$$\text{Re}U = \frac{2}{nT} \int_0^{nT} U \cos(pt + \varphi) \cos(pt) dt, \quad \text{Im}U = \frac{2}{nT} \int_0^{nT} U \cos(pt + \varphi) \sin(pt) dt. \quad (7)$$

For ground tests, accelerations at excitation points are small, as oscillation amplitudes in comparison with the distance from the axis to the mass center of the blade:

$$y \ll r, \quad z \ll r, \quad (8)$$

and excitation forces have the form:

$$F_Y = m\Omega^2 r \cos(\Omega t), \quad F_Z = m\Omega^2 r \sin(\Omega t). \quad (9)$$

Since the force in (5) is applied to the movable exciter coil,  $F$ , the force at the excitation point,  $F_C$ , is determined by the difference:

$$F_C = F - F_B, \quad F_B = m_B \ddot{y} + h_B \dot{y} + k_B y, \quad k = \omega_B^2 m_B \quad (10)$$

where the subscript "B" refers to the mobile exciter system together with the mass of the connecting element. If the force sensor is installed at the excitation point, it measures the value of  $F_C$ .

The influence of mass and stiffness of the exciter can be neglected if the following condition is true:

$$M_K \gg m_B [1 - (\omega / \omega_B)^2], \quad H_K \gg h_B, \quad H_K \approx 2\delta_K M_K, \quad (11)$$

With the sliding suspension of the exciter's mobile system, the right-hand side of the first inequality is equal to  $m_B$ .

The elastic-mass effect of the mobile system and the connecting element can be reduced or completely compensated for by an additional component proportional to  $F_B$ , so that the control signal takes the following form:

$$U = [m\Omega^2 r \cos(\Omega t) - (m_B \ddot{y} + h\dot{y}_B + ky_B)] / E, \quad (12)$$

Practical implementation of such a procedure has a number of difficulties; in this study it has not been applied.

One possible error might be caused by the difference in amplitudes of horizontal and vertical excitation forces (even in the absence of engine vibrations). Their inequality results in the difference in the hodograph of the resultant force from a circle, where the hodograph is referred to as an ellipse. The large ellipse axis is directed along the force with the largest amplitude, and the small axis is directed along the other force.

This type of rotation of the resultant force with its projections, the vertical  $F_Y$  (for example, lesser one) and the horizontal  $F_Z$ , can be presented by a combination of the circular rotation of the force with the amplitude of  $F$  and the horizontal force with the amplitude of  $|F_Z - F_Y|$ :

$$F_Y = F \cos(\Omega t); F_Z = F \cos(\Omega t + \pi/2) + |F_Z - F_Y| \cos(\Omega t + \pi/2). \quad (13)$$

The vertical force and the first summand of the horizontal force simulate the rotating inertial force at the blade loss. The second summand is the force with the amplitude  $|F_Z - F_Y|$  of the additional independent harmonic excitation in the horizontal direction. This corresponds to the simulation of the horizontal excitation together with the rotating inertial force. Forced airplane oscillations with the engine off occur in this case under the action of the rotating inertial force (simulating the blade loss) and are unchangeable in the harmonic excitation force direction.

## 7 EXAMPLES OF RESULTS

Below are illustrations of data obtained during GVT of the civil airplane with two engines on pylons under the wing.

When simulating the engine imbalance, amplitudes of force sensor signals at excitation points do not remain constant, proportional to the control voltages. They are distorted by the influence of moving parts of exciters, and most of all at frequencies near resonances. FRFs of forces at excitation points are given in Figure 1.

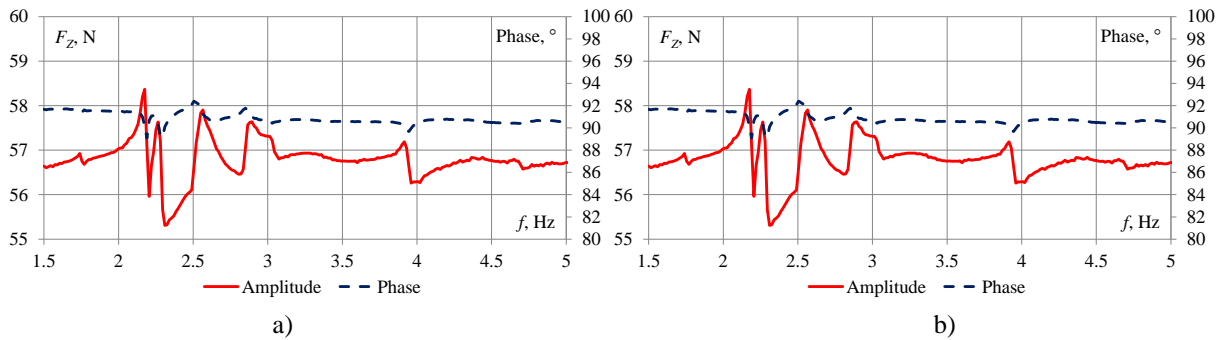


Figure 1: FRFs of forces at the excitation points of engine vibrations: a) vertical, b) horizontal.

Of special interest is the comparison of amplitudes of engine vibrations obtained in the simulation of the rotating inertial force and in the superposition of vibrations under the action of only one vertical or one horizontal force can, in turn. The comparison of accelerations of the engine characteristic point can be seen from FRFs (Figure 2).

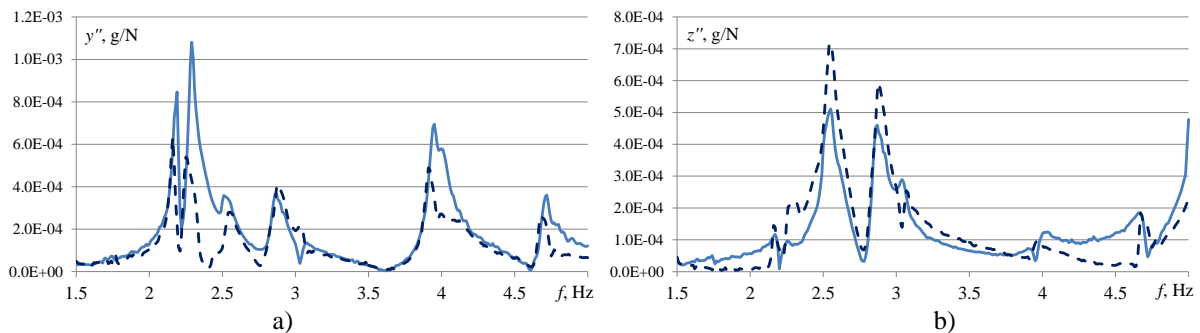


Figure 2: FRFs of accelerations of the engine (near the fan) measured at the excitation of the rotating force and the superposition of vibrations: a) vertical, b) horizontal [solid line: excitation of vibrations by the rotating force  $F$ , dashed line: superposition of vibrations caused by forces  $F_Y$  and  $F_Z$  (excitation of the left engine)].

It demonstrates differences in the vicinity of resonance frequencies. Nonlinearities of rigid characteristics and airplane damping properties can be considered as basic reasons for violation of the superposition principle of vibrations.

## 8 CONCLUSIONS

Both transition processes with variation of rotation speed as well as stationary vibrations take place in case of blade loss. The inertial force and the gyroscopic moment are transferred to the airplane from the damaged engine.

A brief review of the problem is given; the main, refined, relations for the imbalance, estimates of limits of their application in ground tests are shown. including debugging tests of elastic models is presented, and information on airplane testing with imbalance modeling is provided.

The rotating inertial force under imbalance is simulated with a pair of fixed unidirectional forces produced by modal exciters.

The method and the analysis of force excitation system operation, its influence on the tested structure and ways for its reduction are given. In this study the analysis of the accuracy at ground tests with the imbalance simulation is given.

The method and technical means are considered, the analysis of the excitation system operation, their influence on the tested structure, ways for its reduction are provided. In this study the analysis of the accuracy at ground tests with the imbalance simulation is given.

The developed method allows the data to be obtained to adjust analysis model with the required certainty. This procedure is approved by experiments on full-scale airplanes.

Real possibility of measuring airplane forced vibrations with imbalance modeling during GVT is confirmed.

## 9 ACKNOWLEDGEMENTS

The authors are grateful to O.A. Kuznetsov and V.V. Ferapontov for their help in this work.

## 10 REFERENCES

- [1] Kuznetsov O.A., Smyslov V.I. (1996). On the problem of the airplane dynamic loading from the engine imbalance after the blade loss. *Tekhnika vozdušnogo flota Journal*, Volume LXX, No. 3-4.
- [2] Lubrina P. (2003). Ground vibration experiments on large civil aircraft for engine imbalance purpose. *International Forum on Aeroelasticity and Structural Dynamics*, Stockholm, Sweden.
- [3] Leonteva R.V. (2014). Procedure of ground vibration experiments on aircraft with simulating forces due to engine imbalance at blade loss. *29th Congress of the International Council of the Aeronautical Sciences*, St. Petersburg, Russia. Paper 0859.
- [4] Kuznetsov O.A., Leontieva R.V. (2013). Dynamic loads on aircraft due to engine imbalance at blade off. *International Forum on Aeroelasticity and Structural Dynamics*, Bristol, England. Report IF-S5B.
- [5] Leonteva R.V., Smyslov V.I. (2016). Features of simulating the force actions from a damaged engine at ground vibration tests of an airplane. *Uchenyye zapiski TsAGI Journal*, Volume XLVII, No. 6, 61-70.
- [6] Karkle P.G., Pronin M.A., Smyslov V.I. (2011). Aerodynamic force simulation demo model for experiment-calculated flutter research. *International Forum on Aeroelasticity and Structural Dynamics*, Paris, France.
- [7] Prodera modal analysis systems and software. Technical articles. [http://www.prodera.com/uk/prodera\\_articles.htm](http://www.prodera.com/uk/prodera_articles.htm).
- [8] Zharov E.A., Smyslov V.I. (1976). Accuracy of the determination of the vibrational characteristics of an elastic structure in resonance tests with multipoint excitation. *Uchenyye zapiski TsAGI Journal*, Volume VII, No. 5, 88-92.



**COPYRIGHT STATEMENT**

The authors confirm that they, and/or their company or organization, hold copyright on all of the original material included in this paper. The authors also confirm that they have obtained permission, from the copyright holder of any third party material included in this paper, to publish it as part of their paper. The authors confirm that they give permission, or have obtained permission from the copyright holder of this paper, for the publication and distribution of this paper as part of the IFASD-2017 proceedings or as individual off-prints from the proceedings.