EXPERIMENTAL DETERMINATION OF DYNAMIC CHARACTERISTICS OF AIRCRAFT LANDING GEAR AND METHODS OF IDENTIFICATION OF THEIR MODAL CHARACTERISTICS

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Abstract: The fundamentals of the methodology based on the results of calculations, ground vibration tests, laboratory tests of landing gear on a copra with a rotating drum and on the results of airdrome ground tests for the aircraft shimmy safety analysis are presented in this article. The method of calculating stability characteristics on the solution of full eigenvalue problem of shimmy linear equations matrix is developed. The method of dynamic stiffness is developed on the D-partition of torsional dynamic stiffness plane for landing gear strut. Algorithms for landing gear modal characteristics identification on the basis of the spectral analysis of time processes of rolling wheels obtained in laboratory and ground tests are considered.

1 INTRODUCTION

Aircraft field experience shows that in some cases (especially for prototypes) appearance of shimmy of the wheels which leads to the destruction of the landing gear or significant damage, requiring further costly repairs is possible. This is corroborated by the experience of domestic, regional and foreign aviation firms [1-3]. The main reason of shimmy appearance, both during the flight tests of the aircraft and during its further operation are:

- firstly, is that in many cases the design of landing gear is carried out without taking into account the safety requirements of the aircraft from shimmy;
- secondly, because of the practice of refusing in number of cases from carrying out the necessary amount of research to confirm the stability of the wheels shimmy.

Russian domestic methodology of the shimmy research allows a reliable estimate of the aircraft's safety from the landing gear wheels shimmy. It is based on the calculation investigations (CI), laboratory tests (LT) of the full-scale landing gear, ground vibration tests (GVT) of landing gear part of the aircraft and ground full-scale testing of the landing gear of a prototypes aircraft (GT). Experimental determination of dynamic characteristics of the landing gear and methods for the identification of the modal characteristics are the most important part of the methodology for predicting aircraft safety from the landing gear wheels shimmy.

Laboratory tests are carried out with the purpose of modeling aircraft stability from the wheels shimmy in real conditions. These tests are carried out on the rotating drum for the purpose of solving the following main tasks [1].

Ground vibration tests are the main type of tests for determining resonance characteristics of landing gear, which can be carried out in bench testing of attachment and as part of the aircraft [2]. These characteristics can be used to correct parameters of the mathematical model of shimmy. GVT technology and processing methods of its results discussed previously in the article on IFASD 2015 [2].

Aircraft ground tests with the collisions of the wheels on the normalized obstacle conducted with the purpose of obtaining objective data to confirm the results of the CI and LT landing gear, as well as for the correction of the MM shimmy parameters. [2]. This method applies not only to confirm the safety of the aircraft from the wheels shimmy, but on the basis of its results can give a reliable assessment of the effectiveness of modifications landing gear construction, the effectiveness of the dampers and validate safety of the aircraft from the wheels shimmy when failure of the control system wheels.

Reviewed methods of computational studies shimmy, as well as methods of spectral analysis of processes of rolling of the wheels when LT and GT, allow to quantify the stability factor of the aircraft from the shimmy, and to determine the modal characteristics: the spectrum of natural frequencies, modes and damping coefficients depending on the load on the wheels and the speed of their rolling. Experimental modal characteristics used for correction of the MM parameters.

2 MATHEMATICAL MODELS OF SHIMMY AND THEIR RESEARCH METHODS

Computational studies are the primary method of assessment of the safety of the aircraft from the landing gear wheels shimmy at all stages of the design of aircraft [1-4]. The developed shimmy mathematical model of the symmetrical and unsymmetrical landing gear wheel allow a satisfactory assessment of the safety of the aircraft from the shimmy, if the parameters of these models determined with the required accuracy. It is obvious that for reliable estimates of the characteristics of the wheels shimmy requires not only adequate mathematical model, but reliable methods for determining the parameters of these models and their correction, among which the main methods are the ground vibration testing landing gear and methods of spectral analysis results of LT and GT.

To simplify analysis methods, shimmying movement of the landing gear wheels is considered to be subordinate to a system of ordinary linear differential equations with constant coefficients. Such equations, in particular, include the mathematical model (1) shimmy with oriented wheels of the landing gear (figure 1), which is represented in the following physical coordinates:

Ψ- the angle of stay rotation relative to the axis OX passing through the point of attachment of the strut to the airframe arallel to its velocity;

θ - full-rotation angle of the stay is oriented relative to the vertical axis OY orientation;

χ - angle piece θ, caused by the overflow of fluid in the shimmy damper;

λ - displacement of the center of tire contact with the ground normal to the diameter plane of the wheel;

φ - angle rotation axis of tire contact with the ground in a horizontal plane relative to the body of the wheel.

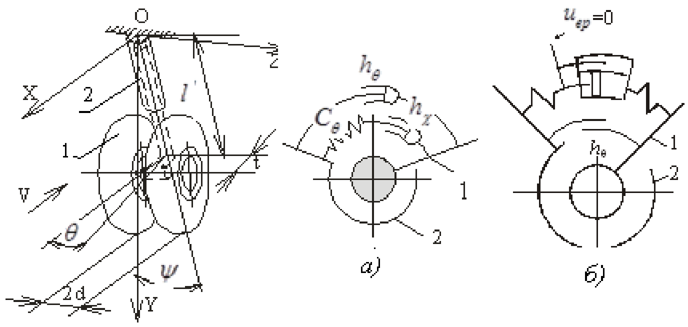

Figure 1: The scheme of oriented (a) and driven (b) wheels.

Then the motion of single or two independently rotating oriented wheels in the symmetric landing gear is characterized by the system of equations:

$$\left.\begin{aligned} I_x\ddot{\psi} + (c_\psi l^2 + nkd^2)\psi + I_{xy}\ddot{\theta} + V\frac{ni}{r}\dot{\theta} + c_{\psi\theta}l(\theta-\chi) - na(l+r)\lambda = 0 \\ I_{xy}\ddot{\psi} - V\frac{ni}{r}\dot{\psi} + c_{\psi\theta}l\psi + (I_y + ni\frac{d^2}{r^2})\ddot{\theta} + c_\theta(\theta-\chi) - nat\lambda - nb\phi = 0 \\ (l+r)\dot{\psi} + t\dot{\theta} + V\theta + V\phi + \dot{\lambda} = 0 \\ \dot{\theta} + \dot{\phi} - \alpha V\lambda + \beta V\phi - \gamma V\psi = 0 \\ h_\chi\dot{\chi} - c_\theta(\theta-\chi) - c_{\psi\theta}l\psi = 0 \end{aligned}\right\} \quad (1)$$

The first two equations of system (1) express the equality to zero of the moments of all the forces acting on the strut about the longitudinal *Ox* and vertical axes *Oy* (figure 1a). The third and fourth equations of system (1) express the law of rolling tires. The fifth equation expresses the law of motion of the piston in the damper shimmy.

In these equations: $\psi,\theta,\lambda,\varphi$, respectively, the rotation angle of the landing gear relative to the axes Ox and Oy, the offset center of tire contact from the diameter plane of the wheel and the angle of the center of the contact patch. α,β,γ - the coefficients of proportionality between the increment of the curvature of the trajectory of the center of tire contact with the ground and increments of components of deformation of the tire λ,φ and ψ, which serve to describe the interaction between elastic tires with the ground with lateral and angular vibrations of the wheel in the shimmy equations (1) under the hypotheses of the M. V. Keldysh [5]:

1. rolling the tire in the around of the contact center there is no lateral slippage relative to the ground;
2. curvature of the trajectory of the contact centers on the ground is a linear function of variables λ and φ and angle ψ.

In the analysis of the shimmy of the steered wheels of the mathematical model presented below by equation (2), takes into account the flow of the working fluid between the cavities of the control mechanism and spool dispenser. This takes into account the damping forces torsional landing gear, the lateral deformation of the tire, which in equations are defined by parameters h_θ and h_λ. For describing the dynamics of hydraulic control system module further establishes a variable , expressing the displacement of the spool given by the rotation of the wheels relative to the axis of orientation, and three parameters control system κ, T_s and D_s, respectively, the proportion of the elastic twisting of the strut, covered by a feedback control system; time constant and Q-factor of the control system.

If the landing gear has two interlocked wheels, the calculation of the wheels to shimmy take into the longitudinal forces acting on the wheel from ground with their vibrations. To describe the motion of the wheels in this case, additionally, there are two variables: 2η – the angle of mutual rotation of the wheels relative to the axis of rotation and ξ – displacement of the center of tire contact with the ground in the longitudinal direction relative to the center of the wheel.

The reaction to the wheel from the ground in the longitudinal direction is assumed to be proportional to the displacement of the center of tire contact with the ground relative to the center of the wheel in the same direction. The effective rolling radius of the wheel is assumed linearly dependent on the displacement of the center contact in the longitudinal direction and the increment of radial compression of the tire. The physical meaning of the first four equations in (1) and (2) the same for all mathematical models of shimmy; oriented, steered and no oriented wheels.
The fifth equation in system (2) expresses the law of motion of the spool drive wheels. The sixth equation in this system expresses the law of fluid flow in the spool-type valve. The penultimate equation in the system expresses the equality to zero of the moments of all the forces acting on each of the wheels relative to the axis of rotation. The last equation presents the mathematical model expresses the law of motion of the center of tire contact with the ground in the longitudinal direction.

$$\left.\begin{array}{c}
I_x\ddot{\psi} + (c_\psi \ell^2 + nkd^2)\psi + I_{xy}\ddot{\theta} + V\dfrac{ni_k}{r}\dot{\theta} + c_{\psi\theta}\ell(\theta-\chi) - na(\ell+r)\lambda - nh_\lambda(\ell+r)\dot{\lambda} = 0 \\
I_{xy}\ddot{\psi} - V\dfrac{ni_k}{r}\dot{\psi} + c_{\psi\theta}\ell\psi + I_y\ddot{\theta} + h_\theta\dot{\theta} + c_\theta(\theta-\chi) - nat\lambda - nh_\lambda t\dot{\lambda} - nb\varphi - ncd\xi = 0 \\
(\ell+r)\dot{\psi} + t\dot{\theta} + V\theta + V\varphi + \dot{\lambda} = 0 \\
\dot{\theta} + \dot{\varphi} - \alpha V\lambda + \beta V\varphi - \gamma V\psi = 0 \\
T\dot{\varepsilon} + \varepsilon + \kappa\theta + (1-\kappa)\chi = 0 \\
\dot{\chi} - D_{su}\varepsilon = 0 \\
ni_k\ddot{\eta} + 2c_\eta\eta - cr\xi = 0 \\
\dot{\xi} + d\dot{\theta} + r\dot{\eta} + \dfrac{V}{r}\mu\xi - \dfrac{V}{r}d\sigma\psi = 0
\end{array}\right\} \quad (2)$$

Calculations shimmy independently rotating wheels can be performed by the system of equations (1), if we take the displacement of the piston of the damper is equal to zero ($\chi(t)$ =0).

In the study shimmy of asymmetric landing gear, which in most cases are used on the main landing gear, it is necessary to consider the elastic and inertial coupling that occurs between lateral, angular and longitudinal oscillations of the wheels. In this case, equation shimmy of the wheels on the asymmetrical strut can be described by the following five degrees of freedom.

With the aim of obtaining general patterns for different designs of landing gear struts equation (1) and (2) is converted to a dimensionless form. The relationship of dimensional and dimensionless quantities for the equations (1) and (2) is determined by the following equations:

$I_x = \bar{I}_x nmr^2$, $I_y = \bar{I}_y nmr^2$, $I_{xy} = \bar{I}_{xy} nmr^2$, $i = \bar{i} mr^2$ - inertia moments of landing gear mount to the relative corresponding axes and inertia moment of wheel the relative to its axis of rotation;
$V = \bar{V} r\sqrt{a/m}$ - the speed of the aircraft;
$h_\chi = \bar{h}_\chi r^2\sqrt{am}$ - the drag coefficient of the damper shimmy;
$h_\theta = \bar{h}_\theta r^2\sqrt{am}$ - damping factor stands for the twist;
$h_\lambda = \bar{h}_\lambda \sqrt{am}$ - coefficient of damping lateral vibrations of the tire;
$D = \bar{D}\sqrt{a/m}$, $T = \bar{T}(\sqrt{a/m})^{-1}$ - Q-factor and time constant of control system by wheels;
κ - part of the elasticity of nose wheels control system, unladen feedback;
$k = \bar{k}a$, $b = \bar{b}ar^2$ - vertical and rolling stiffness of the tire;
$c_\psi = \bar{c}_\psi na$, $c_\theta = \bar{c}_\theta nar^2$, $c_{\psi\theta} = \bar{c}_{\psi\theta} nar$ - lateral, torsional and cross- stiffness landing gear mount;
$\alpha = \bar{\alpha} r^2$, $\beta = \bar{\beta} r$, $\gamma = \bar{\gamma} r$ - the kinematic characteristics of the tire according to the theory of rolling Keldysh;
$t = \bar{t} r$, $l = \bar{l} r$, $d = \bar{d} r$ - wheel axle offset; reduced length stay and half-width of the wheels track;
$\lambda = \bar{\lambda} r$, $\xi = \bar{\xi} r$;
a, m, r and n - the lateral tire stiffness, mass, wheel radius and the number of wheels (scale factors).

To perform computational studies on shimmy are introduced the well-known root locus method and the method of calculation of the boundary region shimmy, i.e. a method for computing combinations of the design parameters of the landing gear that adequate the sustainable movement of the wheels, and the mapping of the specified region with the region of real values of landing gear parameters taking into account their possible changes during operation of the aircraft. On the basis of solution methods for the complete or partial problem of eigenvalues there is a large variety of algorithms for the calculation of the boundary shimmy, among which the method based on the procedure of Routh, enables very effective to solve this problem. To implement these algorithms, equations (1), (2) with non-holonomic constraints represented in matrix form:

$$\dot{X} = A^{-1}B \cdot X, \qquad (3)$$

where ***A*** and ***B*** - the matrix $N \times N$ size; ***X*** is the state vector of the system chosen from the condition of regularity of the matrix ***A***. Depending on the investigated mathematical model shimmy and the choice of vector ***X***, the structure of the matrices ***A*** and ***B*** will be different. Proposed system of equations (1) to transform to the equation (3), using seven component vector (N=7),

$$X = (\dot{\psi}, \dot{\theta}, \psi, \theta, \lambda, \varphi, \chi)',$$

and the system of equations (2) lead to the same equation (3) using eleven component of the vector X (N=11).

$$X = (\dot{\psi}, \dot{\theta}, \dot{\eta}, \psi, \theta, \eta, \lambda, \varphi, \varepsilon, \chi, \xi)'$$

Then coefficients of the characteristic polynomial for the upper almost triangular matrix of Hessenberg ***H***, similar to the matrix $R = A^{-1}B$, are calculation. For finding all the roots s of the polynomial is selected the modified Newton method. The study of the shimmy of various types of landing gear, which are described by linear MM is currently performed using complex of programs ***Shimmy***. This complex allows to investigate the shimmy by root locus method or calculate the boundaries of the region shimmy at any selected plane of dimensional or dimensionless parameters of the equations shimmy, calculate the frequency and mode of shimmy. An example of the calculation of the border region shimmy oriented wheels on the ***Shimmy*** program presented on the plane of the resistance coefficient of the damper shimmy $\bar{h}_\chi$ and strut torsional stiffness $\bar{C}_\theta$ in private channels for the flow of the working fluid of the damper (figure 2).

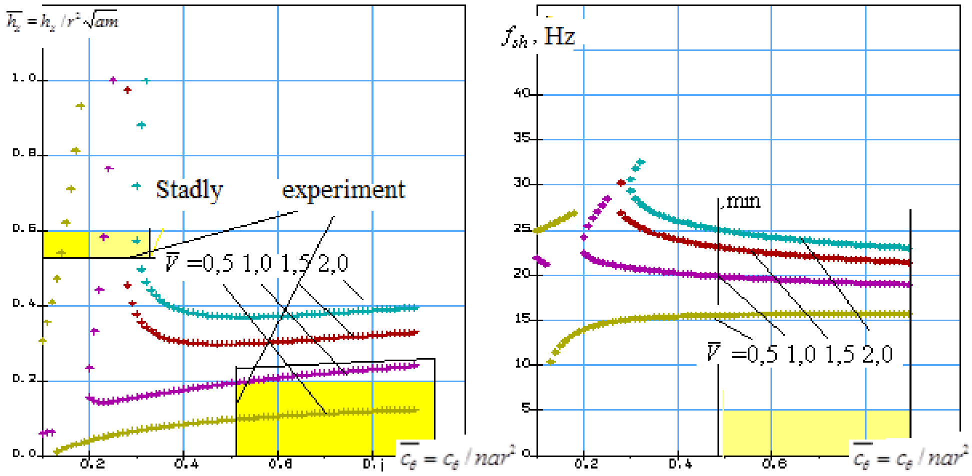

Figure 2: Typical dependence of the resistance coefficient of the damper shimmy $\bar{h}_\chi = h_\chi / r^2\sqrt{am}$ and shimmy frequency f_{sh} by the torsional stiffness of the strut $\bar{c}_\theta = c_\theta / nar^2$ when the speed $\bar{V} = V / r\sqrt{a/m}$

3 THE METHOD OF DYNAMIC STIFFNESS

Here we will mention some peculiarities of the study of the shimmy phenomenon, which significantly complicate the use of the above mentioned methods of research or even make their use impossible.

On real structures landing gear has a large variety of design schemes of relations between the oriented and fixed parts of the strut. This feature is on the one hand greatly complicates their mathematical modeling, and on the other hand development of a new model generally leads to a significant change in not only the design scheme of the studied phenomenon, but the solution algorithm all the tasks [3].
Almost all designs landing gear are nonlinear systems. Therefore, the second significant feature of the study of the phenomena of the wheels shimmy of the aircraft is the necessity of taking into account non-linear factors inherent in real structures landing gear. The main nonlinearity in the design is the local nonlinearity, an example of which can serve as the free-play and dry friction in the fastening elements of the landing gear, and hydraulic system

modules control wheels, and shimmy dampers, etc. [2, 6]. Therefore, the analysis of the stability wheel from shimmying held usually in the time domain.

When solving problems about the study of stability of a rolling wheel landing gear, with the above-mentioned features, the method of dynamic stiffness is extremely effective. This method developed by the author for the study of the wheels shimmy with different models of control systems and damping devices, as well as taking into account the local nonlinear dependencies. It is more efficient compared to the procedures of Routh and numerical methods of solving equations shimmy in the time domain. The method of dynamic stiffness is based on the harmonic linearization of nonlinearities and algorithms of *D*-decomposition [6, 7]. In this method, in particular, equations shimmy (2) using the Laplace transformation are written to a single matrix equation:

$$[K(s)]\cdot\{Q(s)\}=\{0\}, \tag{4}$$

where:

$$\{Q(s)\}=[\psi(s),\theta(i\omega),\lambda(s),\varphi(s),\eta(s),\xi(s)]^T, \quad (s=\delta+i\omega,\ i=\sqrt{-1},)$$

$$[K(s)]=\begin{bmatrix} I_x s^2+c_\psi\ell^2+nkd^2; & I_x s^2+V\dfrac{ni_k}{r}s+c_{\psi\theta}\ell; & -n(\ell+r)(h_\lambda s+a); & 0 & 0 & 0 \\ I_x s^2-V\dfrac{ni_k}{r}s+c_{\psi\theta}\ell; & I_y^* s^2+W(s); & -nt(h_\lambda s+a); & -nb & 0 & -ncd \\ (\ell+r)s & ts+V & s & V & 0 & 0 \\ -\gamma V & s & -\alpha V & s+\beta V & 0 & 0 \\ 0 & 0 & 0 & 0 & i_k s^2+2c\eta & -cr \\ -V\dfrac{d\sigma}{r} & ds & 0 & 0 & rs & s+V\dfrac{\mu}{r} \end{bmatrix}$$

The element $W(s)$ of this matrix at $s=i\omega$ is the frequency characteristic of dynamic stiffness $D(i\omega,\theta_0)$ that defines the relationship between castor landing gear (steered) or not castor landing gear part in the general case can be a nonlinear function, i.e. depending on the amplitude θ_0. To calculate this function can be obtained analytical formulas for the linear $D(i\omega)$ and nonlinear $D(i\omega,\theta_0)$ dynamic stiffness of various control systems and damping devices. Dynamic stiffness $D(i\omega,\theta_0)$ can also be determined and experimental methods.

In particular, for the analysis of shimmy oriented wheels, when the connection between the guided and the stationary part of the stand was simulated using a hydraulic damper (HD) viscous friction with coefficient resistance h_d and with the stiffness of its fixation on elastic basis C_d (figure 1*a*) obtained the following formulas for calculating the components of the dynamic stiffness $D(i\omega)=c^{eff}(\omega)+i\omega h^{eff}(\omega)$: effective stiffness $c^{eff}(\omega)$ and effective resistance coefficient $h^{eff}(\omega)$ of the damper shimmy [6]:

$$c^{eff}(\omega)=c_\theta\frac{\bar{\omega}^2}{1+\bar{\omega}^2},\quad h^{eff}(\omega)=\frac{c_\theta}{\omega}\left(\frac{\bar{\omega}}{1+\bar{\omega}^2}\right),\quad \bar{\omega}=\frac{\omega}{D_d} \tag{5}$$

where $D_d = C_\theta / h_d$ - quality hydraulic damper. The graphs of these functions are presented in figure 3.

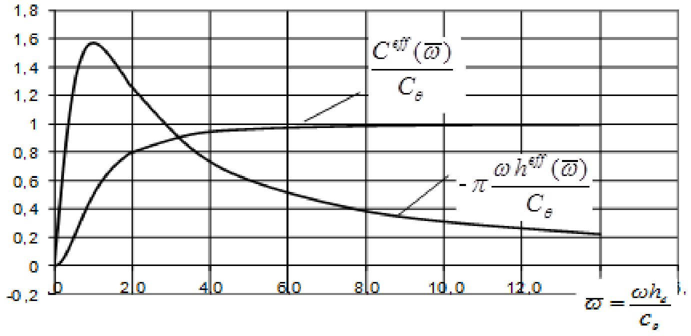

Figure 3: Dependence of components of the dynamic stiffness of HD on an elastic basis from the oscillation frequency

We note here that the maximum level of damping in this case is achieved from the condition of equality of stiffness: $h_d \omega = c_\theta$ - "damper resonance". In the case where the above relationship consists of a dry friction damper (friction damper (FD)), fixed on an elastic basis with stiffness C_θ, and with an effort made to unseat F_{str}, the components of the dynamic stiffness can be calculated by the following approximate formulas (6):

$$\left.\begin{aligned} &C_{eff}(\theta_0) = \mathbf{Re}\, D(i\,\omega, \theta_0) = C_\theta [1 - g(\bar{\delta}_s)], \\ &h_{eff}(\omega, \theta_0) = -\mathbf{Im} \frac{D(i\,\omega, \theta_0)}{\omega} = -\frac{C_\theta}{\omega} g'(\bar{\delta}_s), \end{aligned}\right] \quad (6)$$

where:

$$g(\bar{\delta}_s) = \frac{1}{\pi}\left[\frac{\pi}{2} + \arcsin(1 - 2\bar{\delta}_s) + 2(1 - 2\bar{\delta}_s)\sqrt{\bar{\delta}_s(1 - \bar{\delta}_s)}\right],$$

$$g'(\bar{\delta}_s) = \frac{4\bar{\delta}_s}{\pi}(1 - \bar{\delta}_s), \qquad \bar{\delta}_s = \frac{M_t}{C_\theta \theta_0} = \frac{\theta_{bw}}{\theta_0}.$$

From formulas (6) and graphs in figure 4 shows that the dry friction damper has a maximum damping coefficient when the oscillations amplitude is twice the amplitude θ_{str} the moving of the damper are equal $\theta_{str} = \frac{F_{str}}{C_\theta}$.

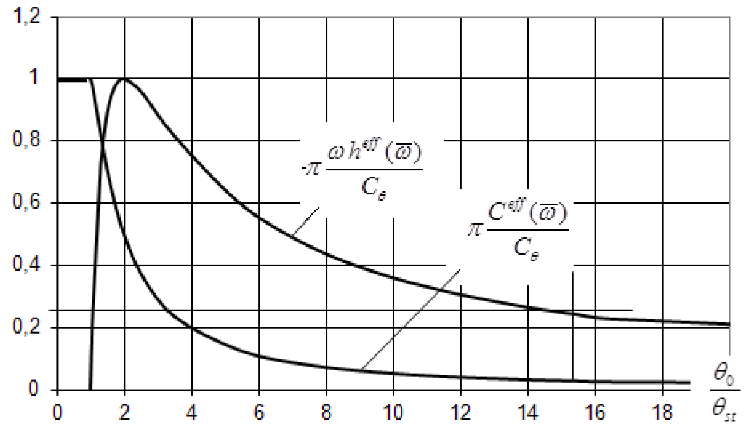

Figure 4: the Dependence of components of the dynamic stiffness of FD on the elastic basis from frequency of the oscillations

In the case with oriented wheels, equipped hydraulic damper with a quadratic characteristic resistance force ($M_d = \bar{h}_\chi \dot{\chi}^2 sign\dot{\chi}$) and taking into account the moment of forces of dry friction in cuffs seal piston rod ($M_t = M_\theta sign\dot{\theta}$) рассмотрен в работе [6].

The main feature of the algorithm is to use *D*-decomposition plane integrated value of the increment to the element *s*-matrix corresponding to the original system of differential equations (2).
To determine the region stability boundaries in the present system introduces an additional element, changing the parameters of which are equivalent to the increment $\Delta K_{jk}(s)$ to element $K_{jk}(s)$ of matrix $K(s)$. To construct the region stability boundaries in the parameter plane of this element, we choose the value $\Delta K_{jk}(s)$ of the plane for *D*-decomposition. The connection of this quantity with the elements $K(s)$ of the matrix is found from the condition of existence of nontrivial solutions of the equation (8):

$$\Delta K_{jk}(s) = -K_{jk}(s) - \frac{\det' K(s)}{\det A_{jk}(s)} = -\frac{H_0(s)}{H_1(s)} \tag{8}$$

where $\det' K(s)$ - the determinant of the matrix $K(s)$ with K_{jk} =0; $\det A_{jk}(s)$ - cofactor of the matrix element $K_{jk}(s)$; $H_0(s)$ and $H_1(s)$ the characteristic polynomials, respectively, in the original mathematical model and the same model, the vector of coordinates which does not contain the q_{k0}-th component. Constructing a hodograph $\Delta K_{jk}(i\omega)\big|_{\lambda=i\omega}$, when values $0 \le \omega < \infty$ get display of the imaginary half-axis $\omega \ge 0$ plane of the roots of the characteristic equation of the system on the plane of the increment ΔK_{jk}.

In studies of the shimmy of the wheel (equally and flutter control surfaces), as a rule, from physical considerations may be determined the degree of stability (or instability) of the system at zero or infinitely large values $\Delta K_{jk}(i\omega)$, which is determined, for example, dynamic stiffness of the control system. This circumstance greatly simplifies the selection of the region of stability on the plane ΔK_{jk}.Varying the parameter of the task p_m, you can also define its critical values of dependence from the parameters of the additional item ΔK_{jk}.

Example of calculation of LCO strut with the wheels oriented with the consideration of nonlinearities a dry friction (9) is presented in figure 5. After calculating the matrix $K(\theta_0, i\bar{\omega})$ for equations (3-7) at $\bar{D}_t(\theta_0, i\bar{\omega}) = 0$ (no dry friction), using the developed algorithm of the method of *D*-partitioning, it is possible to calculate the need for the sustainability movement oriented wheels the values of the damping ratio $\nu_\theta = \pi \operatorname{Im}\Delta k_{22}(i\bar{\omega}_{sh})$, which are determined by the integrated value of the increment to the imaginary part of the matrix element $k_{22}(i\omega)$ and calculated by the formula:

$$\Delta K_{22}(i\bar{\omega}) = -I_y \bar{\omega}^2 + \frac{\det' K(\theta_0, i\bar{\omega})}{\det A_{22}(\bar{\omega})} \quad . \tag{9}$$

Where: $\det' K(\theta_0, i\bar{\omega})$ - the determinant of a matrix $K(\theta_0, i\bar{\omega})$ with $K_{22}(i\bar{\omega}) = 0$; $\det A_{22}(i\bar{\omega})$ - algebraic addition to the element $K_{22}(i\bar{\omega})$ of the matrix $K(\theta_0, i\bar{\omega})$. It is obvious that the intersection points of the calculated function $\nu_\theta = \pi \operatorname{Im}\Delta K_{22}(i\bar{\omega}_{sh})$ with the given function $\pi \operatorname{Im}\bar{D}_t(\theta_{0,}\, i\omega_{sh}) = 4\bar{M}_t / \theta_0$ determines the amplitude and frequency of the LCO of the study system under depending on the changes of given parameters of the nonlinearities $\bar{M}_t, \sqrt{\bar{h}_\chi}$ and $\bar{V}$ (figure 5).

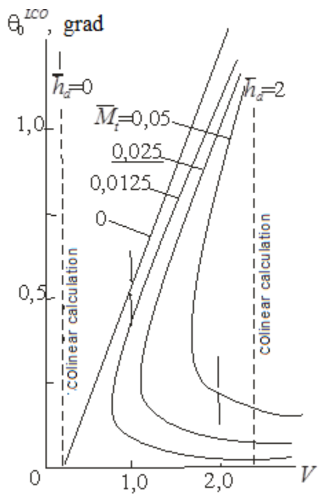

Figure 5: Impact of forces of dry friction on the amplitude of LCO

From the analysis of the received dependences shows that the nonlinearity of a quadratic and the dry friction type significantly change the picture of the emergence and development of shimmy. In the absence of structural friction forces (M_t=0) the movement of the guided wheels can be accompanied by a sustained oscillation, while the level of external perturbations will not cause oscillations of landing gear with amplitudes exceeding the amplitude of the unstable limit cycle. With the growth of the dry friction forces the amplitudes values of the unstable LCO are rising, and sustainable, on the contrary, decreases, i.e., dry friction of strut is able to significantly delay the occurrence of shimmy.
To justify the validity and assess the accuracy of the calculation of the parameters of the LCO and illustrate the capabilities of the developed algorithms research shimmy were made calculations of the shimmy characteristics in the time domain.

In figure 6 for several values θ_0 of the initial angle of strut twist (and the zero initial values of the remaining components of the state vector of investigated system of equations) shows the dependence of the amplitudes θ_N of the landing gear twist angle from the number of periods of oscillations N and the magnitude of the moment M_t.

From the analysis of these dependencies, in particular, it follows that if the level of the external disturbance, acting on the landing gear, leads to a twisting of the strut to the angle θ_0 that is less than the amplitude of the unstable limit cycle oscillation, in most cases, the oscillations of the wheels during rolling are damped, or installed with reduced amplitude stable periodic oscillations (auto-oscillations). If the initial angle θ_0 of twist strut of the is greater than the amplitude of the unstable limit cycle, the oscillations of the wheels can indefinitely increase.

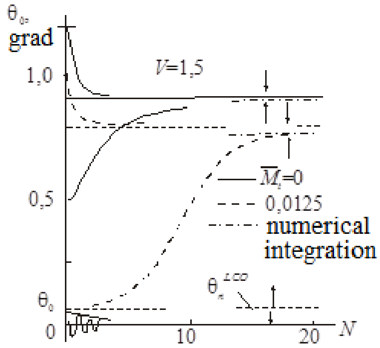

Figure 6: Comparison of method of harmonic linearization with the method of numerical integration

The results of numerical integration of the nonlinear system of equations shims (3) make important conclusion that the accuracy of the approximate method of harmonic linearization of the considered nonlinear dependencies is quite satisfactory for performing engineering calculations of limit cycles oscillations (Figure 6).

To illustrate the practical application of the presented algorithms computational studies oriented shimmy of the wheels of the landing gear and assessing their accuracy on the below figure 7 shows the results of calculation of amplitudes θ_n^{LCO} of the unstable limit cycles oscillations posh light aircraft equipped with a friction damper M_t with time , and their comparison with the experimental data. It is seen that the accuracy of calculation results is sufficient for practice.

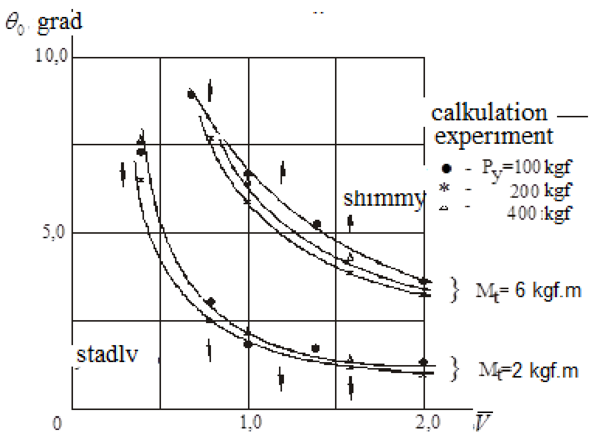

Figure 7: The amplitude of the unstable limit cycle oscillations (LCO) posh with a friction damper to the speed V

4 CORRECTION SETTINGS MM SHIMMY

As stated above, for reliable estimates of the characteristics of the shimmy of the wheels requires not only adequate mathematical model, but reliable methods for determining the parameters of these models and their correction, among which the main methods are the frequency of testing landing gear and methods of spectral analysis results of SI and AI. As practice shows computational studies, the most important parameters is the stiffness of the landing gear and the magnitude of the damping of the oscillations of the wheels with the required accuracy can be determined based on the correction of these parameters on the results of the dynamic (frequency response) tests of the landing gear. With this purpose on the program *Shimmy* calculated roots of the characteristic polynomial equations shimmy (for example, (1) or (2)), the hodographs which are represented, for example, depending on changes in the stiffness of the strut in torsion and variation of the resistance coefficient of the damper shimmy at speed rolling wheels. From figures 8 and 9 it is seen that by the variation of the dimensionless values of stiffness of the strut in torsion $\overline{C}_{\theta} = C_{\theta} / nar^2$, bending stiffness $\overline{C}_{\psi} = C_{\psi} / nar$ and coefficient of damping $\overline{h}_{\theta} = h_{\theta} / nar^2 \sqrt{am}$ their values can be determined from the comparison of the calculated spectrum of roots $s = -\delta \pm i\, f$, $(i = \sqrt{-1})$ of the characteristic polynomial equations shimmy (1) with the experimental data on the frequencies and damping coefficients obtained in the frequency test landing gear.

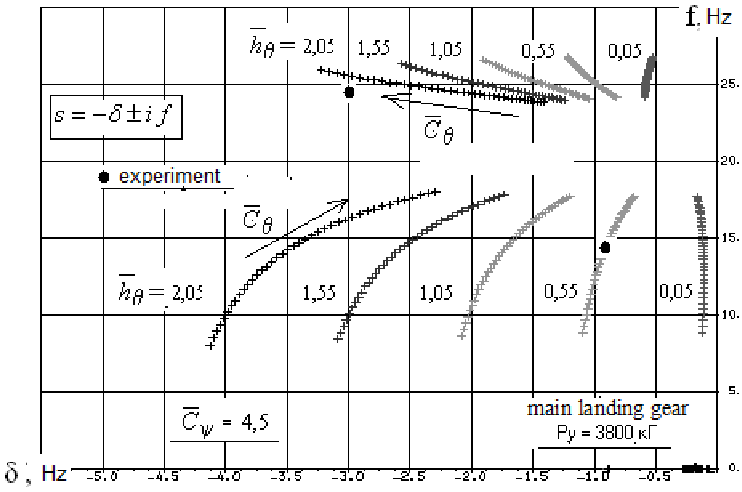

Figure 8:Comparison with experiment hodographs roots of the characteristic polynomial equations shimmy oriented wheels at a variation of parameters of stiffness and damping

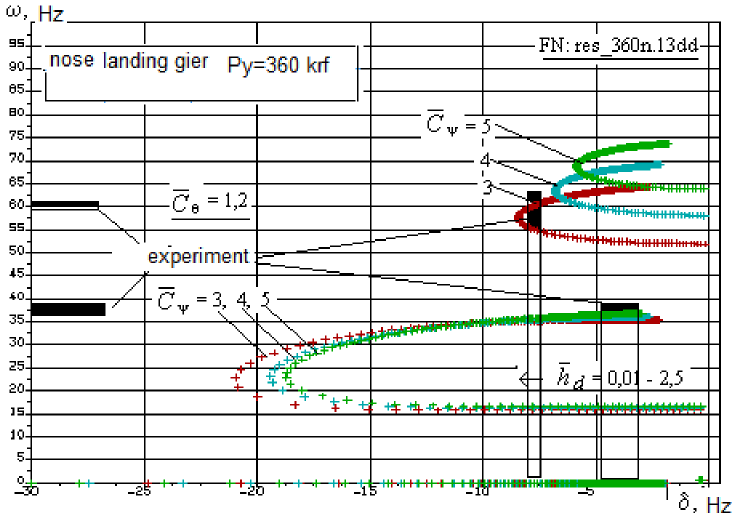

Figure 9:the comparison with the frequency of the test roots of characteristic polynomial equations shimmy oriented wheels with variations of stiffness of the strut in bending and the drag coefficient of the damper shimmy

Parameters $\overline{C}_\theta, \overline{h}_\theta$ landing gear are the most important parameters to establish the safety of the aircraft from the shimmy of guided and unguided wheels, so their values should be specified according to the experimental data. Such data can be directly obtained from processing the results of resonance tests of the landing gear torque through the characteristics of the corresponding dynamic stiffness. Since the resonance frequencies of the torsional support is carried out by agents of fluctuations, creating power F_{1x} and $-F_{2x}$ applied to the ends of the wheel shafts and active in opposition in the longitudinal direction (along the axis of the aircraft, figure 10), then the amplitude of the phase-frequency characteristic (AFCH) of torque $M\text{y}(j\omega)$ and the twist angle of the $\theta(j\omega)$ from these forces are calculated by the formulas [2]:

$$M_y(j\omega) = F_{1x}(j\omega)l_1(\omega) - F_{2x}(j\omega)l_2(\omega)\,;\ \theta(j\omega) = \frac{d_{1x}(j\omega) - d_{2x}(j\omega)}{l_d}$$

Where: $F_{1x}(j\omega)$, $F_{2x}(j\omega)$ - AFCH forces; l_1 and l_2 are the distances from the points of application of forces F_{1x} and F_{2x} to the point relative to which the rotation axis of the wheels when the resonant oscillations are calculated as functions ω:

$$l_1(\omega) = |d_{1x}(j\omega)|/|\theta(j\omega)|, \qquad l_2(\omega) = |d_{2x}(j\omega)|/|\theta(j\omega)|$$

Calculation AFCH the rotation angle $\theta(j\omega)$ of the axis of the wheels is performed by using AFCH response accelerometers, which are located at the ends of the wheel axle at a distance of l_d to measure the corresponding amplitudes of oscillations $d_{1x}(j\omega)$ and $d_{2x}(j\omega)$. Then dynamic support stiffness in torsion can be represented as:

$$D_\theta(i\omega,\theta_0) = \frac{M_y(i\omega)}{\theta(i\omega)} = \mathrm{Re}D_\theta(i\omega,\theta_0) + \mathrm{Im}D_\theta(i\omega,\theta_0) \approx$$
$$\approx [C_\theta^{эфф}(\omega,\theta_0) - J_y^0\omega^2] + i\omega h_\theta^{эфф}(\omega,\theta_0).$$

where J_y^0 is the generalized moment of inertia of the landing gear relative to the axis of orientation of the wheels, which is determined mainly by the mass and diametral moment of inertia of the wheels. θ_0 – the amplitude of the angular oscillation of the wheels relative to the axis of their orientation. From this equation it follows that the desired characteristics C_θ^{eff}, h_θ^{eff} can be calculated according to the processing AFCH $D_0(j\omega,\theta_0)$ and presented in a dimensionless form as follows:

$$\begin{aligned}\bar{C}_\theta^{eff}(\omega,\theta_0) &= \frac{\operatorname{Re} D_\theta(j\omega,\theta_0) + J_y^0}{nar^2} \\ \bar{h}_\theta^{ee}(\omega,\theta_0) &= \frac{\operatorname{Im} D_\theta(j\omega,\theta_0)}{\omega nar^2\sqrt{am}}\end{aligned} \tag{10}$$

As an example, in figures 10 and 11 shows the family of dependencies C_θ^{eff} and h_θ^{eff}, which were obtained on the processing results of the frequency tests of the nose landing gear (NSE) of the aircraft. A family of dependencies defined for the three relative magnitudes of the excitation forces of 0.3; 0.65 to 0.95. Point on the curves "row 4" in these figures correspond to the resonant frequencies f_{rez} and define the required payments on shimmy ranges of parameter changes $\bar{C}_\theta^{eff}(\omega,\theta_0)$ and $\bar{h}_\theta^{eff}(\omega,\theta_0)$.

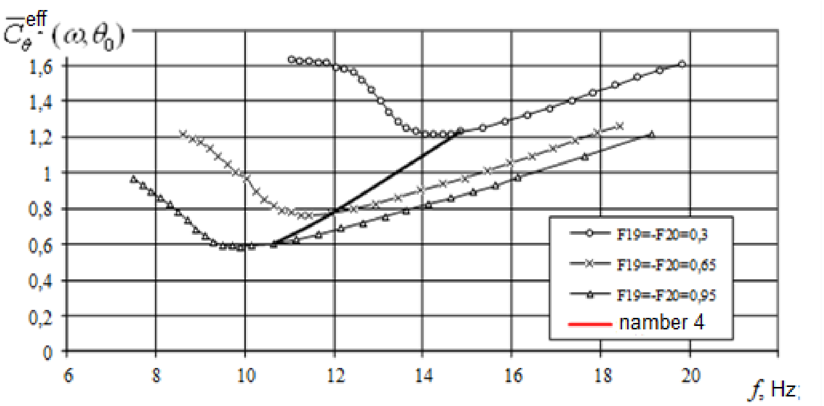

Figure 10: the Dependence of effective stiffness on torsion posh from the frequency oscillation

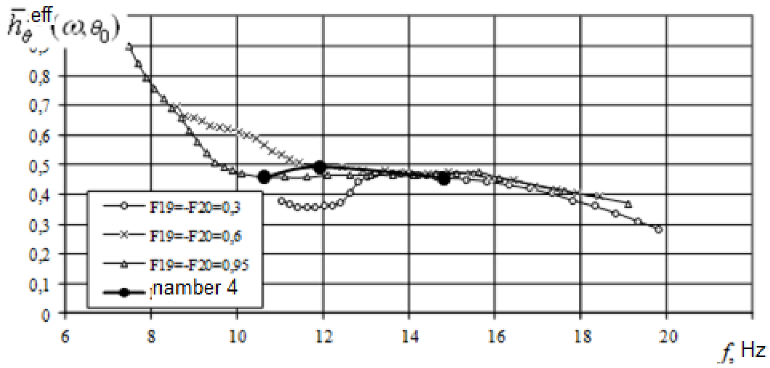

Figure 11: Dependence of the effective damping ratio tone swirl posh by the frequency of oscillation

It is obvious that instead of the resonant characteristics for the correction of the parameters of MM can be used in the experimental modal characteristics that are invited to determine from the results of spectral analysis of temporal realizations of the rolling wheel, measured in SI or AI trials.

For bench testing needs to be addressed, in addition to the basic problem of determining the stability of the rolling wheels, two tasks:

- to determine the stiffness of the rack mount on the stand and set their contrast to the stiffness of the mounting plane;
- to obtain the modal characteristics of a support mounted on the stand for the correction of the corresponding MM.

The solution of these problems can be obtained from the results of spectral analysis of impulse functions obtained by a direct impact with the tire of the wheel (figure 12). An example of impulse functions of the sensor signals from impacts on bus fixed wheels presented in figure 13.

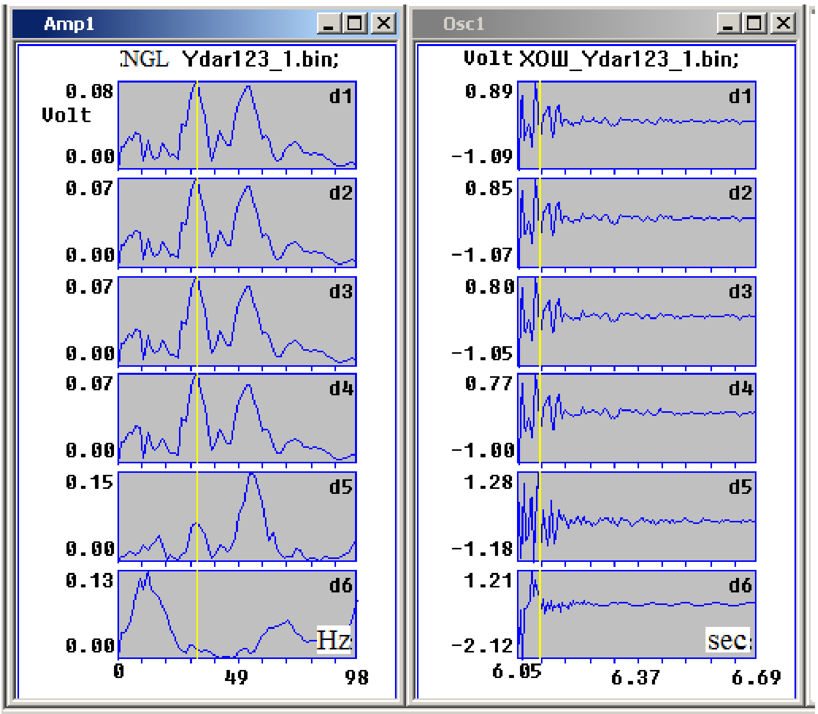

Figure 12: Amplitude Fourier spectrum of impulse functions from a blow to the bus

The processing of impulse functions from impacts on bus you can get the frequencies, shapes and decrements of natural oscillations of the supports and of the comparison with the results of the frequency tests of the landing gear in the structure of the airframe to establish the differences in the strut and use them for the correction of the MM parameters. For the treatment of impulse functions it is recommended to use the program calculate the Fourier spectrum, which for the selected resonance peak are determined by the parameters of the resonant tones, i.e. is the resonant frequency, logarithmic decrement and the amplitude of the oscillations on the signals of all sensors. From a comparison of the phase shifts in time of the sensors is determined by the form of the resonant oscillation of the support. A similar problem is solved in the analysis of the records of free vibrations of a rolling drum stand wheel after the initial deflection of its axis of rotation.

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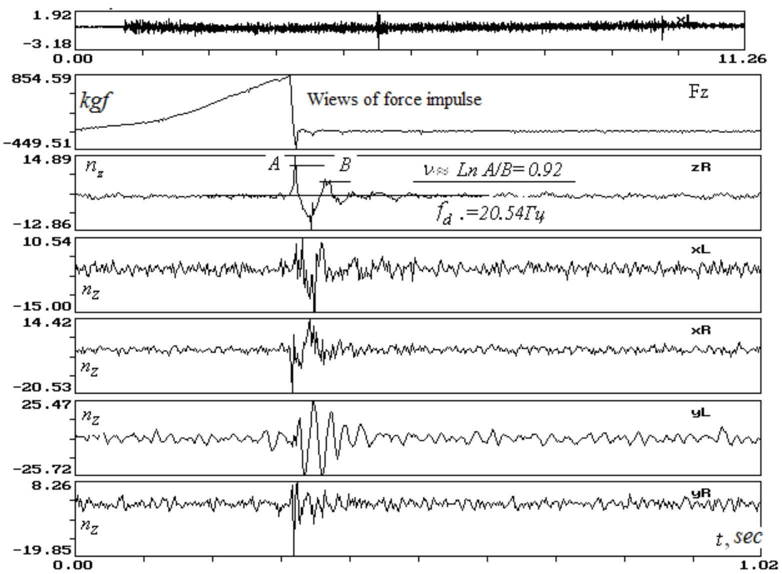

Figure 13: Fragment of a recording pulse function overload n_z at the ends of the axles of the wheels with SI for shimmy

The main characteristics of the rolling process (frequency and decrement of oscillations) can be promptly determined from the waveform of this process, as illustrated in figure 13 and previously in figure 12. From figure 13, in particular, it follows that the form of oscillations in the wheel is a twist wheel about a vertical axis, as the fluctuations of the left (xL) and right (xR) of the ends of the axle of wheels occur in antiphase. Modal characteristics of the landing gear can be obtained from the spectral analysis of the signals of the respective sensors, which register the oscillation of the pillars of the high-speed taxiways, from the collisions of the wheels on the "Board shimmy", with the takeoff and landing of the aircraft. The basic procedures obtain the corresponding oscillation modes following. Covers a typical recording of the sensor signal, for example, lateral acceleration, which describes the oscillations of the wheels of the landing gear when hitting the "Board shimmy" with a constant velocity (figure 14). It should be noted here that these signals are the same as the sensor readings obtained during SI wheels rolling after a disturbance with almost constant speed. Therefore, methods of analysis of such oscillations are common to AI and SI.

It is obvious that the modal parameters of the wheels of the landing gear, such as natural frequency and logarithmic decrement of the oscillations are directly determined by recording the transient process (figure 14) using standard procedures of Microsoft Office Excel to approximate the amplitudes of the oscillations of the exponential function. Form of own fluctuations of the selected tone can be determined by the values of the amplitudes of signals of all the sensors from the Fourier amplitude spectrum and the phase shifts of their time implementations. Such procedures were successfully applied for the analysis of the results AI of the aircraft RRJ-95 for safety substantiation of the first flight, and then to confirm that shimmy of the wheels OOSH when removing HD.

For the analysis of processes of rolling of the wheels during high-speed running, when the takeoff and landing of the aircraft proposed another signal processing sensors. The application of this technology was tested on the example processing time process of the wheel rolling posh for readings of two accelerometers mounted on the front of the chassis near the attachment of the wheels to measure congestion in the direction perpendicular to the diametrical plane of the wheel.

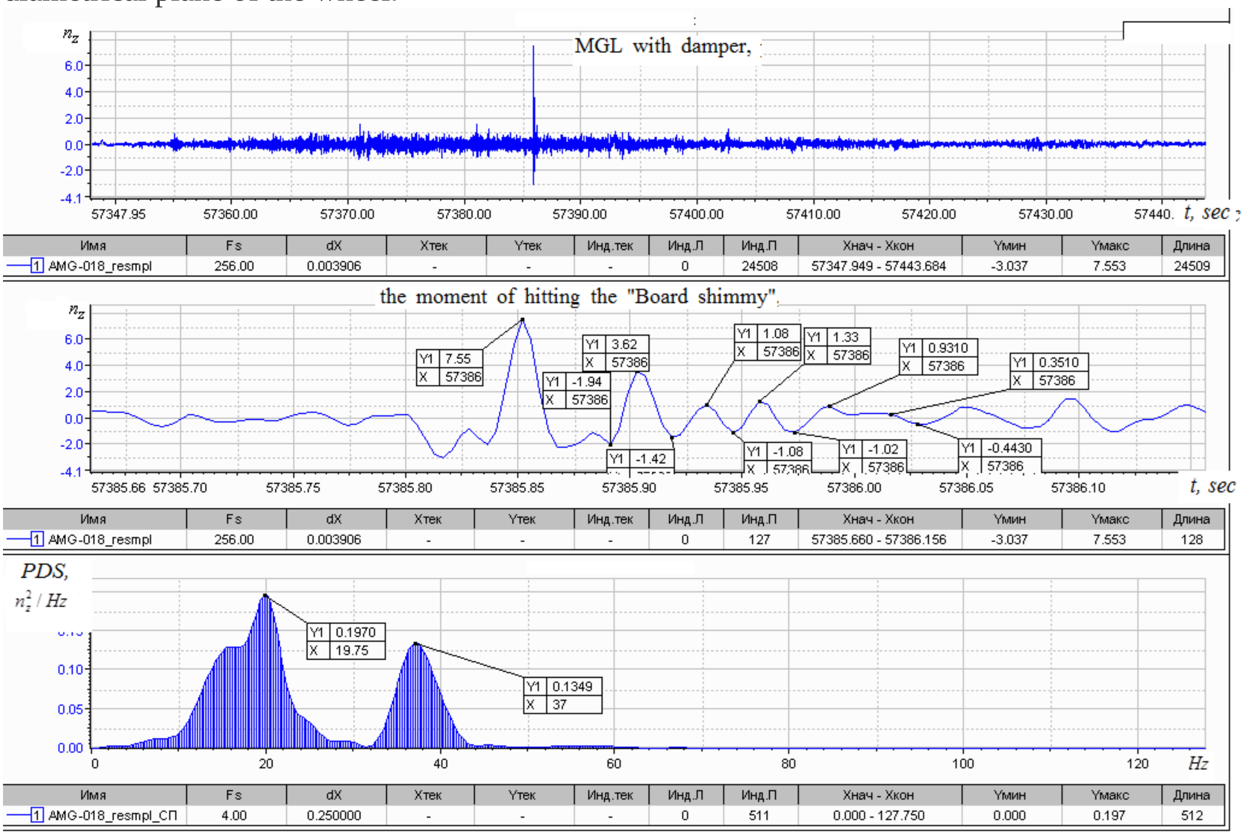

Figure 14: Hitting the "Board shimmy" wheels MLG, free vibrations and their PDS

To obtain modal characteristics of the wheel from the records of the signals of the accelerometers (figure 15) was selected, a 2 second interval of time for which it was possible to make that the movement of the aircraft takes place with constant velocity *Vs*. Further, the signals of two accelerometers registered oscillations of the wheels in the lateral direction, calculates the amplitude Fourier spectrum or PSD.

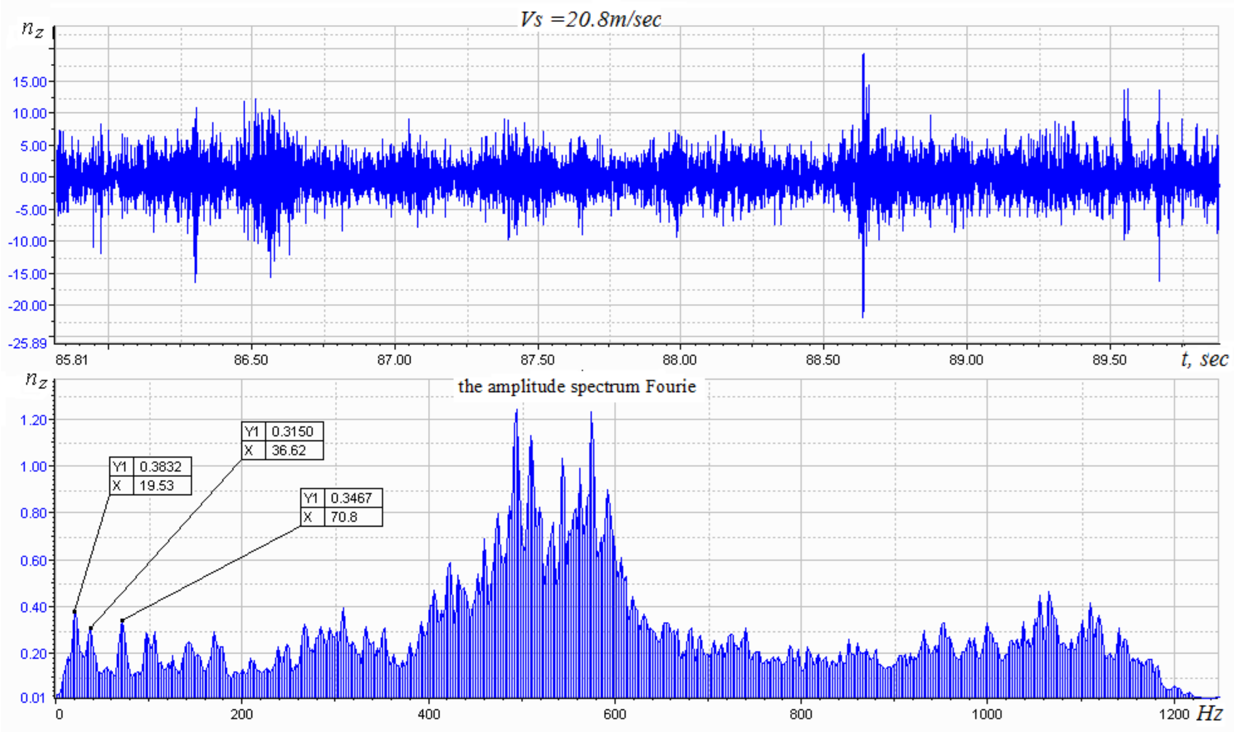

Figure 15: Typical entry overload n_z in lateral direction and its amplitude spectrum

Analysis of the results showed that the process of the wheel rolling posh in the speed range, characterized by the presence of the basic 4-dominant oscillation frequency, which only slightly depends on the speed of movement (figure 16).

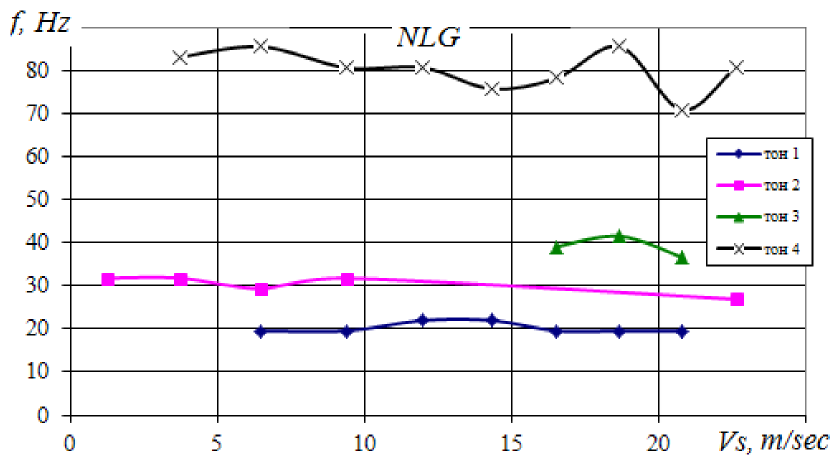

Figure 16:Dominant frequency of oscillation of the wheel sensor signals overload n_z is based on the average rolling velocity of the wheel

Determination of the decrement of the oscillations for each mode of oscillation is more complex. However, an approximate estimate of this parameter can be obtained across the width of the amplitude of Fourier spectrum or PSD in the vicinity of the corresponding resonant frequency. To determine the third modal characteristics of the tone – compute mode shapes of wheels are offered, as in the case of pulsed excitation of wheels at SI and for the case of collision of the wheels to "shimmy Board" to allocate in the vicinity of the dominant frequency of a narrow frequency band. For this range of frequencies proposed a recursive filtering of the signals of all sensors and compare their time of oscillation to the phase shifts between the sensors. From figure 17 it is seen that the signals of the two sensors in a horizontal plane for measuring congestion in a sideways direction to the vibration modes lying in the frequency range *f*=30.0-40.0 Hz the phase shift in the sensor readings was observed. This suggests that in this frequency range is dominated by the amplitude of the oscillations of the lateral bending support.

Figure 17: time signals of the sensors, after recursive filtering in the frequency range *f*=30.0-40.0 Hz. V_{CP}=20.8 m/s

Further, from figure , records are also obtained after recursive filtering of the signal of the same sensors (figure 18) shows that in the frequency range *f* =70.0-80.0 Hz signals of these sensors are in antiphase, i.e. there is a torsion support.

Thus, the possibility of identification of the modal characteristics of OSH by results of processing of signals of the two sensors of the overload, mounted on the front of the chassis away from the unit wheel.

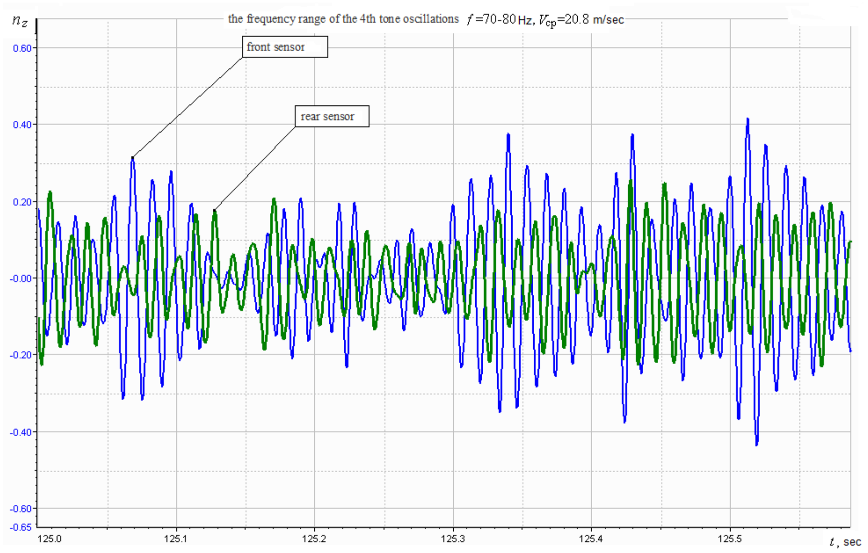

Figure 18: Temporal signals of the sensors after the recursive filter in the frequency range *f*=70.0-80.0*Hz*. Vs=20.8 m/s

4 CONCLUSION

The results discussed above, methods and algorithms show possibilities of the developed research methodology shimmy wheels of different chassis models with local nonlinearities

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