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MODAL PARAMETER ESTIMATE OF TIME-VARYING SYSTEM USING OPERATIONAL MODAL ANALYSIS BASED ON HILBERT TRANSFORM

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Abstract: In this paper an approach for the identification of the modal properties of a time-varying system in the Operational Modal Analysis (OMA) framework is presented. The natural frequencies are estimated using the Hilbert-Huang Transform Method (HHTM) for its capabilities to track the time variations of such modal properties. Then, the Hilbert Transform Method (HTM) is applied for estimating both the damping ratios and mode shapes, not available from the previous method. The developed approach has been numerically assessed considering an aeroelastic system. The accuracy in time-tracking the modal parameters has been investigated by analyzing the numerically simulated time responses from a panel flutter model subjected to random operating loading for which the analytical solution is available.

1 INTRODUCTION

Many engineering applications could be described by time-varying systems. In structural dynamics, such time variations can be suitably represented as a change in the instantaneous modal parameters. For aerospace industry, the identification of the modal parameters of the structure subjected to its actual operating conditions, is of great interest because using such modal parameter, it is possible to validate and consequently update the numerical model of the structure. Indeed, considering the actual operating conditions a more accurate representation of the structural dynamics properties with respect to the real ones is possible. Typical example of the time-varying systems, in aerospace field, are those characterized by the change of their dynamical properties during the different phases of the flight, like an aircraft undergoing changes of the dynamic pressure, or a launcher burning the propellant and losing a large amount of mass. Therefore, an approach capable of estimating such time-varying modal parameters is very appealing when validating the aero-mechanical design or predicting the effects of given structural modifications. The natural frequency time-evolutions are currently analyzed by means of the spectrogram, which is a time-frequency representation of a non-stationary signal, obtained by a limited time-window Fourier Transform centred in separate time-intervals. This approach, although very efficient, lacks in estimates of the damping ratios and mode shapes. Recently, experimental techniques based on the analysis of response-only data recorded from an operating system, named Operational Modal Analysis (OMA), have been developed. These OMA techniques have been introduced relying their basic equations both in time and frequency domain. Among the techniques available in literature, it is worth recalling the frequency domain techniques called Frequency Domain Decomposition (FDD) [1], Hilbert Transform Method (HTM) [2], and Stochastic Subspace Identification (SSI) [3], whereas, in time domain, the well known technique is the Balance Realization Method (BR) [4]. The OMA techniques differs from the one belonging to the so-called Experimental Modal Analysis (EMA) [5], because the measurement of the input excitation is not required, provided that a stochastic broadband uncorrelated (both in time and space domain) dynamic loading excites the system. In this contest, OMA methods are very usefull tool to track the eigen-properties of an aeroelastic system as a function of the flying dynamic pressure or other characteristic flight parameters such as attitude, mass distribution, etc. The objective of this paper is to propose an innovative methodology for the identification of modal parameters of a time-varying system, which is based on Hilbert Transform [6]. In detail, the identification of the natural frequencies is performed by using the Hilbert-Huang Transform Method (HHTM) [7], which is a suitable tool due to its capability of estimating the instantaneous frequencies of the observed time-varying system. It is worth pointing out that the HHTM has been used instead of a spectrogram, because it is shown in literature that the first methodology yields better estimate of instantaneous frequency of an arbitrary signal than the second one [7]. Then, the HHTM has been used on the filtered signals. Once an estimate of the instantaneous frequencies is obtained, a procedure to extract the global time-tracking of the estimated natural frequencies is presented. Thus, using the information derivated from the time-tracking of these quantities, it is possible to estimate the damping ratios and the mode shapes by means of an OMA method, which can be applied only to a time-invariant system. In particular, considering the maximum variation of the natural frequencies, it can be found a time-interval in which the time-varying system can be considered time-invariant and therefore, the whole observation time can be split in such intervals. For this reason, in order to estimate the damping ratios and the mode shapes the Hilbert Transform Method (HTM) [2] has been applied. This method has the advantage to use the Hilbert Transform, which provides the biased FRFs and, in turn, the modal parameters from a residue/poles curve fitting method.

2 THEORETICAL BACKGROUND

In this section, the HHTM and HTM for the estimation of modal parameters of a time-varying system are briefly introduced.

2.1 Natural frequencies estimated by Hilbert-Huang Transform method

The Hilbert-Huang Transform (HHT), has been proposed by Huang et al., [7]. The core of the method is the Empirical Mode Decomposition (EMD) procedure, which can be preceded by a filtering technique[1] [8]. EMD process is aimed to decomposing an arbitrary time series into a finite, often small number of Intrinsic Mode Functions (IMFs). Specifically, the IMFs are continuous, real-valued functions that satisfy two conditions, i.e. the number of local extrema and zero crossings must be equal or differ by at least one and the mean value of the two envelope curves, formed by the extrema (local minima and maxima, respectively), should be zero at any time. Since the decomposition is based on the local characteristic time scale of the data, HHT is applicable to nonlinear and non-stationary processes. Let $x(t)$ the measured signal, N the number of empirical modes, $c_n(t)$ the n^{th} intrinsic function and $r_N(t)$ the residual term, it is

[1]In this paper, the passband filter has been used, as explained in section 3.1.

possible to write:

$$x(t) = \sum_{n=1}^{N} c_n(t) + r_N(t) \tag{1}$$

The extraction of the IMFs is performed via the Sifting Process [7]. This procedure, each local maximum and minimum of the time series are located and two envelope curves connecting all maxima and minima are constructed using spline interpolation. Then, a mean curve is built from the envelope curves and subtracted from the original time series. This curve should be close to an IMF, but it may need some refinement, because it does not satisfy all the requirements to be a IMF, as explained above. Therefore, such a function is treated as input data and the previous steps are repeated k times with an appropriate stopping criterion [7]. The last curve is subtracted from the original time series and a new input is given. Then, the process restarts. This iterative method can be stopped by any of the following predetermined criteria: either when the component, $c_n(t)$, or the residue, $r_N(t)$, becomes costant, or when the residue becomes a monotonic function from which no more IMFs can be extracted. Further details on Empirical Mode Decomposition method are present in [7]. Once the intrinsic mode function components have been obtained, it is possible to apply the Hilbert Transform to each IMF obtaining the associated analytical signal, $x_n(t)$, which is a component of entire analytical signal, Eq.4, and is defined for a real signal $c_n(t)$ as:

$$x_n(t) = c_n(t) + j\mathcal{H}(c_n(t)) = a_n(t)e^{j\theta_n(t)} \tag{2}$$

where $\mathcal{H}(\bullet)$ indicates the Hilbert Transform operator [2], $\theta_n(t)$ is the instantaneous phase of the n^{th} intrinsic mode. The instantaneous frequency [7] can be computed for each IMF as:

$$f_n(t) = \frac{1}{2\pi}\frac{d\theta_n(t)}{dt} \tag{3}$$

Therefore, the analytical signal, $X(t)$, associated to the real one $x(t)$ could be express as a combination of analytical signal obtained by applying Hilbert Transform on each IMF:

$$X(t) = \sum_{n=1}^{N} a_n(t)e^{j\theta_n(t)} \tag{4}$$

Eq.(4) gives both the amplitude and the instantaneous frequency [9] of each component as functions of time. When the Hilbert-Huang Transform is applied on the filtered time series, it is possible to derive the local energy and the instantaneous frequency that give a full energy-frequency-time distribution of the data from the IMFs. In each frequency band, chosen for the filtering technique, a resultant IMF is calculated as a mean of the all IMFs in that band and it may be possible to define a resultant frequency, $\hat{f}_n(t_i)$, as:

$$\hat{f}_n(t_i) = \frac{\sum_{i=1}^{N_{IMF}} {f^B}_n(t_i)[{a^B}_n(t_i)]^2}{\sum_{i=1}^{N_{IMF}} [{a^B}_n(t_i)]^2} \tag{5}$$

[2]The Hilbert Transform of a signal $x(t)$ is defined as the Cauchy principal value of:

$$\hat{x}(t) = \mathcal{H}\left[x(t)\right] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau$$

where, in the considered band, $f_n^B(t_i)$ is the instantaneous frequency of each IMF and the instantaneous power $[a^B{}_n(t_i)]^2$ is assumed as weighting function. Once the instantaneous frequencies are estimated by using HHTM, it is possible to represent the time-tracking of these quantities via a polynomial interpolation of the instantaneous frequency. For the purposes of further investigations, a weighted polynomial interpolation is used for this purpose, where the weights are given by the instantaneous power associated to the considered resultant IMF at a specific time instant.

2.2 Damping ratios and mode shapes estimated by Hilbert Transform method

As already mentioned, the advantage of using the Hilbert Transform is primarily the capability of estimating the imaginary part of a causal function starting from its real part (and viceversa). The polar representation of driving point FRF is given by:

$$H_{ii}(\omega) = |H_{ii}|e^{-j\phi_{ii}(\omega)} \tag{6}$$

or, by introducing the natural logarithm, it can be expressed as:

$$\ln\left[H_{ii}(\omega)\right] = G_{ii}(\omega) - j\phi_{ii}(\omega) \tag{7}$$

in which $G_{ii}(\omega) = \ln\ |H_{ii}(\omega)|$ is the gain function. Considering that the real part of the FRF is an even function and the imaginary part is an odd function, the gain and the phase are therefore even and odd, respectively. As a result, the left-hand side of Eq.(7) can be expressed as the sum of a pair of Hilbert Transform functions:

$$\phi_{ii}(\omega) = -\hat{G}_{ii}(\omega) \tag{8}$$

The gain function is also related to the spectral density function as:

$$\mathbf{G}_{yy}(\omega_k) = \mathbf{H}(\omega_k)\mathbf{G}_{ff}(\omega_k)\mathbf{H}^H(\omega_k) \tag{9}$$

where the input spectral density matrix, defined among N_i inputs, i.e. $\mathbf{G}_{ff}(\omega_k) \in C^{N_i \times N_i}$, is assumed to be derived from a white noise excitation. This implies that $\mathbf{G}_{ff}(\omega_k)$ is frequency independent and $\mathbf{G}_{ff}(\omega_k) = \mathbf{G}_{ff}$ and it is a diagonal matrix when the input excitation is uncorrelated in the space domain. As a consequence, by applying the natural logarithm and performing the Hilbert Transform, Eq.(9) becomes:

$$\mathcal{H}\left[\ln\left(G_{y_iy_i}\right)\right] = 2\mathcal{H}\left[\ln|H_{ii}(\omega)|\right] \tag{10}$$

in which the input spectral density contribution, $G_{f_if_i}$, is null, as the Hilbert Transform of a constant is zero.
Combining the previous Eqs.(8) and (10) it is possible to write:

$$\phi_{ii}(\omega) = -\frac{1}{2}\mathcal{H}\left[\ln\left(G_{y_iy_i}\right)\right] \tag{11}$$

Therefore, the FRF in the ith driving point is available. It is possible to demonstrate that the off-diagonal terms of the FRF are derivable from the comparison between the commonly used H1 and H2 estimators, [10]:

$$\tilde{H}_{ij}(\omega) = \frac{G_{y_i y_j}(\omega)}{\tilde{H}_{ii}(\omega)} \tag{12}$$

Obviously, the estimated functions are biased depending on the unknown input forces, but it is important to underline that the constant bias of the operational FRF does not affect the modal parameter estimates, because they are not dependent on the bias level supposed to be costant in the frequency domain. The modal parameters are evaluated with a least square approximation, considering the expression of the FRF in pole/residue terms. A stabilization diagram is also used in the estimating process to improve the accuracy [11].

2.3 A criterion of applicability of OMA techniques to time-varying systems

The OMA hypotheses require a linear time-independent system excited by white noise in order to identify the considered dynamics in terms of modal damping, natural frequencies and mode shapes. If the system is characterized by time-dependent dynamic properties, the analysis can be carried out by splitting the whole observation time into N_I sub-intervals $I_i, i = 1, ..., N_I$, in which the dynamical features of the system can be considered time-independent. Because the HHTM has been used in order to estimate the trend of frequencies over time, an accuracy criterion is defined considering such dynamical property. Therefore, following the criterion definited in [12] and already used in [13], an estimate of the length of the observation time window is given by:

$$\Delta t \leq \sqrt{\frac{1}{\dot{f}_n}} \tag{13}$$

where $\dot{f}_n$ is:

$$\dot{f}_n = max_I \left[\dot{f}_n(t) \right], \quad t \in [t_I, t_I + \Delta t] \tag{14}$$

The Eq.(13) shows the intuitive condition that the faster the rate of variation of f_n is, the smaller the length of the observation interval shall be in order to apply the OMA techniques to the given signal segment without significantly violating the stationarity assumption.

2.4 The Aeroelastic system

The proposed methodology is assessed considering the response of the structural skin of an aircraft wing subjected to a random load excitation which simulates the operating conditions. A ramp variation in time of the airflow velocity, $U = bt$ with b an arbitrary constant, is further introduced in order to make the system time-varying. The well-known model for "panel flutter" has been used to represent the system dynamics. This system is modeled as one-dimensional body subjected to a loading p, a supersonic flow and a spring bed, as shown in Fig.1. The mathematical model of this problem is proposed in [14]. If the problem is truncated to finite dimensions (e.g. if the first two modes are considered) and normalized to unitary modal mass, it is possible to write:

$$\begin{pmatrix} \ddot{w}_1 \\ \ddot{w}_2 \end{pmatrix} + \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} + \left(\begin{bmatrix} w_1^2 & 0 \\ 0 & w_2^2 \end{bmatrix} + \Lambda \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \tag{15}$$

where $d_n = 2\omega_n\zeta_n$, with ζ_n the modal damping ratio and ω_n the angular modal frequency. Moreover, p_n is the applied load (n = 1,2) and Λ is the aerodynamic term both projected into the model basis.

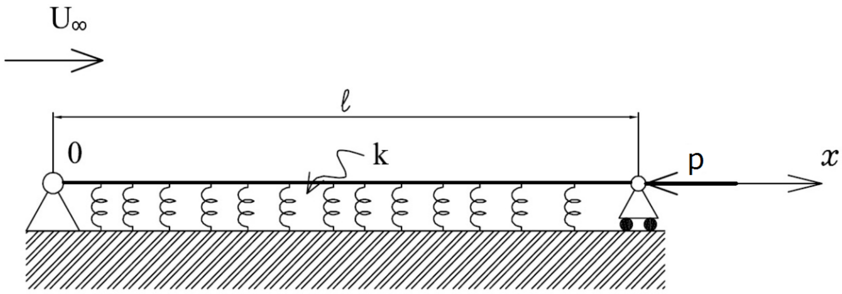

Figure 1: The "panel flutter" model.

In detail, the Λ coefficient, following [14], is linked to the airflow velocity by means of:

$$\Lambda = a_{km}U = a_{km}bt \tag{16}$$

in which k = 1 and m = 2, whereas the coefficient a_{km} is given by [14]:

$$a_{km} = \frac{2m}{k^2 - m^2} \cdot (-m\sin(k\pi)\sin(m\pi) - k\cos(k\pi)\cos(m\pi) + k) \tag{17}$$

and for this case, $a_{12} = a_{21} = 2.667$ and $a_{11} = a_{22} = 0$.
It is worth noting that the pure structural model, i.e. without aerodynamics, is at t = 0 when Λ = 0.

3 NUMERICAL RESULTS

The numerical model of the panel flutter, illustrated in the previous section, has been implemented in order to simulate the responses in terms of acceleration of four points of the structure. Then, the numerical data were generated by resolving the Eq.(15). The applied load is a random excitation in order to reproduce the flight conditions with the mean value and standard deviation equal to 0 and 1, respectively, and the modal parameters which have been used are shown in Tab.1.

Mode	f_n, Hz	ζ_n, %
1	10.0	1.0
2	35.0	2.0

Table 1: Modal Parameters used for the simulation.

Moreover, four time-histories, belonging to four different positions on the beam, have been used in order to have a sufficient number of measurement locations to identify the first two mode shapes properly. In particular, considering the normalization per unit for length, the chosen points are: 0.1, 0.3, 0.7 and 0.8. Moreover, in order to find the accelerations of these positions, the solution obtained in the modal basis have been projected into the inertial reference system and the corresponding vertical components have been used. The all analyses refer to a stable system, characterized by a flight velocity U lower than the flutter speed [3]. In Fig.2 the stable response time-histories, corresponding to the different locations are sketched.

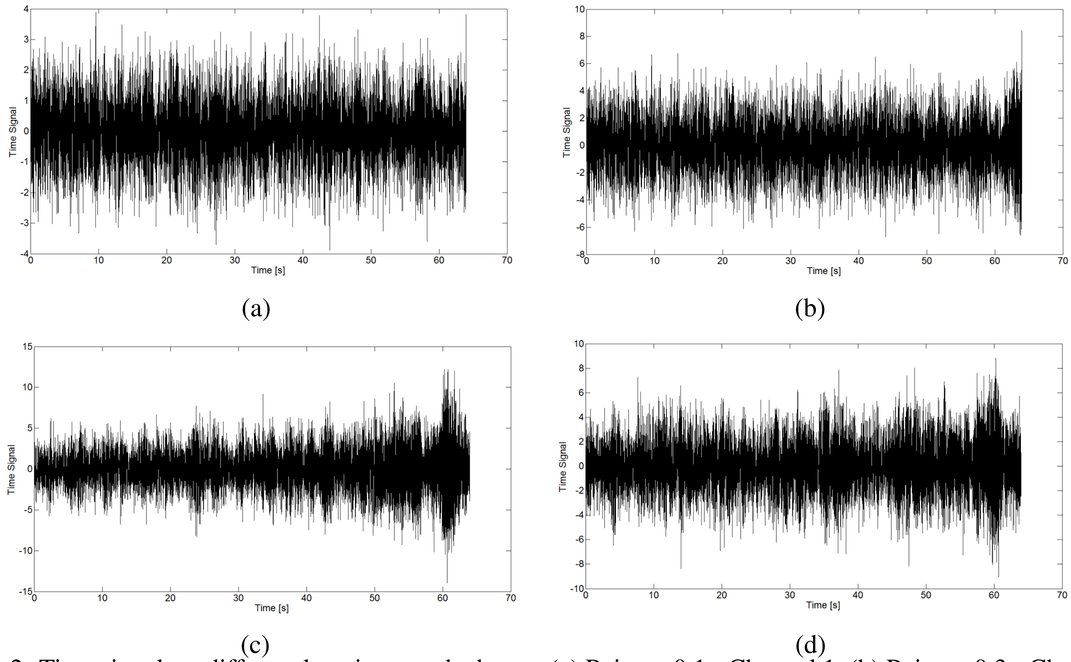

Figure 2: Time signals at different locations on the beam: (a) Point at 0.1 - Channel 1, (b) Point at 0.3 - Channel 2, (c) Point at 0.7 - Channel 3, (d) Point at 0.8 - Channel 4.

The theoretical solution, in terms of frequencies and damping ratios, is obtained resolving the characteristic polynomial of fourth order associated to system written in Eq.15 [14]. The resulting time-tracking of the analytical natural frequencies is shown in Fig.3.

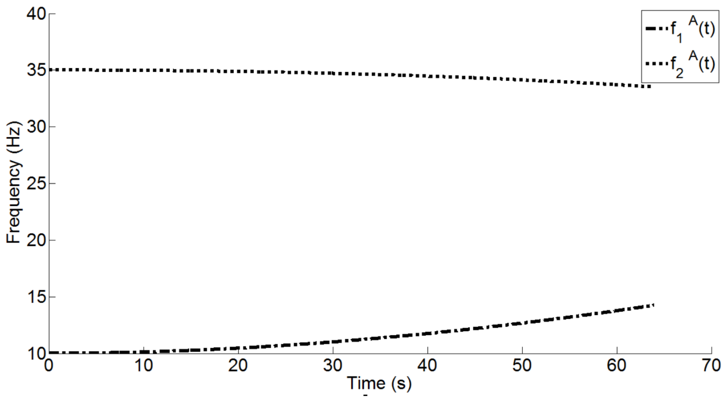

Figure 3: The time-tracking of analytical natural frequencies.

3.1 Estimate of natural frequencies by using HHT

Once the accelerations have been obtained, by resolving the Eq.(15), for each time-history, the Empirical Modal Decomposition and then the Hilbert-Huang Transform have been carried out, by using a passband filters. Such tools have been applied on all time signals. Thus, the natural frequencies have been approximately estimated by means of a spectrogram, in order

[3] The flutter speed is the speed beyond which the structure has dynamic instability. Thus, a frequency, which is belonging to the flutter mode, is associated to such velocity.

to identify the correct frequency bands to filter the time series. Then, two of IMFs have been recognized as each having the frequency content of one vibration mode. The selection of the these IMFs has been done by the user by observing that energy content of the original signal is practically contained in these IMFs. Recalling the Sifting Process, see section 2.1, the IMF with the content of the first frequency has been identified before the IMF with the content of the second one. The estimated behaviour of the natural frequencies is illustrated for each time response in Figs.4, 5, 6 and 7. The polynomial fit of the instantaneous frequencies has been performed by using the amplitude of the analytical signal, Eq.4, as from section 2.1. Fig.4 to Fig.7 show the results obtained respectively for signal 1 to 4. The black dots represent the instantaneous natural frequencies, whereas the green solid lines are the second order polynomial fitting of these quantities.

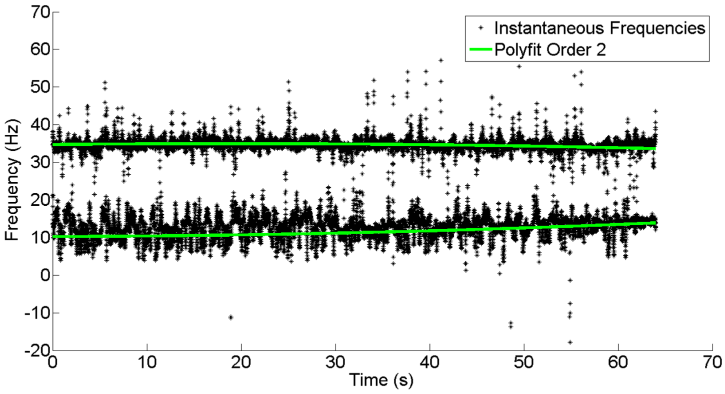

Figure 4: Time-tracking of natural frequencies using HHT, channel 1.

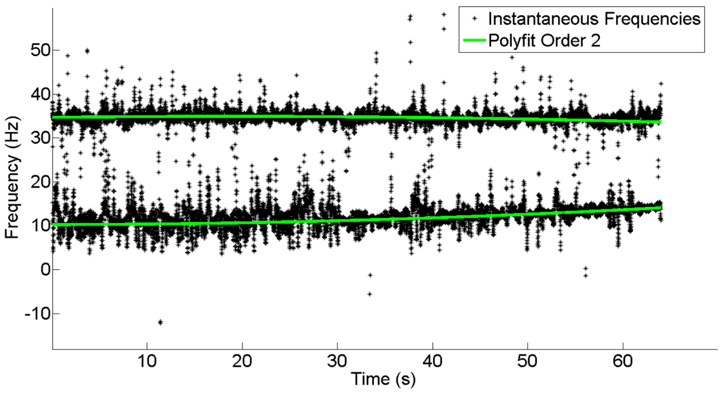

Figure 5: Time-tracking of natural frequencies using HHT, channel 2.

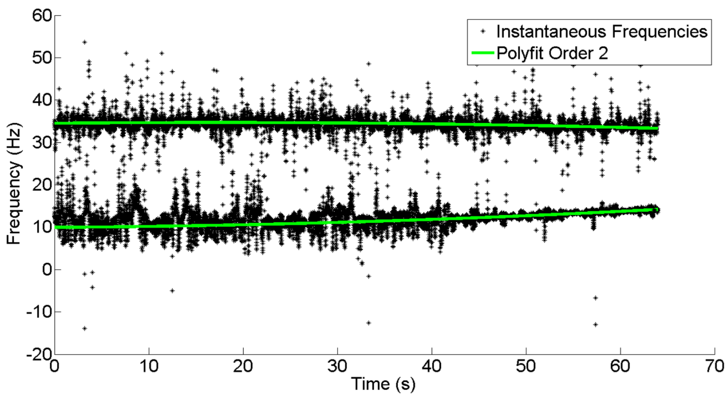

Figure 6: Time-tracking of natural frequencies using HHT, channel 3.

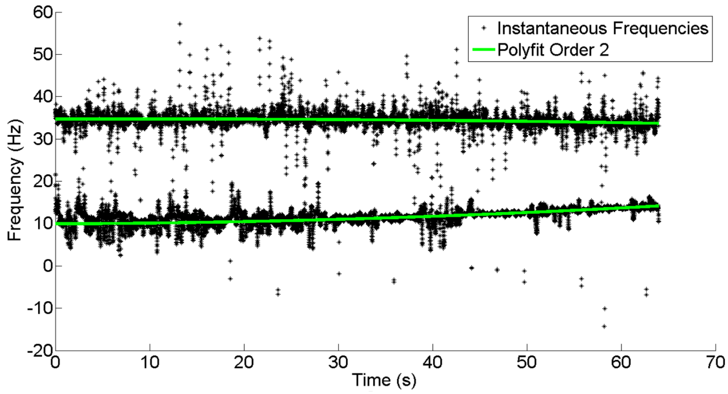

Figure 7: Time-tracking of natural frequencies using HHT, channel 4.

From the obtained results, it is possible to note that the behaviour of natural frequencies is well approximated by a polyfit curve of order 2, for each time-history. Moreover, a comparison with the analytical results has been carried out in order to demonstrate the accuracy of the estimates obtained by using the Hilbert-Huang Transform. Thus, by looking at Figs.8 and 9, it is possible to note that the time-tracking of the estimated natural frequencies are almost the same with respect to the analytical one.

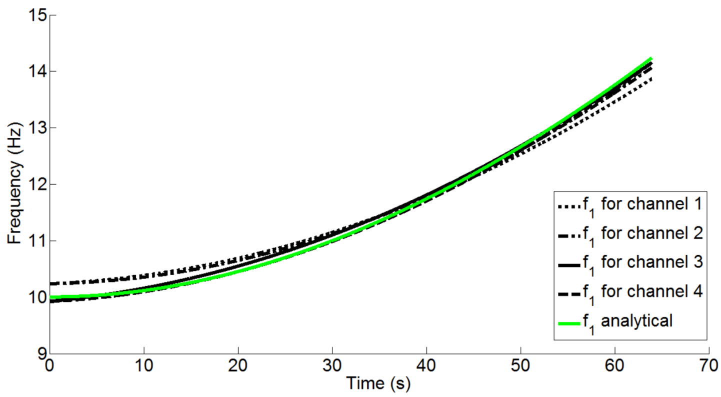

Figure 8: Time-tracking of natural frequencies: comparison between the first natural frequencies and analytical one.

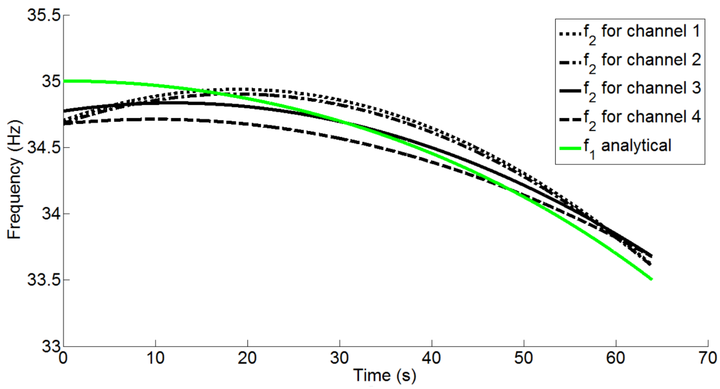

Figure 9: Time-tracking of natural frequencies: comparison between the second natural frequencies and analytical one.

As it is possible to see in Figs.8 and 9, the natural frequency estimates are practically coincident with the analytical predictions. The corresponding negligible errors are mainly due to numerical issues related to the time discretization process. For this reason, a further correlation between the results of the proposed method and the analytical ones is shown from Fig.10 to Fig.13, where the eigenfrequency shifts, defined as in Eq.18, are reported as a function of time and for each of the analyzed channels, respectively.

$$\varepsilon_k(t) = \frac{f_k^A(t) - f_k^E(t)}{f_k^A(t)} \tag{18}$$

in which k = 1,2.

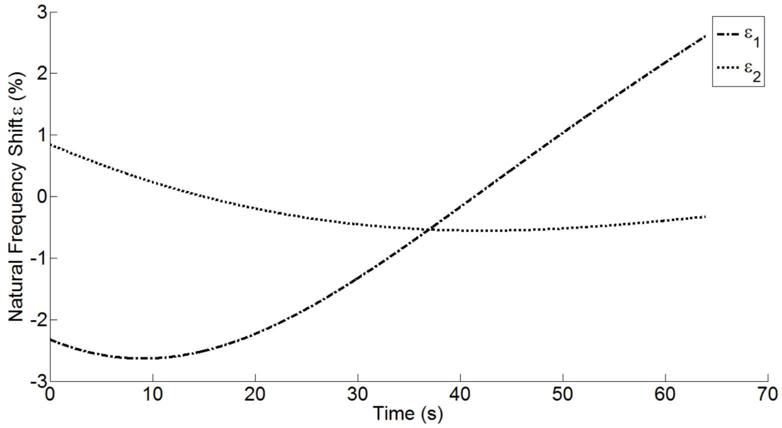

Figure 10: Natural frequency shift for channel 1.

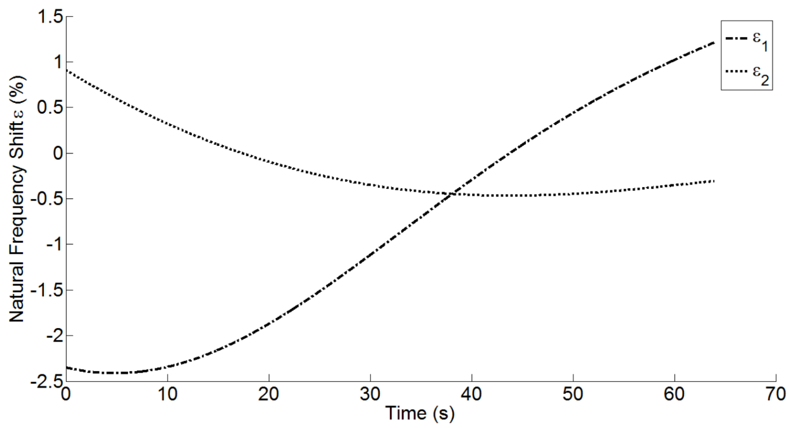

Figure 11: Natural frequency shift for channel 2.

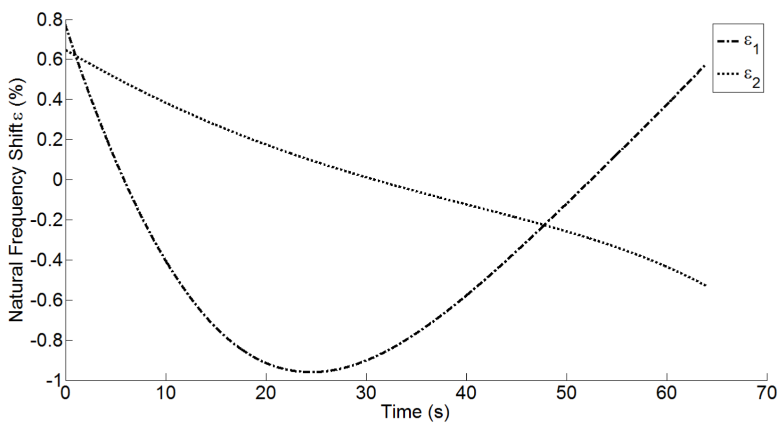

Figure 12: Natural frequency shift for channel 3.

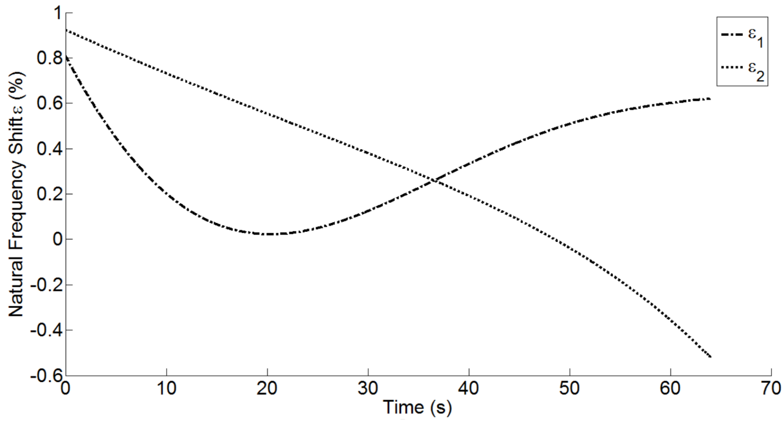

Figure 13: Natural frequency shift for channel 4.

From Fig.10 to Fig.13, it seems that the Hilbert-Huang Transform provides good estimates of the time-tracking of the natural frequencies of the considered aeroelastic system and the errors of the estimates are negligible. In fact, the natural frequency shifts show the same behaviour for all channels. In particular, the analytical first natural frequency is underestimated in the first part of the observation time and it is overestimated in the second part, whereas the second natural frequency shows an opposite trend. This behaviour is not found in the 4^{th} signal, because it is slight different from the others regarding the estimate of the first frequency.

3.2 Estimate of damping ratios and mode shapes by using HTM

Damping ratios and mode shapes are finally estimated using the HTM assuming the aeroelastic system properties as those corresponding at the selected time for which the natural frequency estimate has been already carried out using HHTM. In such an operating condition, the system is assumed to be time-invariant and linear behaving. Therefore, the HTM could be applied to analyze the time histories, which are derived from the sum of IMFs, in the time-interval for which the time-invariance is valid. Therefore, the criterion, presented in section 2.3, has been applied in order to split the whole observation time. The maximum values of derivative of time-tracking of the frequency and the corresponding observation time-window for each channel are shown in Tab.2.

N. Channel	$\dot{f}_n$, 1/s^2	Δt, s
1	0.1067	3.06
2	0.1171	2.92
3	0.1172	2.92
4	0.1179	2.91

Table 2: Parameters used to find the correct observation time-window for HTM.

Therefore, the length of the observation time window is a smaller value than those in Tab.2, so as to ensure that the system is time-invariant at all sub-intervals for all the analyzed signals. For this reason, the chosen value for the length of sub-interval is:

$$\Delta t = 2.90\ s \tag{19}$$

Then, the whole observation time has been separated into 22 sub-intervals having 512 samples and for each of them the HTM has been used the estimate both the damping ratios and the mode shapes. It is worth underlining that the time-history used for the estimate of modal parameters is not the original signal, but the sum of IMFs. Specifically, the sum consists of the IMFs which have been chosen to find the trend of natural frequencies. The trend of damping ratios over time, obtained from HTM, is shown in Fig.14. It is possible to note that one estimate of the damping of first mode is not accurate. In fact, in sub-interval I_{10} = [26.1 s - 29 s], the damping ratio has been estimated 1.24 %; instead in this sub-interval the analytical value is around 0.87 %. This result is due to the low accuracy of estimating. For this reasons, the polyfit has been performed without considering this low accurate estimate. As shown in Fig.14, the trend of the polyfit curve shows good correlation with the analytical one.

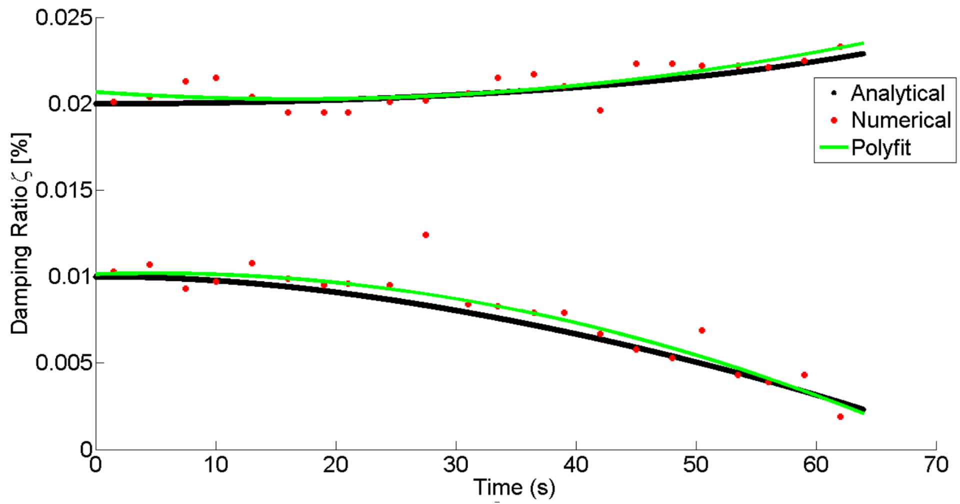

Figure 14: Time-tracking of damping ratio.

In order to evaluate the evolution of natural mode shapes, all the identified modes corresponding to the different sub-intervals of analysis have been compared with the reference mode corresponding to the one at the initial time-interval I_1 = [0 s - 2.9 s], using the Modal Assurance Criterion (MAC). The results of the comparison for the first and the second mode shapes are shown in Figs.15 and 16, respectively.

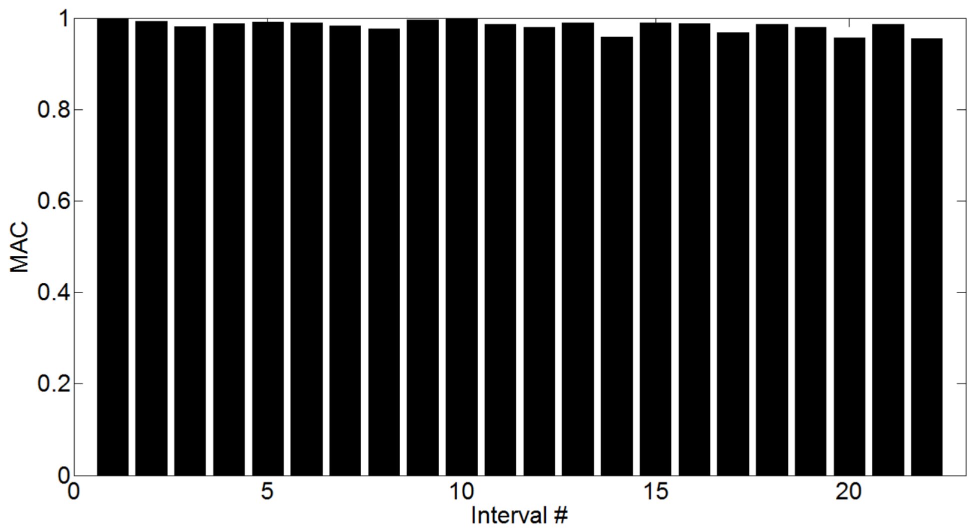

Figure 15: Time-evolution of MAC value for first mode shape with respect to the time-interval 1 one.

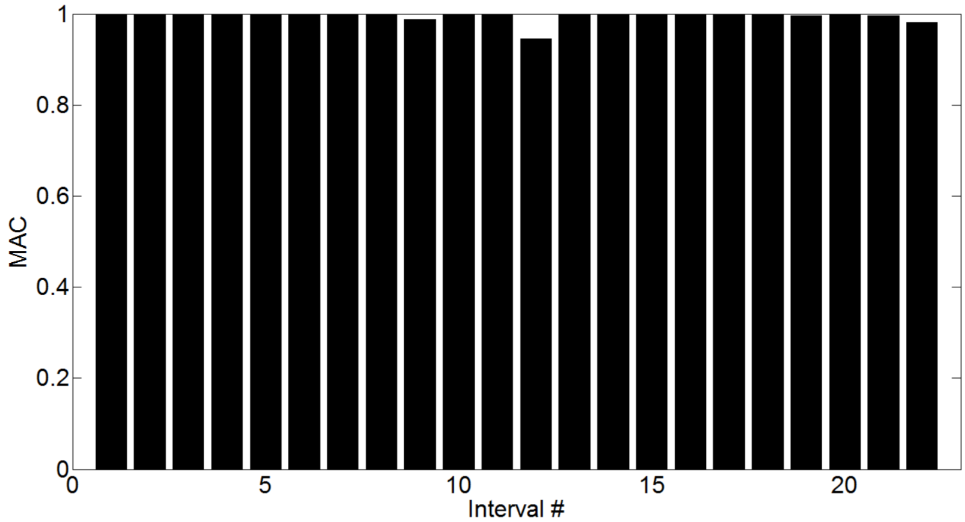

Figure 16: Time-evolution of MAC value for second mode shape with respect to the time-interval 1 one.

It is possible to note that the first mode shape exhibits an evident change than the second one. This is probably due to the intrinsic properties of the analyzed system because a corresponding a large variation of the first natural frequency and damping ratio over time. In fact, the first frequency changes in a range [10.0 Hz - 14.24 Hz] and the damping ratio between [1 % - 0.23 %]. Instead the second frequency and damping ratio change more slowly and respectively in ranges [35.0 Hz - 33.5 Hz] and [2 % - 2.29 %]. Moreover, the number of the used measurement points, i.e. four, is the minimum value in order to identify the second mode.

4 CONCLUSION

A method based on the Hilbert Transform is proposed to deal with time-varying system. The method requires first the use of the HHTM for the time-tracking of the natural frequencies. Then, the HTM is considered for the estimate of the damping ratios and the mode shapes corresponding to the natural frequencies previously identified. This last phase assumes the system is well represented by a linear and time-independent model, at least in the time-intervals, in which the natural frequencies are evaluated. The proposed approach has been validated considering a structural skin of an aircraft wing, under its actual operative conditions. Indeed, an excellent correlation with the analytical results has been achieved. Therefore, the developed method is found to be a promising tool for the system identification using real flight test data.

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