

EFFICIENT AEROELASTIC RESPONSE ANALYSIS INCLUDING GEOMETRIC NONLINEARITIES BASED ON STRUCTURAL ROM

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Abstract: In this paper, efficient reduced order model (ROM) is established for nonlinear aeroelastic response analysis. The method can be used to solve the aeroelastic response problems of wing containing geometric nonlinearities. Traditional methods of aeroelastic analysis can't reflect the nonlinear characteristics of structures. And their results can't satisfy the precision demand of engineering analysis. The approach for structure modeling presented here is based on a combined modal/finite element (MFE) approach that describes the stiffness nonlinearities. We apply that structure modeling method as ROM coupled with nonlinear unsteady vortex lattice method (UVLM) to aeroelastic response analysis. The results show that aeroelastic response analysis of wing based on structure ROM can achieve a good agreement compared to analysis based on the nonlinear finite element method (FEM). The method in this paper is suitable for the preliminary design and the aeroelastic response analysis of the large-aspect-ratio wing efficiently.

1 INTRODUCTION

As the representative of the very flexible airplane, high-altitude long-endurance (HALE) aircrafts usually attract extensive attention. Because of its light weight and large flexibility, the wing of HALE will produce large deformation during the flight. Meanwhile, significant elastic deformation of wing will bring significant changes in aerodynamics configuration and stiffness character, which makes the aeroelastic problem of large flexible aircraft. Geometric nonlinearity becomes a very important factor that affects aeroelastic stability and response. For the design requirement of HALE, Hodges, Cesnik and Patil proposed the concept of fixed wing aircraft geometric nonlinear aeroelastic problem in 1999^[1-2]. Then lots of research with diverse content of geometric nonlinear aeroelastic related to large flexible HALE has been carried out^[3-8]. The coupled effects between large flexibility and aeroelastic must be properly accounted for in a nonlinear aeroelastic framework. Recently, C.Howcroft et al. discussed five aeroelastic modeling methods applied to the aeroelastic analysis including geometric nonlinearities in recent research. Predictions of static aeroelastic equilibria from five modeling method were compared. Discussions of aerodynamic modeling choices, orientation of aero forces and drag effect they made in the paper have important influence in related research^[9].

Gust response analysis is a serious problem for aircraft especially for very flexible aircraft whose large deformation may significantly change structural dynamic characteristics and aerodynamic features. Su and Cesnik studied the dynamic response of a highly flexible wing under a spatially-distributed discrete gust model in the time domain^[10]. Guo et al. studied the nonlinear gust response of free flexible aircraft using a CFD/CSD method^[11]. Liu et al. established a theoretical geometrically nonlinear aeroelastic analysis framework and validated results with wind tunnel tests^[12]. Bi et al. solve flexible wing response solution coupled nonlinear finite element method with double lattice method and complete gust load alleviation with piezoelectric patches^[13].

Nonlinear finite element method (FEM) is used to calculate the stiffness of model and displacement under aerodynamics in geometric nonlinear aeroelastic analysis usually. There should be large amount of freedom in solving of aeroelastic problem including geometrical nonlinearity. Compared to FEM, reduced order model (ROM) can reduce the scale of the problem and can analyze the characteristics of large flexible aircraft geometric nonlinearity easily. It shows us computational inexpensive mathematical representation of structure analysis in nonlinear aeroelastic problem and offers the potential for real-time domain analysis. Demasi et al. reconducted function of load step with structural tangent modes via procedure of Proper Orthogonal Decomposition (POD) to reduce freedom of structure with planar and non-planar structural configurations^[14]. Worth focused on, there have been some approaches investigated by utilizing commercial FEM software package to obtain structure ROMs. McEwan et al. performed the modal/FE method (MFE) by static analysis with numbers of specified static load cases^[15]. Cooper et al. implied the MFE approach for modeling the geometric nonlinearity of a large-aspect-ratio wing model^[16]. An et al. improved MFE in geometric nonlinear static aeroelastic problem so that follower force effect and spanwise deformation can be considered to get a more exact solution^[17].

Research in this paper aims to solve gust response aeroelastic problem including geometric nonlinearities based on MFE improved as ROM. It is more fully reflect the influence of large deformation of wing under load in the analysis of aeroelastic. The prescribed load cases with follower force and corresponding displacements obtained from the nonlinear FEM static analysis are transformed into modal coordinates by using the modal transformation of underlying linear system where more of useful modes have been applied to participate in modelling. We use a regression analysis to curve fit the sets of test load and nonlinear displacement maps for the sake of finding the unknown nonlinear stiffness coefficient. Aeroelastic response computation based on nonlinear ROM is combined with non-planar unsteady vortex lattice method (UVLM) marched in the time domain to describe the real physical response of a flexible wing under gust load. Surface spline interpolations approach is performed for structure/aerodynamics coupling. Method presented describes the nonlinear stiffening effects, and achieves a good agreement compared to the approach of aeroelastic response analysis based on the FEM.

2 FORMULATION

2.1 Nonlinear ROM

2.1.1 Nonlinear structure equation

Consider the case of an initially straight, geometrically nonlinear beam subject to forced vibration. Structural equation of motion in physical co-ordinates for forced vibration in the transverse direction is of the form:

$$[\mathbf{M}]\{\ddot{\mathbf{w}}(x,t)\} + \alpha[\mathbf{M}]\{\dot{\mathbf{w}}(x,t)\} + [\mathbf{K}_L]\{\mathbf{w}(x,t)\} + [\mathbf{K}_{NL}(\mathbf{w}(x,t))]\{\mathbf{w}(x,t)\} = \{\mathbf{F}(x,t)\} \quad (1)$$

Here $\{\mathbf{w}(x,t)\}$ is the transverse deflection vector, $[\mathbf{M}]$ is the mass matrix, α is the coefficient for damping, $[\mathbf{K}_L]$ is the linear stiffness matrix, $[\mathbf{K}_{NL}] = [\mathbf{K}_{NL}(\mathbf{w}(x,t))]$ is the assembled nonlinear stiffness matrix where the stiffness is dependent upon displacement, and $\{\mathbf{F}(x,t)\}$ is the external force vector.

The spatial and temporal components of the beam motion can be separated by expressing the equations of the beam motion can be separated by expressing the equations of motion in terms of modal amplitudes as:

$$\{\mathbf{w}(x,t)\} = [\phi(x)]\{\mathbf{p}(t)\} \quad (2)$$

Here $\{\mathbf{p}(t)\}$ is a time-dependent vector of modal amplitudes, and $[\phi]$ is a time-independent modal matrix of the N underlying linear mode $\{\phi(x)\}_r, r = 1, 2, \dots, N$, which may be obtained by solving the eigenvalue problem for free vibration:

$$[\mathbf{K}_L]\{\phi(x)\}_r = w_{Lr}^2[\mathbf{M}]\{\phi(x)\}_r, r = 1, 2, \dots, N \quad (3)$$

Here w_{Lr} is the linear natural frequency of r mode. Substituting the truncated modal expansion into the system equations of motion and pre-multiplying by $[\phi]^T$, then we can get nonlinear structural equation in modal co-ordinates:

$$[\phi]^T[\mathbf{M}][\phi]\{\ddot{\mathbf{p}}(t)\} + \alpha[\phi]^T[\mathbf{M}][\phi]\{\dot{\mathbf{p}}(t)\} + [\phi]^T[\mathbf{K}_L][\phi]\{\mathbf{p}(t)\} + [\phi]^T[\mathbf{K}_{NL}(\mathbf{p}(t))][\phi] = [\phi]^T\{\mathbf{F}(t)\} \quad (4)$$

Upon completion of the modal transformation the new system equations of motion in modal space are:

$$[\mathbf{A}]\{\ddot{\mathbf{p}}(t)\} + \alpha[\mathbf{A}]\{\dot{\mathbf{p}}(t)\} + [\mathbf{E}_L]\{\mathbf{p}(t)\} + [\mathbf{E}_{NL}(\mathbf{p}(t))]\{\mathbf{p}(t)\} = \{\mathbf{f}(t)\} \quad (5)$$

Here $[\mathbf{A}]$ is the modal mass matrix, $[\mathbf{E}_L]$ is the linear modal stiffness matrix and $\{\mathbf{f}(t)\}$ is the modal force vector. It should be noted that all of the matrices in the modal equation of motion are now diagonal apart from the non-linear stiffness matrix $[\mathbf{E}_{NL}]$, which may contain cross-coupling terms and will be a function of $\{\mathbf{p}(t)\}$.

2.1.2 Regression analysis for ROM

Backwards to equation (5), modal mass matrix $[\mathbf{A}]$ and linear modal stiffness matrix $[\mathbf{E}_L]$ can be obtained from linear modal analysis of model, damping coefficient α , modal force vector $\{\mathbf{f}(t)\}$ is known already. Only nonlinear modal stiffness matrix $[\mathbf{E}_{NL}]$ including nonlinear stiffness coefficient is unknown.

Considering the beam in a static sense only, with velocity and acceleration terms set to zero, and all of the geometric and material properties being time invariant, then equation (5) can be simplified as:

$$[\mathbf{E}_L]\mathbf{p} + [\mathbf{E}_{NL}(\mathbf{p})] = \{\mathbf{f}(t)\} \quad (6)$$

The left-hand side of equation (6) can be regarded as a stiffness restoring force, with a linear and a non-linear component. The right-hand side of equation (6) may be regarded as a statically applied load. It follows that if there has a set of applied static loads, and corresponding displacements, then the unknown stiffness coefficients which relate the applied

load to the resultant displacement may be determined using regression analysis. The set of applied loads and corresponding displacements are denoted as “static test case”, and can be solved for using MSC.Nastran finite element software package.

An ordinary polynomial approach has been used to curve fit the nonlinear force and corresponding displacement relationship. The polynomial expression of the nonlinear restoring forces, same as nonlinear stiffness coefficient is derived as the following series of up to third order:

$$\mathbf{E}_{NL}(p_r, p_s, p_t)_r = \sum_{j=0} \sum_{k=0} \sum_{l=0}^{j+k+l=3} A_r p_r^j p_s^k p_t^l, (j \times k \times l = 0) \quad (7)$$

Here, $\mathbf{E}_{NL}(p_r, p_s, p_t)_r$ is the formulation of nonlinear stiffness coefficient for r mode, $\mathbf{A}_r(r, i)$ is the i th element in nonlinear structure equation corresponding to r mode, same as the nonlinear stiffness coefficient of modal polynomial combined main modal co-ordinate p_r with nonlinear stiffness coefficient under s -th and t -th modal co-ordinate $p_r^j p_s^k p_t^l$ in nonlinear equation. Nonlinear stiffness coefficient matrix \mathbf{A}_{NL} is what we want to obtained by regression analysis.

Consider that there are NT sets of static test load cases, complete static FEM analysis of NT sets of test loads on the model in commercial software package MSC.Nastran. Then we can get NT sets of corresponding displacement. Translate the loads and displacement to modal space. The nonlinear restoring force for each of the test cases can now be fitted to find the unknown nonlinear nodal stiffness coefficients in a least squares sense. The nonlinear restoring forces for a certain mode r can now be shown in matrix form to be:

$$\begin{Bmatrix} f_{r(1)} - E_{L_r} p_{r(1)} \\ f_{r(2)} - E_{L_r} p_{r(2)} \\ \vdots \\ f_{r(NT)} - E_{L_r} p_{r(NT)} \end{Bmatrix} = \begin{Bmatrix} p_{r(1)}^2 p_{1(1)} & p_{r(1)}^2 p_{2(1)} & \cdots & p_{r(1)}^2 p_{NA(1)} & \cdots \\ p_{r(2)}^2 p_{1(2)} & p_{r(2)}^2 p_{2(2)} & \cdots & p_{r(2)}^2 p_{NA(2)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{r(NT)}^2 p_{1(NT)} & p_{r(NT)}^2 p_{2(NT)} & \cdots & p_{r(NT)}^2 p_{NA(NT)} & \cdots \end{Bmatrix} \begin{Bmatrix} A_{NL}(r, 1) \\ A_{NL}(r, 2) \\ \vdots \\ A_{NL}(r, NA) \\ \vdots \end{Bmatrix} \quad (8)$$

Problem has been translated to standard least square problem, complete regression analysis to each order modal, we can get all of the nonlinear stiffness coefficient. Simplify equation (8), regression problem can be present in matrix short notation as:

$$\{\mathbf{q}\}_r \approx \{\hat{\mathbf{q}}\}_r = [\mathbf{D}]_r \{\mathbf{A}_{NL}\}_r \quad (9)$$

Here $\{\mathbf{q}\}_r$ is an $NT \times 1$ vector containing the mode r nonlinear stiffness restoring force for each of the data sets, $\{\mathbf{A}_{NL}\}_r$ is an $NA \times 1$ vector containing all of the unknown stiffness coefficients for each mode r . The matrix $[\mathbf{D}]_r$ is an $NT \times NA$ matrix, known as the design matrix as the fitting problem for each mode r . Nonlinear stiffness coefficients matrix $\{\mathbf{A}_{NL}\}_r$ can be obtained by solving regression formulation (8)(9).

2.1.3 Strategy for generating test load cases

Through the analysis presented before, the regression analysis is provided by the actual deformation and load testing after FEM analysis by commercial software package, so the accuracy of nonlinear stiffness coefficient directly depends on the rationality of selection of

the test load case, which is related to the success of recovery of nonlinear structural equation. Selection of test load cases needs to meet the following conditions:

- (a).The selected cases must be able to reflect the linear and nonlinear factors of the structure
- (b).The selected cases must meet the characteristics of aerodynamics in aeroelastic analysis
- (c).The selected cases must be reasonable and interested in our research
- (d).The selected cases must the requirements of nonlinear FEM calculation cost and complexity, the account of cases should be as fewer as possible.

It should be noted that, in condition (d), aerodynamic force on the wing is follower force, which is more fit the actual characteristic of aerodynamic force. That is to say, take oriented load as the load test cases can't meet the requirement. In this paper, aerodynamics force under the wing's deformation combined bending modes and torsion modes is chosen as test load case, meanwhile regard bending modes and torsion modes as normal modes to realize analysis of approximate follower force load. The formulation of wing's deformation which makes aerodynamics forces should be:

$$f_{AIR}\{\mathbf{w}\} = f_{AIR}\left(\sum a_i\{\phi_i\}_{bend} + \sum a_j\{\phi_j\}_{torsion}\right) \quad (10)$$

Here $\{\phi_i\}_{bend}$ and $\{\phi_i\}_{torsion}$ are bending modes and torsion modes interested, $a_{i,j}$ are scalar modes weight factors, which can make the selected test cases reflect the nonlinear factors of the structure and interested in our research. It should be noted that deformation combined with linear modes can't reflect spanwise deformation, which is not fit actual condition and can't meet requirement of nonlinear analysis. This paper assume the beam can't be extended and shortened, solve the real spanwise deformation with geometric relationship after model wing's flapwise deformation and edgewise deformation under linear modes combination is obtained, then, make test load cases under the corrective nonlinear deformation. The correction presented above can make the test load cases fit actual condition and meet the requirement of nonlinear analysis.

2.2 Non-planar unsteady vortex lattice method ^[12]

The unsteady vortex lattice method, as a time-domain aerodynamic computation, is efficient method to calculate aerodynamic loads for aircraft. It can be easily combined with structural dynamic computation such as ROM to get response results for aeroelastic system. Additionally, the exact boundary condition is satisfied on the actual wing surface, which can be conveniently used for very flexible wings whose aerodynamic surfaces are subjected to large spatial deformations ^[18]. The UVLM is based on full potential equations without any linearization and can well reflect the unsteadiness effects of the 3-D low-speed flow around a flexible lifting surface.

Vortex ring elements are used to discrete the boundaries of the aerodynamic domain in the UVLM, for both the wing and the wake in Figure.1. Leading segment of the vortex ring is placed on the panel's quarter-chord line and collocation point is located at the center of the three-quarter chord line. The whole flow domain is represented by vortex rings and the aerodynamic influence coefficient can be obtained via Biot-Savart law.

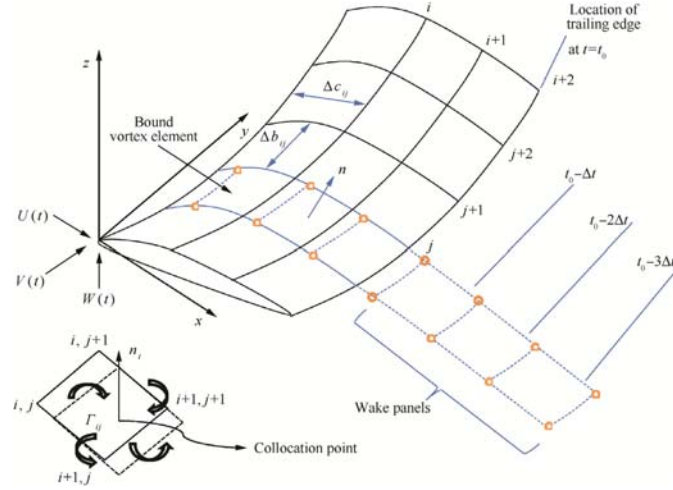


Figure 1: Non-planar unsteady vortex lattice method model

In each discrete time step computation, the wing is moved along its flight path and each trailing edge vortex panel sheds a wake panel with the vortex strength equal to its circulation in the previous time step [19]. Since the vortex wake is for-free, each vortex must move with the local stream velocity. A free wake model and a fixed wake model are two wake models in UVLM. For a single wing considered in the paper, a fixed wake model is used in which the wake panels follow the motion of the trailing edge and move with the local flow velocity ignoring the influence of induced velocities on the wake by the bound vortex and the wake vortex. According to the comparison of these two models, the aerodynamic loads are well consistent with each other. For a gust response problem, gust load is the main factor that matters for structural response in an aerodynamic way, so the fixed model is good enough for the single flexible wing response problem and it brings tremendous computation efficiency compared with the free wake model.

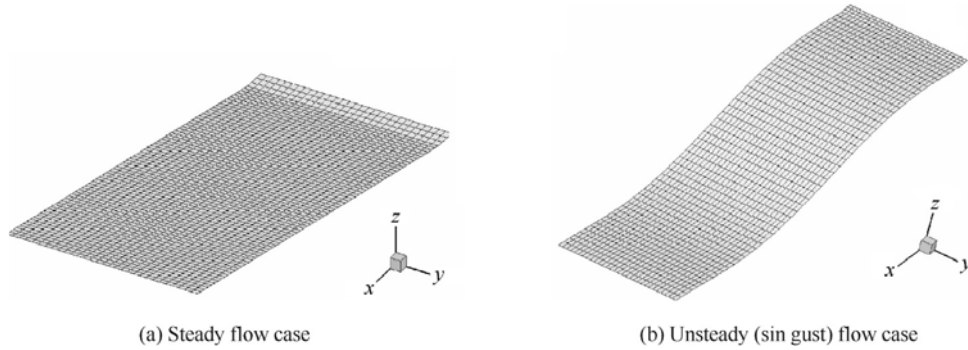


Figure 2: Fixed wake model

2.3 Surface spline interpolation

The surface spline is used for the coupling of aerodynamics and structure. The configuration of structure is usually considered to be embedded in a 3-D space. The undeformed configuration could be 1-D, 2-D, or 3-D, and the deformed configuration is usually 3-D.

Consider n given structural grids and the corresponding deformation vector U_S , then the deformation vector U_A of m aerodynamic grids could be interpolated [20].

$$U_A = GU_S \quad (11)$$

Here G is the spline matrix for displacement interpolation between the aerodynamics grids and structure grids. In aeroelastic analysis, the transformation between the aerodynamics and the structural force systems requires structure equivalence rather than static equivalence. Structure equivalence means that the two force systems deflect the structure equally. When the aerodynamics forces F_A and their equivalent structure forces F_S do the same virtual work on virtual deflections, the structure equivalence of the two force system is guaranteed:

$$\delta U_A^T F_A = \delta U_S^T F_S \quad (12)$$

Here δU_A and δU_S^T are the arbitrary virtual deflections, satisfying equation (11). So:

$$F_S = G^T F_A \quad (13)$$

2.4 Gust response analysis methodology

Gust response analysis is implemented in the discrete time domain. At the beginning of each time step computation, the unsteady aerodynamic load is computed by non-planar unsteady vortex lattice method. The structural displacement and velocity at the end of last computation step will be treated as the initial condition in the next structural transient dynamic analysis, which can guarantee the continuity of structural response analysis. Each structural transient dynamic analysis is carried forward for a time step, during which the unsteady load is kept unchanged. Structural transient dynamic analysis is calculated by Nonlinear ROM described above. This will make sense if the time step is small. The resultant structural displacement and velocity are used to update the aerodynamic surfaces and exact geometric boundary conditions for next step aerodynamic computation. Although the computation is implemented in the discrete time domain, the structural displacement and velocity are continuous and the updated aerodynamic computation helps the unsteady aerodynamic computation accurate and practical. The legible analysis procedure is shown in Figure.3.

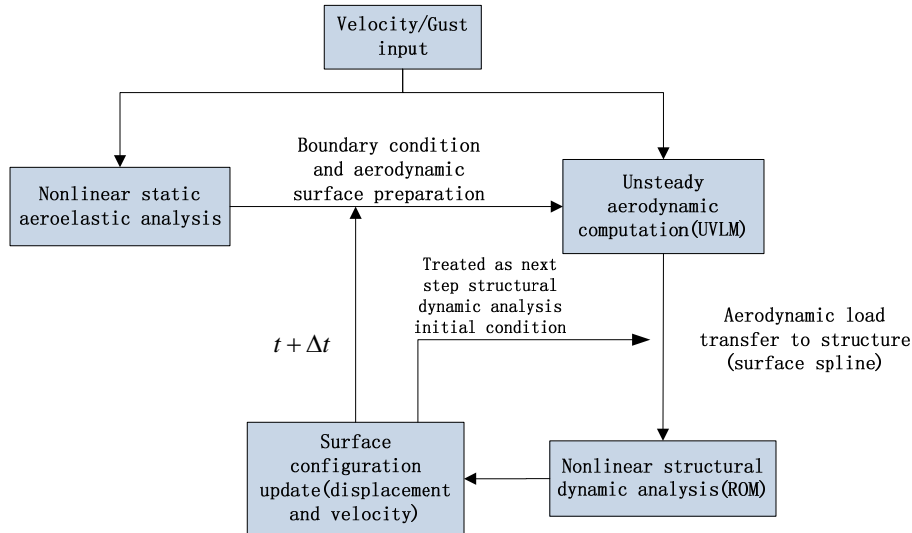


Figure 3 Time-domain nonlinear gust response analysis flow chart

Nonlinear dynamic solution would be solved by equation (14):

$$[\mathbf{A}]\{\ddot{\mathbf{p}}(t)\} + [\mathbf{E}_L]\{\dot{\mathbf{p}}(t)\} + [\mathbf{E}_{NL}(\mathbf{p}(t))]\{\mathbf{p}(t)\} = \{\mathbf{f}_q(t)\} + \{\mathbf{f}_g(t)\} \quad (14)$$

$\{\mathbf{f}_q(t)\}$ and $\{\mathbf{f}_g(t)\}$ are the generalized unsteady aerodynamic loads and the atmosphere gust. It has to be noted that, before the gust response analysis, it is necessary to investigate the static

aeroelastic characteristics. Large static structure deformation may change aerodynamic loads and structural dynamic features, which may have big influence on gust response results. The legible analysis procedure is shown in Figure.4.

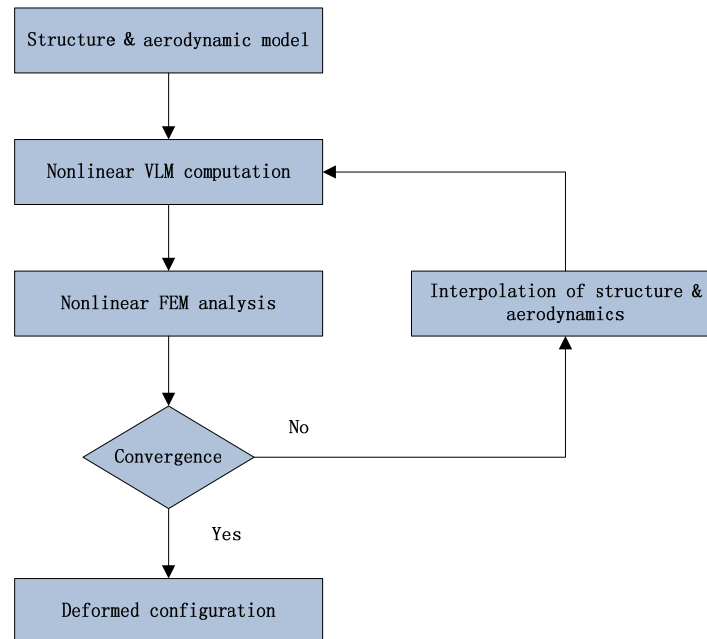


Figure 4: nonlinear static aeroelastic analysis flow chart

3 NUMERICAL EXAMPLE

3.1 Wing model

A typical very flexible wing, which large deformation can be guaranteed has been conducted. The wing model has large-aspect-ratio, the design parameters are shown in Table1. It is used to validated the accuracy of ROM and solve aeroelastic response problem considering geometric nonlinearity.

Item	Value
Span(mm)	1000
Chord(mm)	100
Airfoil of the wing	NACA0015
Beam location	50% of local chord
Material of beam	Spring steel
Weight of the structure(kg)	1.958
Elastic modulus of beam	219Gpa
Section of beam	35mm × 1.5mm rectangular

Table 1: Design parameters of wing model

Beam of wing is a steel ruler, which was selected to provide the main bend and twist stiffness and has large flexibility in flapwise. The wing shape is simulated by wing sections made from balsa wood and coton paper. Each section attaches to the wing beam at a single point. 2mm

space between adjacent sections was for eliminating the additional stiffness effect since the theoretical structural dynamic analysis only concerned about the beam effect. There is a wingtip store to regulate the flutter characteristics. The wingtip store is 200mm long and weight 63.5g. An aeroelastic analysis model of the wing is established, the 3-D CATIA model and the structure FEM shown in Figure5 which uses beam elements and lumped mass elements for the stiffness and mass simulation.

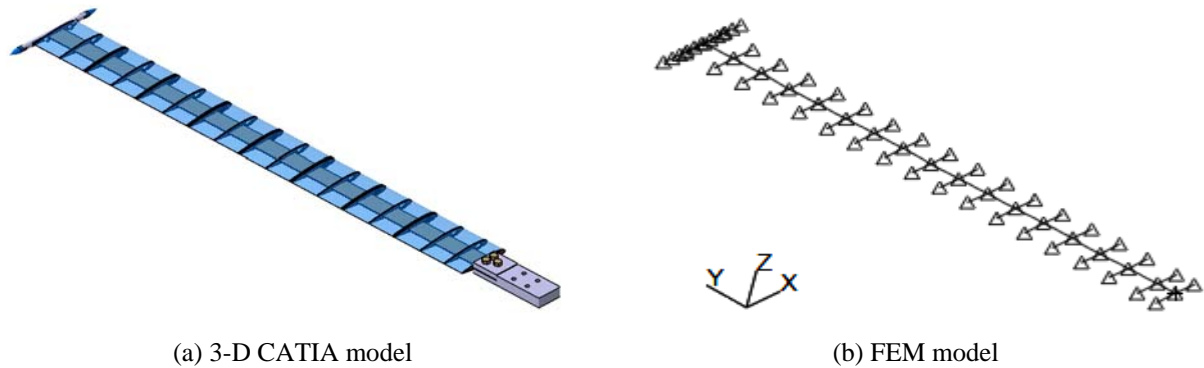


Figure 5: CATIA and FEM model

Nonlinear structural ROM would be described in modal space. Main modes of wing model have been presented in Table.2. It can be seen that, frequencies of first two modes are very low, which means the flexibility of model is very high.

Mode	Description	Frequency/Hz	Modal stiffness	Frequency in GVT/Hz
1	1 st bending	1.141	5.134E+01	1.14
2	2 nd bending	7.474	2.203E+03	7.58
3	1 st torsion	22.92	2.072E+04	22.17

Table 2: Main modes of wing model

We chose seven modes to participate established ROM and two orders of telescopic mode are used for recovery of spanwise deformation. Telescopic modes are solved by analytical method.

3.2 Validation of nonlinear structural ROM

The accuracy of nonlinear ROM can be validated by comparing results of ROM and nonlinear FEM after applying the validating load on the wing model.

Nonlinear ROM has been obtained by regression analysis from test load cases chosen before and corresponding deformations. The nonlinear ROM calculation from structural equation must fit all of the test load cases and corresponding deformation. More important, nonlinear structure equation should fit any other aerodynamic load and their deformation. Then the ROM can be applied to structural and aeroelastic response analysis reasonably. Validation procedure is illustrated in Figure.6

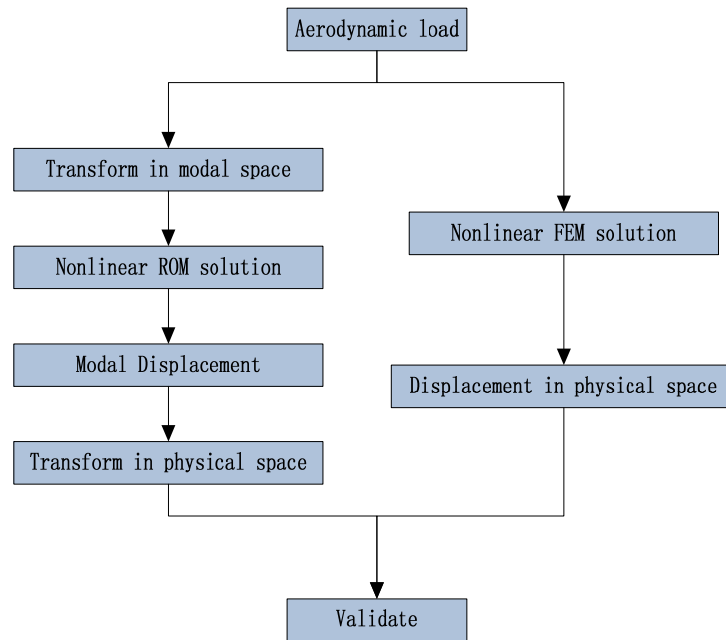


Figure 6: Validation Procedure

Take 50 sets of growing validation load as examples which all have distribution of actual aerodynamics load. Displacement of wing tip under these sets of validation load has been presented in Figure.7. The choice of validation loads makes the displacement of wing tip within the range of 10% to 25% which is in the demand of nonlinear analysis. Spanwise displacement of wing tip is also considered. It is important in geometric nonlinear aeroelastic analysis which will change aerodynamic distribution.

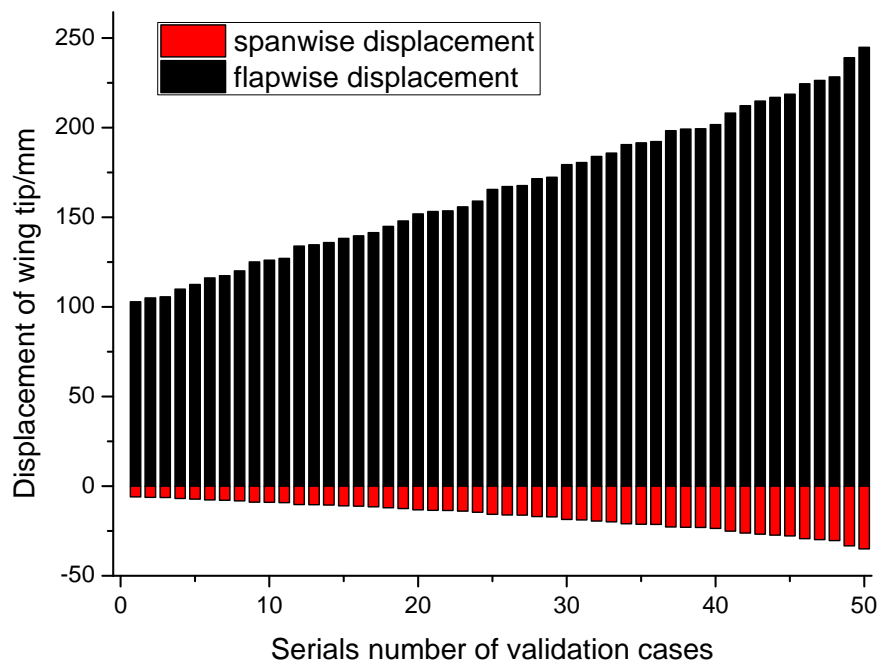


Figure 7: Displacement of wing tip under validation loads

Calculation results of those 50 cases are shown in Figure.8 with comparing of ROM solutions and FEM solutions. Here black line with circle represent relative deviation of wing tip flapwise displacement and red line with rectangle represent relative deviation of wing tip spanwise displacement.

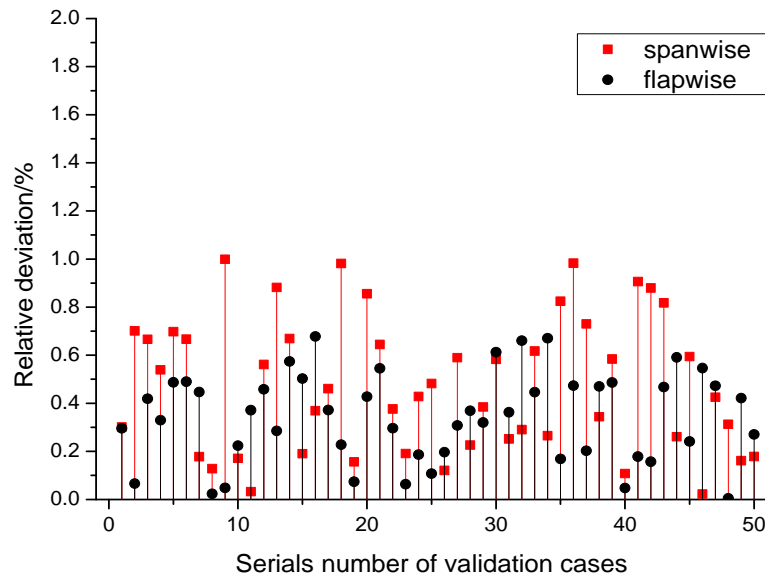


Figure 8 : Relative deviation under validation loads

It can be seen that relative deviation between ROM solutions and nonlinear FEM solutions under these validation loads are no greater than 1%. ROM solutions can meet a great agreement to nonlinear FEM solutions. The structure ROM established is reliable.

Worth noting that, overall deviation level of flapwise displacement is lower than spanwise displacement. Flapwise deformations were larger than spanwise deformation in regression analysis and the former is closer to the nonlinear analysis range. The rationality of recovery spanwise deformation with telescopic modes in ROM is also in consideration. Those may be the reason of flapwise displacement is more accuracy in ROM solutions.

3.3 Static aeroelastic analysis

Before gust response analysis, static aeroelastic analysis is implemented. Large static structure deformation may change aerodynamic loads and structural dynamic features, which may have big influence on gust response results.

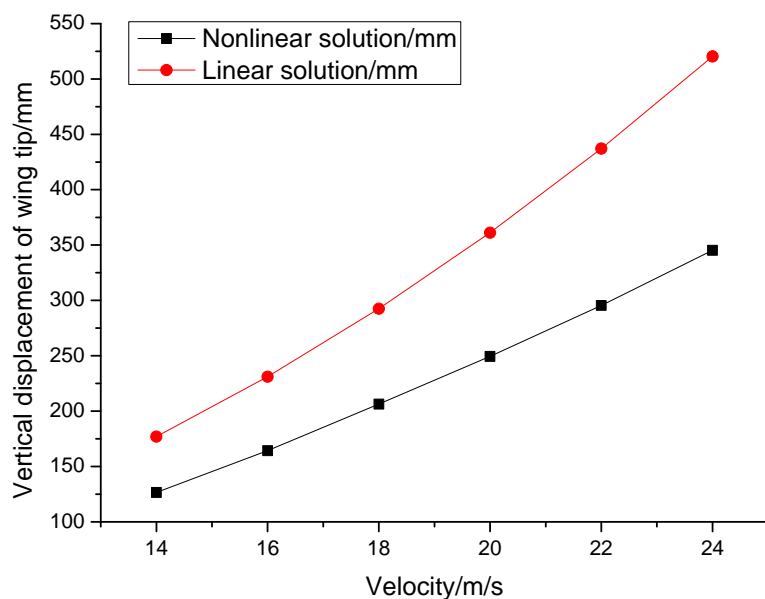


Figure 9 : Static aeroelastic analysis

The wing is fixed under 3° angle of attack and velocity of airflow was in range of 14m/s to 24m/s. Vertical displacement of wing tip is shown in Figure.9 and large deflection is produced under that condition. It should be noted that tip displacement of wing in linear solution is 520.37mm when velocity of airflow is 24m/s, large than semi-span of the wing, which is apparently wrong. In contrast, tip displacement in nonlinear solution is 345.23mm. Huge difference between linear and nonlinear solution has been illustrated in large deformation state.

3.4 Gust response analysis

In this paper, both the nonlinear gust response analysis based on ROM and nonlinear gust response analysis based on FEM are presented and compared to illustrate the consistency.

Nonlinear gust response analysis based on FEM has the same procedure with analysis based on ROM. Nonlinear FEM is coupled with UVLM, and surface spline is used for the coupling of aerodynamics and structure. Similar procedure can be seen in Figure3.

A numerical example is chosen to illustrate the accuracy of nonlinear aeroelastic response analysis based on ROM established in this paper. The wing is fixed under 3° angle of attack and the continuous sin gust. Frequency of gust is 4.0Hz. Velocities of airflow are 14m/s, 16m/s and 18m/s. Results between FEM and ROM solutions has been shown in Figure.10.

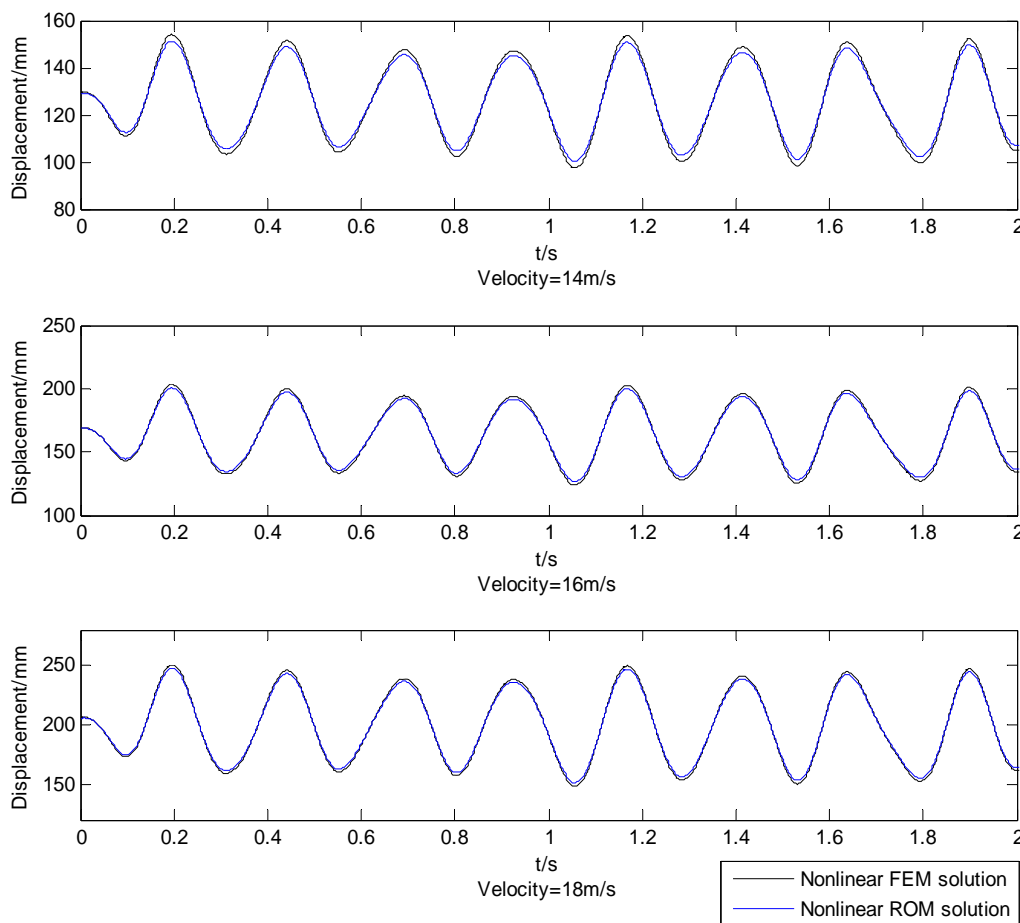


Figure 10: Nonlinear gust response analysis results between FEM and ROM solutions

It can be seen that end of wing is affected heavily by downwash airflow. Although the modal frequency changed under different wind speed cases, the increased wind speed and the resulting increased aerodynamic load became the main reasons that made the wing tip deflection increased along with the wind speed. The gust response analysis solutions based on nonlinear ROM has a good agreement to the gust response analysis solutions based on nonlinear FEM, which prove the process established is accuracy and reasonable.

4 CONCLUSIONS

A method for the nonlinear aeroelastic response analysis of wing has been presented in this paper. Main work in this paper is to couple the improved modal/FE method as structure ROM with nonlinear UVLM for applying in nonlinear aeroelastic response analysis. Once the procedure is established, gust response time-domain solution can be implemented.

The ROM presented here innovatively use seven modes to recovery the large deflection of wing to reflect the characteristic of wing's large deflection, that is in order to identify the nonlinear stiffness coefficients more exactly also. Meanwhile, the method set the aerodynamic follower forces under certain deformation as test load cases in ROM which made analysis close to more real flight condition and makes result more reasonable. At last, aerodynamic influence coefficient matrix is changing for considering the changing of wing's deformation instead of a constant value based on the initial deformation. Though that makes solution more complexity, the nature of aerodynamics under the large deformation has been more considered. It can be seen from the calculated result of the aeroelastic response analysis of wing based on ROM can achieve a good agreement to analysis based on FEM. It is valuable to theoretical analysis and engineering application in aeroelastic with geometric nonlinearities.

Research is continuing to apply the method to flight dynamic aeroelastic analysis of flexible aircraft and coupling with control characteristics of system.

5 REFERENCES

- [1] Patil M.J, Hodges D.H, On the Importance of Aerodynamics and Structural Geometrical Nonlinearities in Aeroelastic Behavior of High-Aspect-Ratio Wings. 41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference and Exhibit, Atlanta, GA, April 3-6, 2000.
- [2] Patil M.J, Nonlinear Aeroelastic Analysis, Flight Dynamics, and Control of a Complete Aircraft [dissertation]. Atlanta, GA: Georgia Institute of Technology, 1999.
- [3] Darecki M. et al, Flightpath 2050: Europe's Vision for Aviation Maintaining Global Leadership & Serving Society's Needs, Publications Office of the European Union, 2011, Luxembourg, Belgium
- [4] Cesnik C.E.S, Hodges D.H, Patil M.J, Aeroelastic Analysis of Composite Wings. 37th Structures, Structural Dynamics and Materials Conference, Salt Lake City, Utah, April 15-17, 1996
- [5] Patil M.J, Hodges D.H, Limit Cycle Oscillations in High-Aspect-Ratio Wings. 1999, AIAA-99-1464
- [6] Xie C.C, Static/dynamics coupling theory and test study of aircraft aeroelastic stability. Beijing: Beihang University, 2009 [dissertation]. [Chinese]

- [7] Tang D.M, Dowell E.H, Effects of geometric structural nonlinearity on flutter and limit cycle oscillations of high-aspect-ratio wings. *Journal of Fluids and Structures*. 2004, 19: 291-306
- [8] Yang C, Wang L.B, Xie C.C, Liu Y, Aeroelastic trim and flight loads analysis of flexible aircraft with large deformations. *SCIENCE China* 2012: 55(10): 2700-2711
- [9] Howcroft C, Cook R, Calderon D, Lambert L, Castellani M, Cooper J.E, Lowenberg M.H, Neild M.A, and Coetzee E, Aeroelastic Modeling of Highly Flexible Wings. 15th Dynamics Specialist Conference, San Diego California, January 4-8, 2016. AIAA 2016-1798
- [10] Su W, Cesnik C.E.S, Dynamic response of highly flexible flying wings, Proceedings of the 47th AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics and materials conference. Reston: AIAA, 2006: 412-435
- [11] Guo D, Xu M, Chen S.L, Nonlinear gust response analysis of free flexible aircraft. *Int J Intell Syst App (IJISA)* 2013, 5(2): 1-15
- [12] Liu Y, Xie C.C, Yang C, Cheng J.L, Gust response analysis and wind tunnel test for a high-aspect ratio wing. *Chinese Journal of Aeronautics*, 2016, 29(1): 91-103
- [13] Bi Y, Xie C.C, An C, Yang C, Gust load alleviation wind tunnel tests of a large-aspect-ratio flexible wing with piezoelectric control. *Chinese Journal of Aeronautics*, 2017, 30(1): 292-309
- [14] Demasi L, Palacios A, A Reduced Order Nonlinear Aeroelastic Analysis of Joined Wings Based on the Proper Orthogonal Decomposition, 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference, Orlando Florida April 12-15, 2010. AIAA 2010-2722.
- [15] McEwan M.I, Wright J.R, Cooper J.E, Leung A.Y.T, A combined modal/finite element analysis technique for the dynamic response of a nonlinear beam to harmonic excitation. *Journal of Sound and Vibration*, 2001, 243(4): 601-624.
- [16] Harmin M.Y, Cooper J.E, Aeroelastic behavior of a wing including geometric nonlinearities. *The Aeronautical Journal*, 2011, 115(1174):767-777.
- [17] Xie C.C, An C, Liu Y, Yang C, Static aeroelastic analysis including geometric nonlinearities based on reduced order model. *Chinese Journal of Aeronautics*, 2017, 30(2): 638-650
- [18] Hense H, Consistent aeroelastic linearization and reduced-order modeling in the dynamics of maneuvering flexible aircraft [dissertation]. London: Imperial College London, 2013
- [19] Katz J, Plotkin A. *Low-speed aerodynamics*. New York: Cambridge University Press, 2011.
- [20] Wang L.B, Xie C.C, Yang C. Static Aeroelastic Analysis of Flexible Aircraft with Large Deformations. 54th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference, Boston Massachusetts, April 8-11, 2013. AIAA 2013-1893

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