CROSS-MODAL DAMPING MODEL: AN EXPERIMENTAL EXTRACTION APPROACH AND AIRCRAFT DYNAMIC LOADS APPLICATION

Oscar Castro Alvarez[1] and John A. Coote[1]

[1] Loads and Aeroelastics Engineer at Airbus
Pº John Lennon s/n Getafe 28906
oscar.castro-alvarez@airbus.com

[2] Consultant Loads & Dynamics Engineer
110 Longmead Ave, Bristol, BS78QQ
john.coote@blueyonder.co.uk

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Abstract: This paper reviews several possibilities for improving the standard use of experimental data to model damping by the aircraft industry. Starting from the simplistic proportional damping assumption, more sophisticated models and methods are suggested to extract damping data from ground vibration tests (GVT) and update the damping model.

At aircraft level, experimental data is processed to identify off-diagonal elements of the modal damping matrix. The potential benefits of this approach are put to test with two dynamic analyses.

At component level, a dedicated ground vibration test was conducted on an aircraft elevator to explore different techniques of damping modelling, using experimental data to extract information and benchmark numerical results. The merits and difficulties of different methodologies are discussed, opening up possibilities for further investigation.

1 INTRODUCTION

1.1 Scope

The current aircraft industry standard for dynamic analyses generally accounts for damping with a simplistic modal approach: each normal mode has a single value to account for all the energy dissipated across the whole structure at each modal frequency. These modal damping terms, elements of the diagonal damping matrix C, can be either obtained from GVT (and extrapolated to other mass cases), or simply conservatively estimated depending on the type of studies being addressed.

This formulation is, to some extent, an expansion of the concept of proportional or, also called, Rayleigh's damping, and it can be traced back to the seminal work on modal analysis and damping [1]. Rayleigh's classical assumption forces the damping matrix to be a linear combination of mass and stiffness matrices, greatly simplifying the dynamics formulation by uncoupling the modal equations. A formal elaboration on the generalized proportional damping theory can be found in [2].

Apart from the global damping characteristics, in general, there are very limited dedicated dampers locally modelled (elastomers on engines or APU) and usually considered only for very specific analyses. Despite its many advantages, this modal approach also comes with a price:

- Lack of spatial damping distribution: modal approach does not account for local effects in most of the cases.
- Cross-coupling modal damping terms: intermodal damping coefficients are missing, giving up benefits in loads alleviations.
- Frequency and non-linear behaviors: not capturing effects as the well documented increment of damping with high amplitude, only used in industry as a last resort in very specific cases.

All these previous effects, and others like friction, are still to be included in the current damping model for standard aircraft dynamic analyses. Moreover, the new materials already arriving to A/C production are adding improved performance with less weight and new capabilities and properties. The tools and methodologies for their simulation in order to fully benefit from them are still to be implemented in industry practices.

An additional application of a robust spatial identification of damping is the so-called vibration-based health monitoring technique. The energy dissipation usually associated with local damage in the structure, could lead to its detection by identifying changes in the damping distribution.

The work here-in presented aims at improving the standard damping modelling by exploring different possibilities, based on experimental data, and applying it to industrial aircraft analyses:

1. Improving the current standard of damping modelling for a commercial aircraft by adding cross-modal terms while ensuring its consistency with experimental results.
2. Application of the improved damping model in representative full aircraft dynamic analyses to estimate potential benefits.
3. Review several enhanced damping identification techniques applied to experimental data from a ground vibration test of an A/C elevator, with and without added dampers.

1.2 Basic damping theory

The fundamental equation of the structural dynamics (1), with all or some of its components, is the starting point for most of the analyses of practical interest in aircraft engineering. In that equation, the dynamics characteristics of the structure are completely defined by mass, stiffness and damping. The last one is, by far, the poor relative of this family.

$$M\,\ddot{\boldsymbol{x}} + C\,\dot{\boldsymbol{x}} + K\,\boldsymbol{x} = \boldsymbol{f}(t) \quad (1)$$ [1]

There is a clear and enduring lack of understanding of the actual physics behind damping, with so many different theories and models that puts into question the very unified concept of damping. A non-exhaustive list of effects covered under this umbrella includes material

[1] Equations convention: matrices in capitals, vectors in lower case bold and scalars with lower case letters.

damping (with as many models as types of materials), joint effects (air-pumping and friction), fluid-structure interaction (of special relevance in flutter analyses), cable vibration and different sorts of non-linearities.

The only common feature among these very diverse phenomena is the dissipation of energy. Moreover, and unless mass and stiffness, detailed damping properties are not easily derived by matching experimental results from dedicated tests.

The classical equation in (1) assumes a viscous behavior, with the damping force proportional to velocity. The other main formulation, hysteretic or structural damping, makes it vary accordingly to the displacement.

Viscoelastic materials, combining viscous and elastic behaviors, also admit several models with different applications: Maxwell, Kelvin-Voigt and other combinations of springs and dashpots.

Under any of these formulations, it is of the maximum interest to describe the response of any point of the structure under a sinusoidal excitation. The frequency response functions (FRF) so derived offer a very valuable insight into the damping characteristics that, of paramount importance, can be compared and validated against GVT data. Equation (2) shows the FRF between any two points with a modal hysteretic approach.

$$H_{kj}(\omega) = \sum_{r=1}^{N} \frac{\phi_{kr}\phi_{jr}}{{\omega_r}^2 - \omega^2 + i\eta_r {\omega_r}^2} \tag{2}$$

There is an extensive bibliography covering the description, characterization and modelling of damping for engineering applications, including classic and comprehensive texts to be used as introduction and reference [3].

2 DAMPING MODEL UPDATE FROM EXPERIMENTAL DATA

2.1 Review

The pivotal impact of damping on static and fatigue loads, as well as other vibrations topics (noise, passenger comfort, pilotability) affecting the aircraft design, combined with the difficulties already described to create predictive theoretical models, explain the great interests in experimental data. Vibration tests give the opportunity of validating, or directly populating, the parameters of the damping models chosen for a specific structure. Ewins reference work [4] is a very didactic introduction to the subject.

The subject of experimental identification of damping, in a modal or spatial distribution, has been addressed in an ever-growing number of texts. A wide range of techniques and methodologies, with their specific applicability, are summarized in [5].

A classic hurdle for the extraction of detailed damping data from vibration tests is the habit of assuming normal modes to be real. Apart from very undamped structures, experience shows them complex in most of the cases. The practicality of this simplification, already described above, leads to usually force the experimental modes into the real field, losing precious information for damping identification in the process. A revision of techniques using directly the FRF data instead of modal approach are presented in [6].

The precision of these techniques to extract damping information is to be evaluated taking into account their practicality in engineering applications. In some cases their requirements could include a complete set of normal modes, excitation and measurements on all d.o.f. or perfectly clean signal without any noise. The relative merits and success of the different methods in test validation leave the question of which is best still open, depending heavily on the application targeted.

For the practical cases described in Sections 3 and 4 a few methodologies were considered in order to illustrate the process of damping extraction from test data, as well as to evaluate the potential of the usually discarded damping information for dynamics loads analyses.

2.2 Deterministic damping model

The most simplistic, although widely used, damping model for dynamic analyses takes a constant modal damping along the whole range of frequency. Applying a value of 3% structural damping (1.5% critical) is a common rule of thumb and, very often, no further refinement is sought unless the results are challenged due to design constraints. As a matter of fact, this value is accepted for the airworthiness authorities in the absence of better information [7].

As introduced in Section 1.1, the next step in damping modeling is to use the modal damping from vibration tests to tailor this flat rate, adjusting it for each normal mode frequency. Again, in most of the engineering applications for aircraft industry, this is the most refined damping model to be found.

Taking the modal damping, either from assumptions or tests, as a baseline, a first improvement in local damping characteristic can be done base on specific knowledge of high-damping components. This is the case for elastomers, widely used in rubber mounts of engines and other vibrating devices. The experimentally determined dynamic characteristics of these materials, applied to specific geometries, can be directly implemented into the models. Section 4.3 describes this approach in a practical case.

2.3 Adhikary’s method

The complex nature of normal modes is related to the presence of non-negligible off-diagonal terms in the modal damping matrix. Assuming low-damped structures($\varsigma_j \ll 1$), Adhikary uses the first order perturbational method to target this information using the experimental complex modes [8].

The cross modal damping is derived from the amplitude of the other modes in the complex part of each measured mode. At each modal resonance the dampers in the structure will generate an out of phase force which forces the other modes at that frequency depending on the level of coupling. The method analyses all these out of phase responses to derive a fully populated damping matrix, to replace the current diagonal matrix.

Equation (3) represents each complex eigenvector as the addition of a real part from the corresponding undamped normal mode, plus an imaginary part calculated as a linear combination of the rest of the undamped eigenvectors.

$$\boldsymbol{z}_j \approx \boldsymbol{x}_j + i \sum_{\substack{k=1 \\ k \neq j}}^{N} \frac{\omega_j \acute{C}_{kj}}{{\omega_j}^2 - {\omega_k}^2} \boldsymbol{x}_k \tag{3}$$

The diagonal terms of the modal damping matrix $\acute{C}$ are extracted from the complex eigenvalues (4), following the usual viscous formulation with the assumption of low damping.

$$\lambda_j = \mp\omega_j + i\,{}^{\acute{C}_{jj}}/_{2} \tag{4}$$

Finally, the full modal damping matrix can be transformed back to spatial coordinates with a pseudo-inverse operation using the not-complete real eigenvectors matrix X (5). This is normally the case in real aircraft GVT where the number of accelerometers greatly exceeds the number of modes obtained:

$$C = [(X^T X)^{-1} X^T]^T \acute{C}\, [(X^T X)^{-1} X^T] \tag{5}$$

The resulting damping matrix is not necessarily symmetric, which may be explained by existing non-viscous damping mechanisms. Although the reciprocity in real structures makes a non-symmetric damping non-physical, it would still deliver valid response function predictions.

In such a case, Adhikary proposes an alternative method to identify a non-proportional non-viscous damping matrix [8]. The only additional requirement is the availability of the mass matrix condensed at the accelerometer points, easy to obtain from a FEM model.

2.3.1 Alternatives Method

Within the many damping extraction techniques reviewed, it is worth mentioning the Minas-Inman methodology due to the possibility of updating the damping matrix without a complete set of normal modes [9]. The main difficulty, although surmountable, is that the stiffness and mass matrices need to be condensed at points with experimental data, which could require a substantial number of accelerometers.

Pilkney's iterative version of the Lancaster's classic theory for damping extraction with a modal method is also interesting [10] , although it needs comprehensive testing, imposing limits in practice on the size of the structural component analyzed.

Lee and Kim's simplification of Chen, Ju and Tsuei's method FRF-based technique [6] is very easy to implement, although with similar limits due to the requirement of a full matrix of excitation-measurements along the structure.

3 A/C ENHANCED DAMPING MODAL MATRIX: OFF-DIAGONAL ELEMENTS

3.1 Extraction from experimental data

The available data of a complete ground vibration test of an airliner, labelled here as AC-4, included complex eigenvectors which allowed using Adhikari's methodology [8]. Off-diagonal elements were extracted to populate the modal damping matrix. As explained above, this theory also allows recovering a spatial distribution of the damping using the cross-modal information. Figure 1 shows the damping obtained from test data for AC-4. Obviously, the topology as represented is not physical in a strict sense, but hints at the modal interactions in play.

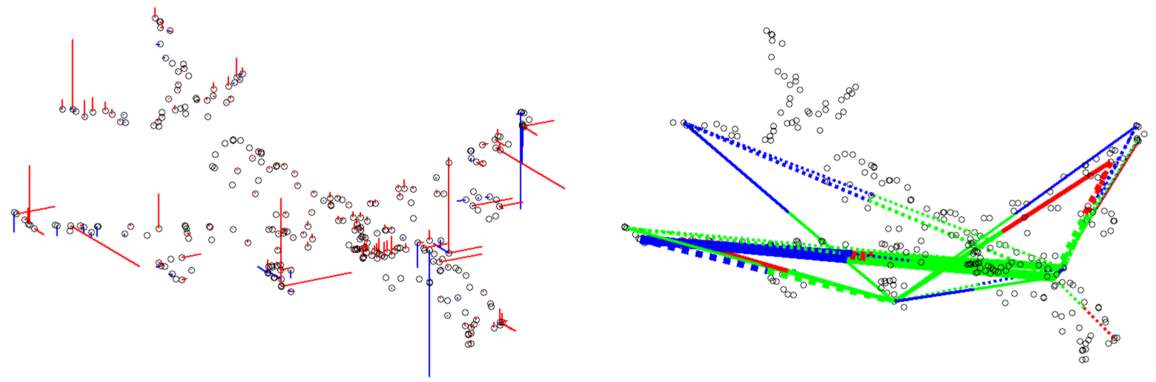

Figure 1: AC-4 accelerometers set-up during GVT. Damping direct terms (left) and spatial damping connections (right) for modes up to 5.2 Hz.

Only a few viscous couplings of some identified modes were updated into the model. This limited enhancement of the damping matrix gives the opportunity of a specific evaluation on the potential effect of these off-diagonal damping terms in standard dynamics loads applications.

3.2 Application on aircraft dynamic analyses

3.2.1 Introduction

The main purpose of obtaining a GVT-validated complete damping matrix is to tap into the benefits of additional damping from the cross-modal terms. These off-diagonal elements, discarded with the proportional damping assumption, offer an additional alleviation during dynamic analyses.

In order to give a first assessment of this damping potential, two sets of modes (labelled X1 and Y1) were selected based on their relevance for outboard wing and rear fuselage respectively.

Using a calibrated FEM of the AC-4 aircraft (Figure 2), two types of dynamic analyses were performed to evaluate the impact of these cross-modal damping terms:

- **Windmilling**: sinusoidal excitation was applied on the portside inner engine to obtain the response at different locations in the A/C. Special attention was given to the range of frequencies covering outer-wing normal modes.
- **Dynamic landing**: the FEM was excited at the pintels of the front and main landing gears with loads time-histories representing a landing operation.

Numerical simulations were run in a FEM code, using the modal frequency and modal transient solutions respectively. The modal damping matrix was extracted and updated during the calculations. It is worth mentioning that special attention needs to be taken to the coupled-uncoupled solution algorithms available in the solver. Any comparison, to be meaningful, needs to ensure consistency on this aspect.

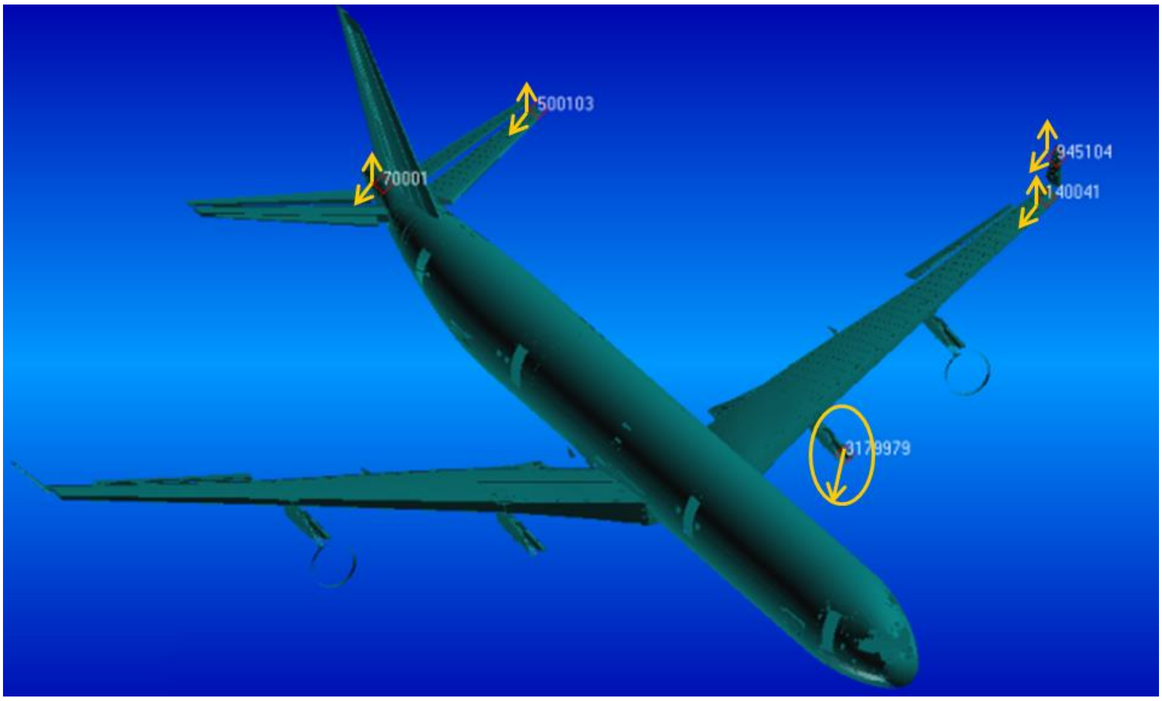

Figure 2: AC-4 FEM with windmilling excitation at portside inner engine.

3.2.2 Windmilling

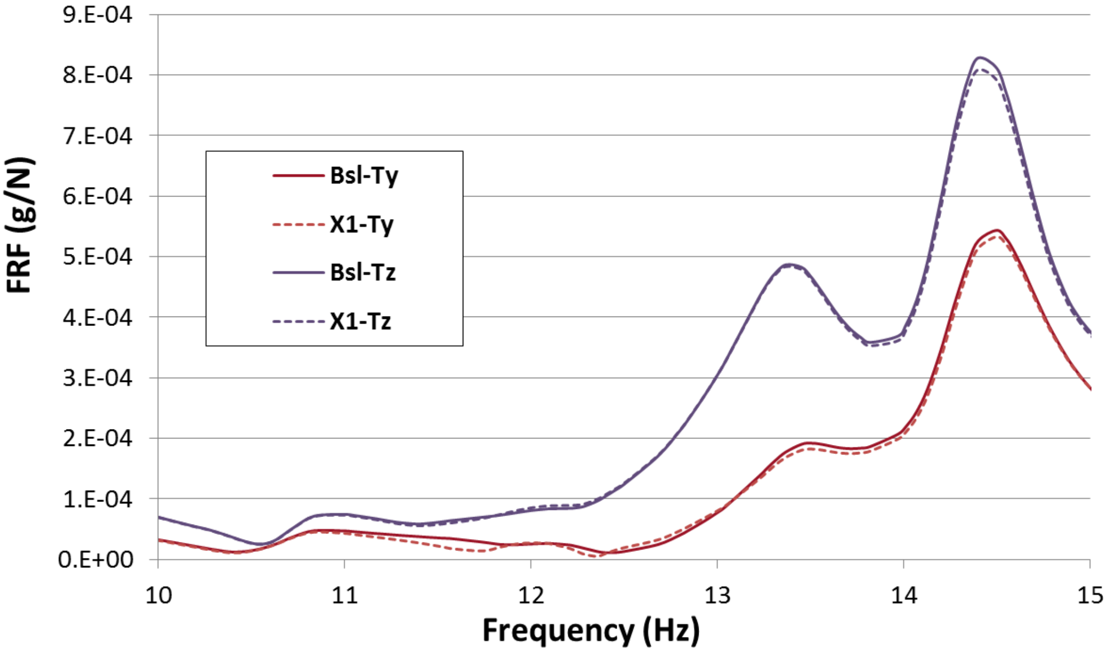

Figure 3: AC-4 winglet response during windmilling simulation, with (X1) and without (Bsl) cross-modal damping terms.

The enhancement of the diagonal modal damping with selected cross-damping terms shows a clear peak reduction in the FRFs at the wingtip (Figure 3). More significant than the 3% decrease is the fact that it came practically from just 2 small cross-damping terms, and that it was achieved with no added costs in terms of design or additional tests required. The data simply was there to be used.

Although more comprehensive analyses could shed more light on the mechanism of modal interactions, it seems clear from the results that dynamic loads can be alleviated on specific components by targeting normal modes with significant displacements on that area, and modal frequencies relevant for the dynamics load case. There are also some indications that cross-modal terms between symmetric and antisymmetric modes are also effective.

3.2.3 Dynamic landing

The purpose of this analysis was to perform a representative assessment of the potential of modal damping off-diagonal terms in dynamic landing loads calculation. A specific set of 7 modes with rear fuselage and HTP components, symmetric and antisymmetric, were selected to improve the damping modal matrix with their cross-modal terms. Figure 4 shows the time-history of the accelerations at the rear fuselage. Again, a valuable reduction of 1.2% and 4.9% in the first two positive acceleration peaks was achieved with the enhanced damping matrix.

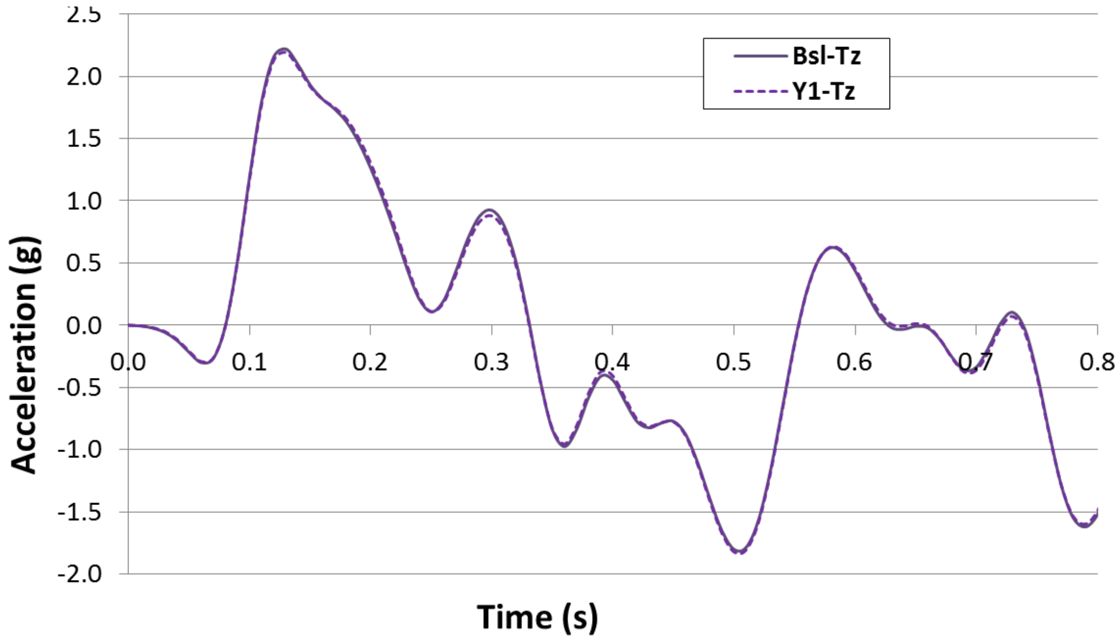

Figure 4: AC-4 rear fuselage response during dynamic landing numerical simulation, with (Y1) and without (Bsl) cross-modal damping terms.

For the HTP tip vertical response (Figure 5) the decrease of the acceleration peaks is quite remarkable with 4.6% and 6.7% reductions in the first two peaks.

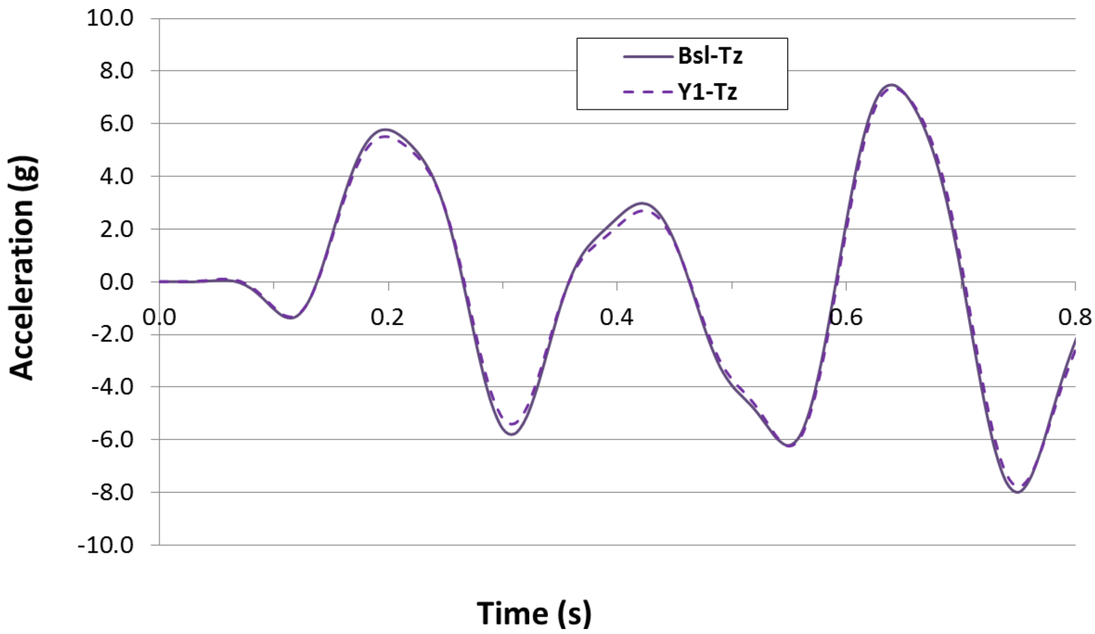

Figure 5: AC-4 HTP tip vertical response during dynamic landing numerical simulation, with (Y1) and without (Bsl) cross-modal damping terms.

4 DETAILED DAMPING MODEL FOR AN ELEVATOR

4.1 Introduction

The experimental identification of damping at aircraft level is the final goal in order to benefit from additional loads alleviation. Nevertheless, the size and complexity of a modern aircraft makes it very difficult to single out specific estimated damping contributions, although joints are the usual suspects. Without this knowledge beforehand, it is very complicated to assess and validate the different methodologies to extract damping from GVT data.

To overcome this problem a dedicated vibration test was designed and conducted on an airliner elevator. The aim is to use the results as benchmark, taking advantage of its relative simplicity, while remaining in the area of real aircraft components. Any technique validated at this component level should be applicable for industrial purposes, what it is not the case for most of the academic models.

Two configurations have been run during the GVT: baseline elevator, and the same specimen with added damping layers. The presence of a known source of damping will be extremely valuable to assess the accuracy of the different methodologies.

In the following sections the vibration tests and the status of the on-going investigations are described.

4.2 Test description

The specimen used for the GVT is a standard starboard elevator, hanging with elastics bands from its hosting points to achieve almost free-free boundary conditions. A vertical excitation of 10N was applied by a shaker on the inboard side of the specimen, achieving a good response all across the elevator (see Figure 6). Up to 50 accelerometers were attached to the surfaces to capture a fine representation of the modal shapes.

For the configuration with added dampers, a series of SBR rubber layers were firmly glued to the upper surface. SBR was selected after a review of viscoelastic materials based on its efficiency as a free-surface damper [11]. The specimen, including accelerometers, cables and dampers were carefully weighted to ensure controlled mass conditions. A detailed FEM model of the elevator was also available to provide a good simulation of the stiffness characteristics.

Figure 6: A/C elevator GVT with SBR damping layers.

4.3 Deterministic damping model

The results of the GVT with a clean configuration of the elevator provided a baseline with the actual dynamic characteristics of the specimen, particularly relevant for the damping parameters. As expected, the structure is lightly damped, with a maximum of 1.01% of equivalent critical damping for the 1st torsion mode.

The first enhanced damping model was set-up for the configuration with added SBR rubber. The update of damping and stiffness after adding damping layers was done at each element following methodologies from [3], while masses were also adjusted to weighted values. Figure 7 displays the elements enhanced, all in the top cover.

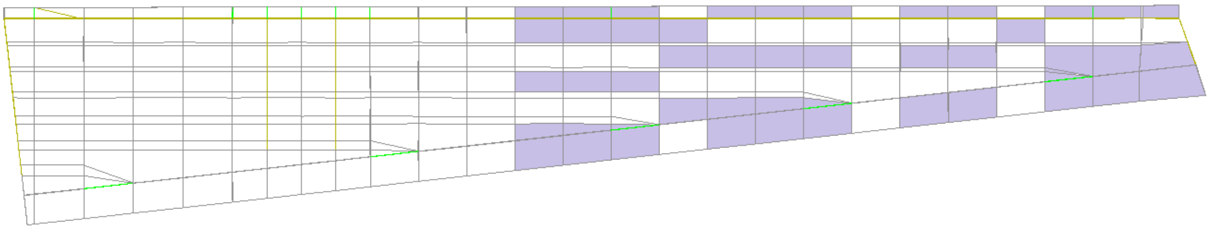

Figure 7: Elevator FEM with areas updated in damping/stiffness to reflect SBR damping layers (purple).

In this case, by superimposing local dampers contribution in the model, while keeping the clean elevator GVT modal damping values, we are able to evaluate if this approach can be predictive and improves the matching to the experimental frequency response functions.

Knowing in advance specific damping characteristics in a component, gives the opportunity of benefiting from a deterministic damping modelling, and suggest design modifications with dampers for specific problems. Practical examples in aircraft industry are rubber mounts for engines and other elastomers.

4.4 Cross Modal Damping

Adhikari's method was used to extract the cross-modal damping terms for the elevator, with and without the SBR layers. Data from three different levels of excitation was used to ensure robustness and identify potential non-linearities. Figure 8 shows the results in both cases.

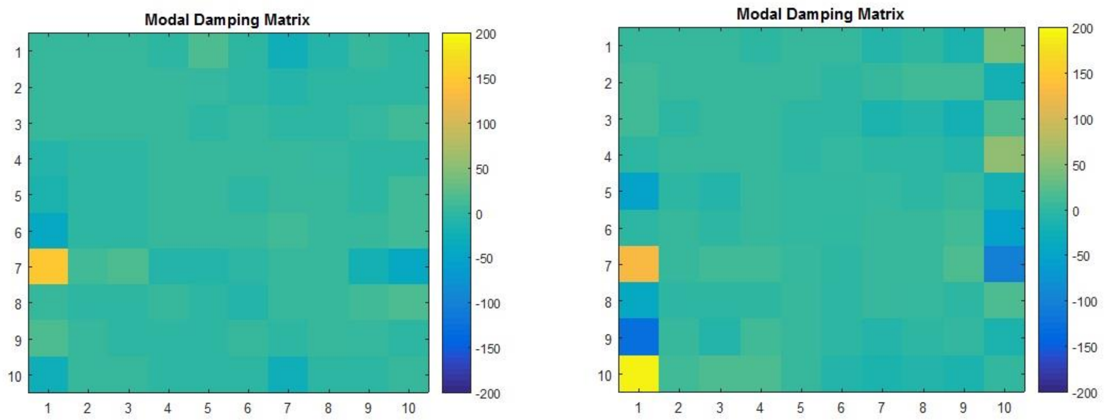

Figure 8: A/C elevator damping cross-modal terms. Baseline (left) and with SBR rubbers added (right).

The most obvious feature is the significant increase in cross damping in modes 1 and 10. The inclusion of these terms in frequency response calculations would significantly affect the results, working in addition to the direct modal damping terms. These values showed an evolution with the amplitude of the excitation, although not significant.

As a summary, results show a substantial consistency, which provides confidence in a future use in calculations.

4.5 Methodologies benchmarking

The FRAC correlation criterion [12] was selected to compare FRF from GVT data to different FEM simulations in a consistent way. In the plot below it can be noticed that no shift in frequency was introduced to align the response peaks, although the conclusions would not change.

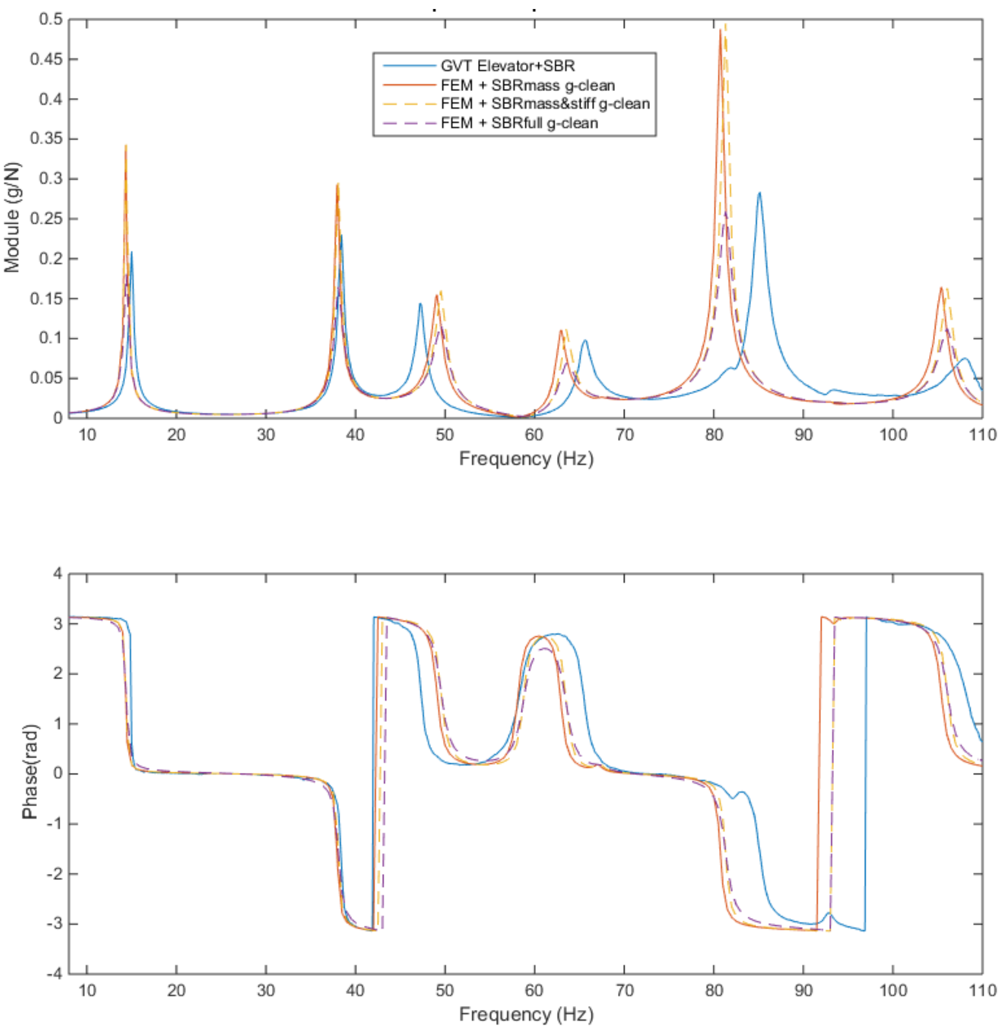

Figure 9: SBR-damped Elevator. FRF at the outboard tip. GVT data and several FEM models.

The main conclusion from Figure 9 is that every step in refining the modelling of the SBR dampers improves the FRF matching to the GVT data. The successive addition of mass effect, stiffness and finally local damping, done in a predictive way, improves the FRAC coefficient by up to 40%. Clearly, there are margins of enhancement with local damping modelling.

Methodologies detecting and locating damping in the structure (i.e. Adhikari's) could be used to identify where to act in terms of local damping modelling. The technique used here to model it locally was via the material properties (MAT2 cards in the Nastran FEM). This is just one possibility, although dedicated damping elements could be taken into consideration.

A process of optimization, targeting FRF matching, could be also very practical to find the local damping parameters improving the global response [13].

5 CONCLUSIONS

Damping is the area of structural dynamics modelling less developed by far and, for this very same reason, with great potential. New aircraft designs and materials are pushing the limits of the loads requirements, strengthening the case of more sophisticated damping models.

The dynamic loads simulations for a commercial aircraft in Section 3 are a good proof of the potential in loads alleviation of these enhanced damping models, even with a limited number of cross-modal terms added to the damping matrix. The evaluation of a full modal damping matrix from GVT data will be the logical next step. As part of these investigations, existing GVT data will be used to better understand the cross-modal damping mechanisms and, in the future, even to provide suggestions leading to tailored designs.

The on-going work on the elevator GVT tests is a push on this direction, opening up the possibility to evaluate other techniques extracting damping information from experimental data.

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