HiSST: 3rd International Conference on High-Speed Vehicle Science Technology

14 -19 April 2024, Busan, Korea

Design of a nozzle for hypersonic wind tunnel with optimized hyperbolic geometry

M. Iida[1]*, K. Shimamura*[2]

Abstract

An expansion tube is a type of shock tube that can generate a high enthalpy flow simulating atmospheric entry. However, hypersonic flows have the problem that the diameter of the test flow becomes smaller due to significant boundary layer development. We designed a nozzle with a contour shape based on the hyperbolic equation to suppress the development of the boundary layer. We obtained the geometry of a hyperbolic nozzle using an optimization algorithm and evaluated its performance using CFD. Then we discussed the design method of the nozzle using multi-objective optimization with genetic algorithm.

Keywords: *hypersonic nozzle, expansion tube, optimization*

Nomenclature

$\boldsymbol{Q}$ – Conservative variables
$\boldsymbol{E}, \boldsymbol{F}, \boldsymbol{H}$ – Inviscid flux vector
$\boldsymbol{E_v}, \boldsymbol{F_v}, \boldsymbol{H_v}$ – Viscous flux vector
x – Axial coordinate
r – Radial coordinate
f – Objective function
p – Static pressure
p_i – Component of p
p_{mean} – Mean static pressure
θ – Angle of flow
θ_i – Component of θ
M – Mach number
M_i – Component of M
M_{design} – Design mach number
i – Grid index, radial direction
a,b,L – Nozzle parameter

1. Introduction

Sample return capsules such as Hayabusa and OSIRIS-Rex re-enter the Earth's atmosphere at hypersonic speeds. During this process, the capsule is exposed to an extremely hot environment due to aerodynamic heating, so appropriate thermal protection is required. For this reason, experimental devices have been studied to simulate the re-entry environment by generating a hypersonic flow. In experiments, the scale of the sample is important and a larger sample can provide more detailed data. However, hypersonic wind tunnels require large amounts of energy to operate, making it difficult to increase the size of the apparatus itself. In addition, hypersonic flows have a highly developed boundary layer and the core of the test flow is small, so the sample must be small. Hypersonic nozzles have been designed to solve these problems. However, few studies have addressed the design in terms of flow uniformity and core flow size, and there are still unresolved issues.

The most basic hypersonic nozzle is a bell nozzle designed by the method of characteristics (MOC). However, this design method has the disadvantage that the total nozzle length increases as the Mach number increases, so the core flow decreases. Stewart [1] applied the characteristic curve

[1] *Tokyo Metropolitan University, 6-6 Asahigaoka, Hino, Tokyo, 191-0065, Japan, iida-masaki@ed.tmu.ac.jp*
[2] *Tokyo Metropolitan University, 6-6 Asahigaoka, Hino, Tokyo, 191-0065, Japan, shimamura@htgl.sd.tmu.ac.jp*

method to the nozzle design of RHYFL-X and designed a 5 m Mach 13 nozzle. However, the nozzle had to be shortened to 3.75 m because the boundary layer at the nozzle outlet was 50% of the diameter. Toniato[2] designed a Mach 12 bell nozzle using CFD and a parallel simplex algorithm, but there is room for improvement in the flow uniformity near the central axis at the nozzle outlet.

Thus, current hypersonic nozzles are designed based on the bell nozzle, but the problems of boundary layer development and flow uniformity have not been solved. In contrast, Chue [3] developed a hyperbolic nozzle using the hyperbolic equation as the contour and designed the Mach 12 nozzle. This design method is shorter than the bell nozzle, which reduces the development of a boundary layer. The nozzle wall expands continuously, and there is no inflection point to create compression waves at the inflexion point, so a uniform flow can be obtained at the nozzle outlet. The hyperbolic nozzle solves the problems of the bell-shaped nozzle, but its nozzle is designed according to empirical rules, and a proper design method has not been established.

Therefore, the purpose of this study was to establish a design method for a hyperbolic nozzle that is superior to a bell nozzle. We proposed a design method for a hyperbolic nozzle using CFD and an optimization algorithm. The nozzle was designed for the flow conditions of an expansion tube (Fig. 1) owned by the university.

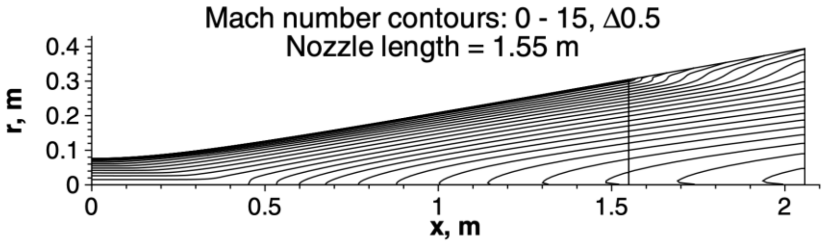

Fig 1. Hyperbolic nozzle by Chue[3]

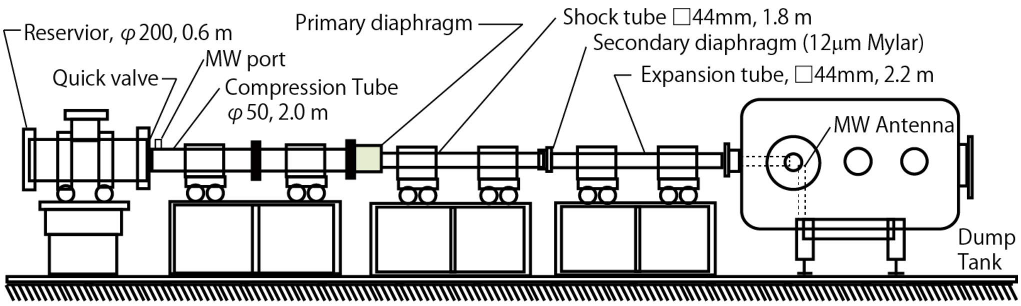

Fig 2. 9m Expansion Tube

2. Computational Methods

2.1. CFD

In this study, the expansion tube was assumed to be two-dimensional axisymmetric. The governing equations are the two-dimensional axisymmetric Navier-Stokes equations for laminar flow and thermochemical equilibrium in all regions (Eq.1).

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{\partial \boldsymbol{E}}{\partial x} + \frac{\partial \boldsymbol{F}}{\partial r} + \boldsymbol{H} = \frac{\partial \boldsymbol{E}_v}{\partial x} + \frac{\partial \boldsymbol{F}_v}{\partial r} + \boldsymbol{H}_v \tag{1}$$

where $\boldsymbol{Q}$ is a conserved quantity vector, $\boldsymbol{E}$, $\boldsymbol{F}$, and $\boldsymbol{H}$ are inviscid flow vectors, and $\boldsymbol{E}_v$, $\boldsymbol{F}_v$, and $\boldsymbol{H}_v$ are viscous flow vectors. In this study, to solve for supersonic flow fields, the characteristic velocity is the sum of the flow and sound velocities. The time term in the equations is solved explicitly by Euler, the numerical flux is solved by the AUSM-DV scheme, and the MUSCL method is used as a second-order spatial refinement method. The viscosity term is evaluated by the second-order accurate central difference. The symmetry condition was assumed on the central axis, and the wall surface was assumed to be isothermal and no-slip.

In the method using Nelder-Mead of the two optimization methods, the computational domain is from the first cornea to the nozzle outlet (Fig. 3). Therefore, we refer to this method as "NM-TestFlow". In the axial direction, the expanded waveguide mesh was divided using the moving mesh method (Fig. 4) to create a mesh with a minimum size of 0.5 mm. The radial direction was divided into 40 unequally spaced sections. The nozzle portion was divided at intervals of 0.5 mm in the axial direction without using the moving mesh method (Fig. 5).

In the analysis using the genetic algorithm, on the other hand, only the nozzle was used as the computational domain. And we refer to this method as "MOGA-Nozzle". This was done to reduce the computation time for an increase in the number of computations. Calculations were performed assuming a uniform flow of air entering the nozzle inlet. The flow conditions were obtained from the CFD results from the first cornea. The grid was divided equally in the axial direction by 0.5 mm and unequally spaced in the radial direction.

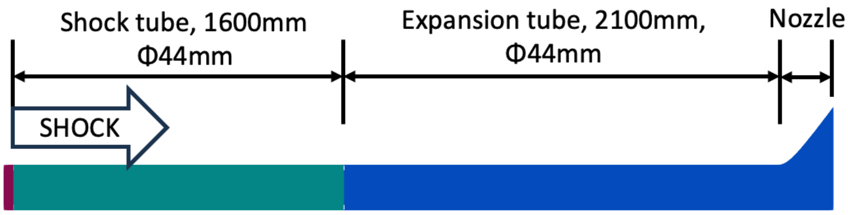

Fig 3. Calculation region

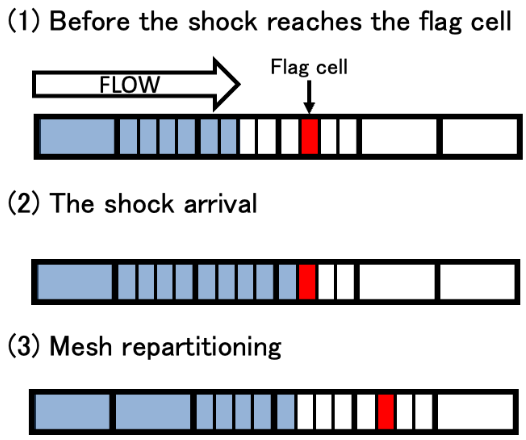

Fig 4. Process of moving mesh method

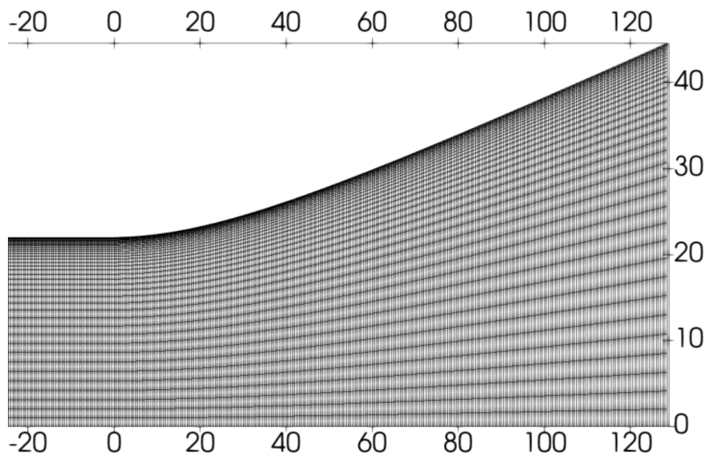

Fig 5. Nozzle grid

2.2. Design and Optimization

In a hyperbolic nozzle, the nozzle contour is determined by the hyperbolic equation. In this study, an axisymmetric nozzle was designed. Eq.2 shows the equation. The shape of the nozzle is determined by three parameters in the equation: a, b, and the length of the nozzle, L.

$$\frac{(r-r_0)^2}{b^2}-\frac{x^2}{a^2}=1 \quad (2)$$

Two optimization methods were used in this study: the Nelder-Mead algorithm[4] and a genetic algorithm. This was done in order to perform a multifaceted analysis by using two different methods with different properties.

The Nelder-Mead algorithm is a method for finding the point with the minimum objective function value in the space enclosed by the simplex (Fig.6). In other words, the simplex is repeatedly moved in the design variable space. The disadvantage of this method is that the computational cost tends to increase due to the need to perform CFD calculations many times in the process. On the other hand, this algorithm does not require differentiation of the function during optimization. Therefore, it has the advantage that optimization can be performed even when the causal relationship between the design variables and the objective function is a black box.

The optimization flowchart is shown in Fig.7. CFD calculations are performed on the flow in the nozzle with curves defined by parameters to quantify the variation of the test flow. The parameters are then adjusted using the Nelder-Mead algorithm. The flow in the nozzle defined by the adjusted parameters is then analyzed. This process is repeated until convergence conditions are met.

In this study, the three design variables are a, b, and the nozzle length in Eq.2. The objective functions were the dispersion of the axial static pressure distribution of the airflow at the nozzle outlet, the dispersion of the flow angle distribution, and the deviation of the Mach number distribution from the designed Mach number, as in Eq.3. These were weighted to calculate the objective function f. The airflow at 30 μs after the arrival of the shock wave was used as the evaluation target, and the core flow region was evaluated. Core flow was defined as a continuous region satisfying $\boldsymbol{\partial M/\partial r} \geq -\mathbf{20}$ according to the work of Chan [5].

$$f=\frac{\alpha}{n}\Sigma(p_i-p_{mean})^2+\frac{\beta}{n}\Sigma(\theta_i-\theta_{mean})^2+\frac{\gamma}{n}\Sigma(M_i-M_{design})^2 \quad (3)$$

Genetic algorithms have the property of preserving solution diversity. Therefore, Pareto solutions can be derived and trade-off relationships can be investigated. Using this property, the performance characteristics of hyperbolic nozzles can be investigated to determine the direction and criteria of nozzle performance. In this study, a, b, and nozzle length L were used as design variables. As in the NM algorithm, the objective functions were the variance of the axial static pressure distribution of the airflow at the nozzle outlet, the variance of the flow angle distribution, and the deviation of the Mach number distribution from the designed Mach number, and three-objective optimization was performed. The number of generations and individuals were set to 35 and 50, respectively.

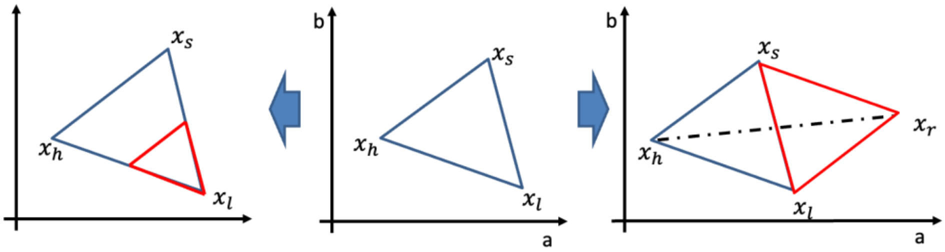

Fig 6. Process of Nelder-Mead

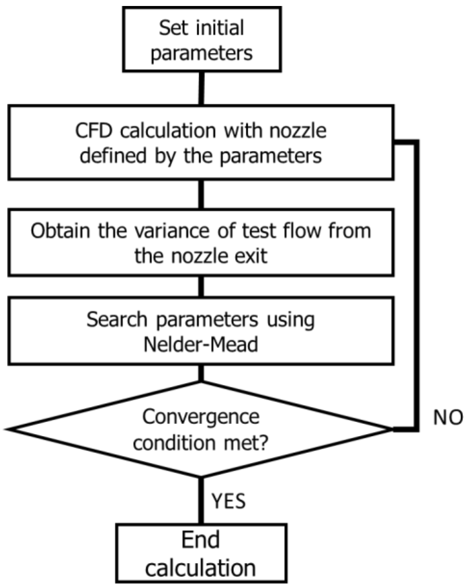

Fig 7. Process of nozzle design

3. Results and Discussion

3.1. NM-TestFlow

First, the calculation results of the model reproduced from the first diaphragm are presented. Fig.8 and 9 compare the experimental and CFD results for the static pressure at the wall of the tube. On the other hand, in Fig.9, the values at the start of the test are consistent, and the results are reproduced up to 50 μs, after which they deviate from the original values. Next, the test flow is examined by physical quantities other than static pressure, and the density history of the test gas and compressed pipe gas at the nozzle inlet and the temperature history of the gas are shown in Fig.10. From this figure, it can be seen that the test gas exists only inside the shock layer. Fig. 11 shows the density distribution of the airflow before it enters the nozzle. From this figure, it can be seen that He expands by pushing out Air, and the test air flow disappears. These results suggest that the interaction between He and air has not been completely reproduced.

The Mach number of the hyperbolic nozzle designed by NM-TestFlow was 3.834. The resulting nozzle is shown in Fig. 12. The length, outlet radius and evaluation values are shown in Table 1.

Fig.13 and 14 show the static pressure distribution and Mach number distribution at the nozzle inlet and outlet. It can be seen that the non-uniform static pressure distribution at the nozzle inlet has been made uniform at the nozzle outlet. On the other hand, there is a turbulence in the form of a step, which is considered to be an effect of the test airflow not being reproduced. The fact that the Mach number does not increase through the nozzle can be attributed to the fact that the flow being evaluated is the high-temperature flow inside the impact layer, as described above.

On the other hand, in Fig.15, the flow angle is suppressed to 5[deg] in the core flow. The deviation of a conical nozzle with a similar nozzle length is 13.5 [deg]. Compared to this, the optimization seems to have been effective.

3.2. MOGA-Nozzle

Next, the computational domain was restricted to nozzles, and the genetic algorithm was used to examine their characteristics. The final population could be divided into two groups according to nozzle length. Fig.16 shows the variance of the static pressure and flow angle, as well as the static pressure and deviation from the designed Mach number. A Pareto front was obtained for the static pressure - flow angle. Fig.17 shows the dispersion of core flow radius and nozzle exit radius, and the dispersion of core flow radius and flow angle. As the nozzle radius is increased, the core flow radius also increases, but the core flow radius reaches its maximum at 30 mm when the nozzle radius is around 40 mm. There is a similar correlation between core flow radius and flow angle. The nozzle length does not seem to affect the dispersion of the core flow radius or flow angle, since the results are the same for the two groups.

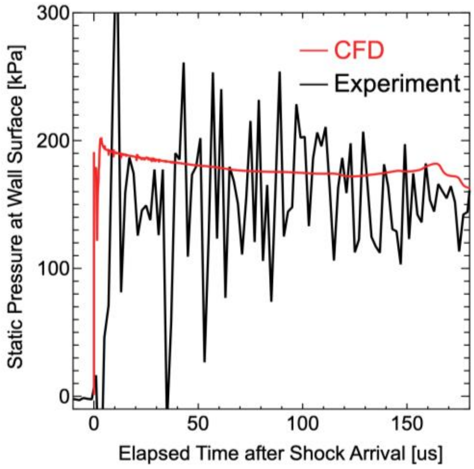

Fig 8. Comparison of wall static pressure history at 1.1m behind the first diaphragm

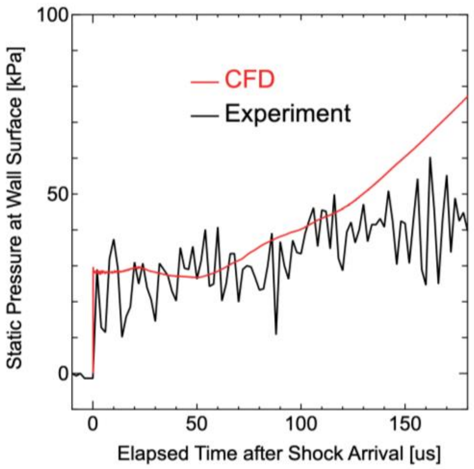

Fig 9. Comparison of static pressure history at 1.1m behind the second diaphragm

Table 1. Nozzle parameter and performance

Length [mm]	Outlet radius [mm]	Core radius [mm]	M	Pressure variance[kPa^2]	Angle variance[deg^2]	Mach deviation[-]
250.0	35.05	18.64	3.834	0.1813	0.3855	1.823

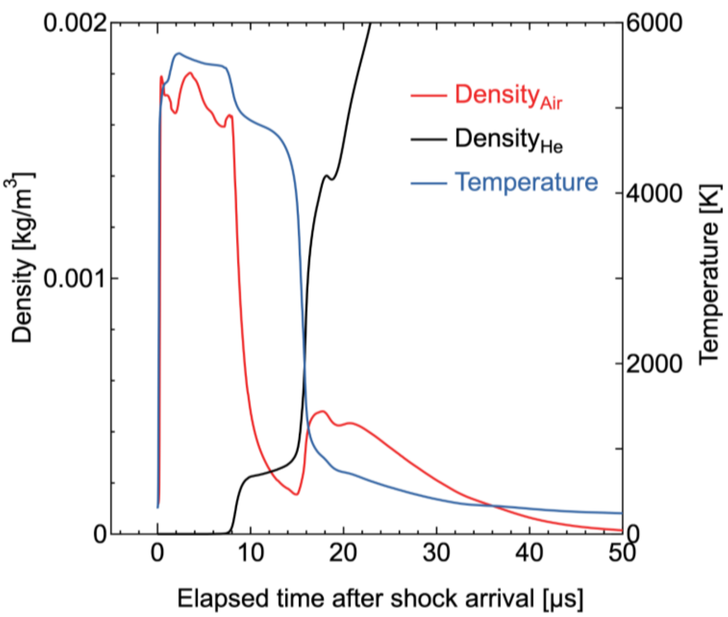

Fig 10. Density and temperature histories of test gas (Air) and compression tube gas (He) at nozzle inlet

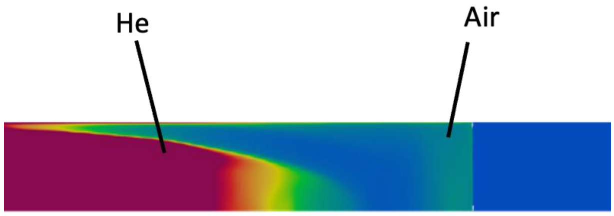

Fig 11. Density distribution at expansion tube

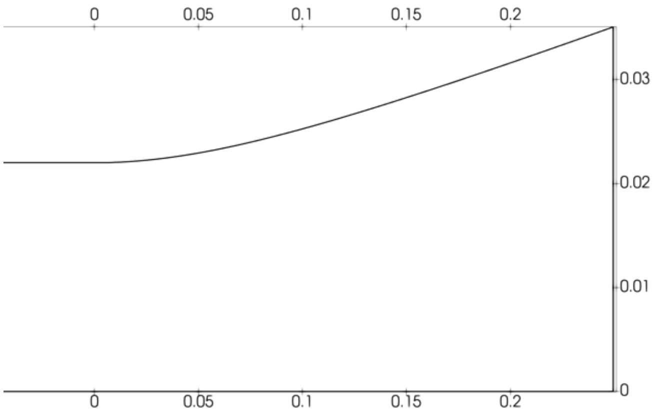

Fig 12. Hyperbolic nozzle designed by NM

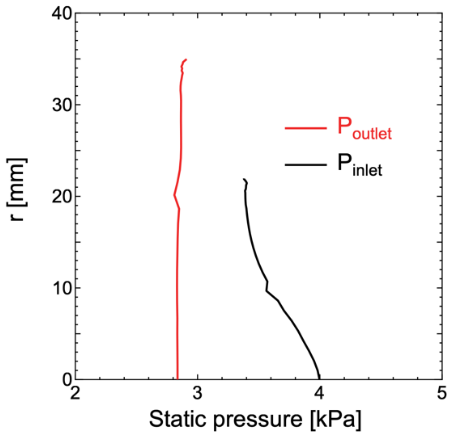

Fig 13. Static pressure distribution at nozzle inlet and outlet

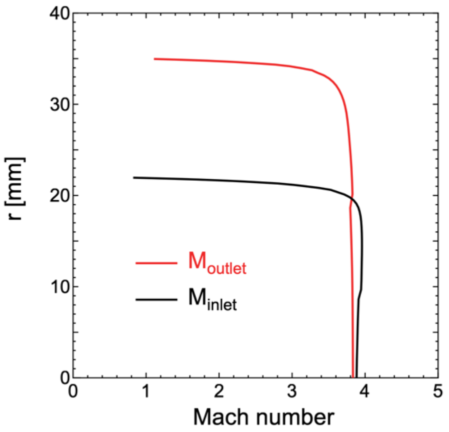

Fig 14. Mach number distribution at nozzle inlet and outlet

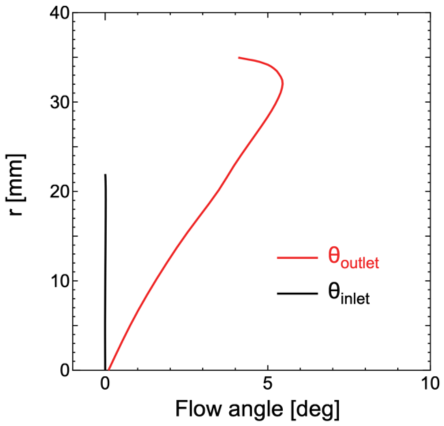

Fig 15. Flow angle distribution at nozzle inlet and outlet

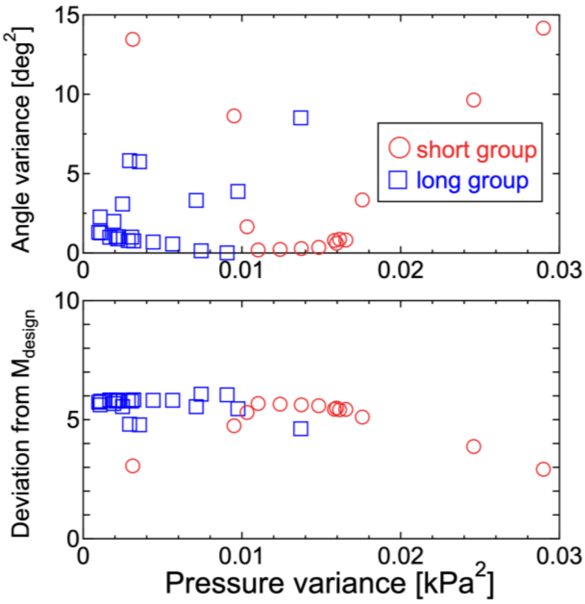

Fig 16. Pressure variance and Angle variance, and Pressure variance and Deviation of $\boldsymbol{M_{design}}$

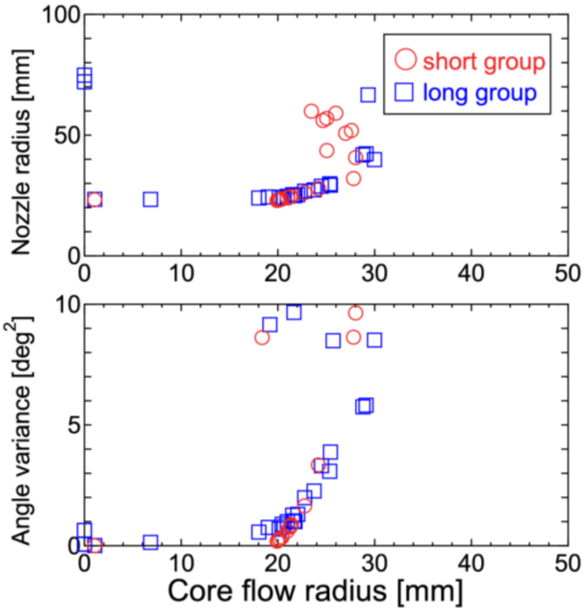

Fig 17. Core flow radius and Nozzle radius, and Core flow radius and Angle variance

4. Conclusion and Future Work

The purpose of this study was to establish a design method for a hyperbolic nozzle, and to clarify the design method for a supersonic nozzle superior to a bell-shaped nozzle. As a method, a design method combining CFD and the Nelder-Mead algorithm was proposed. The performance characteristics of the nozzle were investigated using CFD and the genetic algorithm. As a result, a hyperbolic nozzle with a dispersion value of 0.1853 [kPa^2] for the static pressure distribution, 0.3855 [deg^2] for the flow angle, and 1.823 deviation from the designed Mach number was designed by CFD. It was also found that the maximum value of the core flow radius was 30 mm at a nozzle radius of 40 mm. It was also found that the nozzle length correlated with the static pressure distribution but had no effect on the flow angle or Mach number of the airflow. The off-design characteristics of the nozzle will be investigated in the future, and the nozzle will be produced. After that, we would like to evaluate the nozzle by measuring the pitot pressure distribution in the radial direction.

References

1. Stewart, Benjamin S. Predicted scramjet testing capabilities of the proposed RHYFL-X expansion tube. (2004)
2. Toniato, P., et al.: Expansion tube nozzle design using a parallel simplex algorithm. Shock Waves, 30.2: 185-199 (2020)
3. Chue, R. S. M., et al.: Design of a shock-free expansion tunnel nozzle in HYPULSE. Shock Waves, 13.4: 261-270 (2003)
4. Nelder, John A; Mead, Roger.: A simplex method for function minimization. The computer journal, 7.4: 308-313 (1965)
5. Chan, Wilson YK, et al. Aerodynamic design of nozzles with uniform outflow for hypervelocity ground-test facilities. Journal of Propulsion and Power, 34.6: 1467-1478 (2018)