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A Design Method for All-movable Rudder Structure with Lattices and Stiffeners by Thermo-elastic Topology Optimization under Mass Constraints

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Abstract

In high-speed vehicles, the all-movable rudder structures are typically subjected to the dual effects of aerodynamic pressure and thermal loads. In this paper, the all-movable rudder structure with lattices and stiffeners is optimized using the thermo-elastic topology optimization method. The main thought of the design method can be summarized as follows: First, the representative lattice units of the selected lattices are equivalent to the virtual homogeneous materials whose effective elastic matrixes are achieved by the energy-based homogenization method. Meanwhile, the stiffeners are modelled using the solid material. Subsequently, the multi-material thermal-elastic topology optimization formulation is established for both the virtual homogeneous materials and solid material to minimize the structural compliance under mass constraint. Thus, the optimal layout of both the lattices and stiffeners could be simultaneously attained by the optimization procedure. Finally, the effectiveness and reliability of the proposed method were verified through the design of a typical all-movable rudder structure.

Keywords: *Topology optimization, thermo-elastic, rudder, lattice, stiffener*

1. Theoretical statements

1.1. Equivalent mechanical properties of the lattice unit cell

In this paper, the energy-based homogenization method is employed to calculate the elastic matrix of the virtual homogeneous material. Based on homogenization theory, the average properties of the virtual homogeneous material are equal to the average properties of the lattice unit cell. The average stress and strain can be expressed as

$$\begin{aligned} \bar{\sigma} &= \frac{1}{V_\Omega}\int_\Omega \sigma \mathrm{d}\Omega \\ \bar{\varepsilon} &= \frac{1}{V_\Omega}\int_\Omega \varepsilon \mathrm{d}\Omega \end{aligned} \tag{1}$$

the relationship between the average stress and strain can be stated as

$$\bar{\sigma} = \mathbf{D}^{\mathrm{H}}\bar{\varepsilon} \tag{2}$$

herein, $\mathbf{D}^{\mathrm{H}}$ represents the equivalent elastic matrix. The elastic strain energy stored in the lattice unit cell can be expressed as:

$$\bar{E} = \frac{1}{2}\bar{\sigma}_{ij}\bar{\varepsilon}_{ij}V_\Omega \tag{3}$$

From eq.1. and eq.2., the strain energy of the lattice unit cell can be can be directly calculated by the average strain and the equivalent elastic matrix. In finite element analysis, nine linearly independent

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test strain fields are applied to the lattice unit cell and the values of corresponding strain energies are obtained. Then, the equivalent elastic matrix can be calculated.

1.2. Topology optimization formulation

The thermo-elastic topology optimization problem is formulated to minimize the global compliance C or the partial compliance $C_{\Omega s}$ subject to constraints on mass of the stiffener structures:

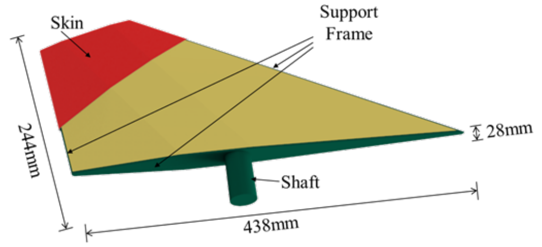
$$
\begin{aligned}
&\text{find:} \quad \mathbf{x}=\{x_{ij}\} \quad (i=1,\ldots,n; \quad j=1,\ldots,m)\\
&\text{minimize:} \quad C \text{ or } C_{\Omega s}\\
&\text{subject to:} \quad \mathbf{KU}=\mathbf{F}^{\mathrm{m}}+\mathbf{F}^{\mathrm{th}}\\
&\qquad M=\sum_{i=1}^{n}\rho_i V_i=\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}\rho^{(j)}V_i \le \bar{M}\\
&\qquad 0<x_{\min}\le x_{ij}\\
&\qquad \varphi_q \le 0
\end{aligned}
\tag{4}
$$

Herein, n and m are the number of designable elements and candidate materials, respectively. For example, m=2 in the case of one considered lattice structure and the stiffener structure. $\mathbf{K}$, $\mathbf{U}$ and $\mathbf{F}^{\mathrm{m}}$ are the global stiffness matrix, nodal displacement vector and the design-independent nodal force vector, respectively. In this formulation, $\mathbf{x}$ denotes the set of design variables and x_{ij} represents the presence (1) or absence (0) of the jth candidate material in the ith finite element. A lower bound for the design variables of $x_{\min}=10^{-3}$ is introduced in order to avoid the singularity of the structural stiffness matrix in the finite element analysis. V is the total volume of all designable elements and V_i is the volume of the ith element, ρ_i denotes the density of the ith element and $\rho^{(j)}$ denotes the density of the jth candidate material. $\bar{M}$ is the upper bound of the mass of overall structure. φ_q represents other optimization constraints that may be involved in the optimization process.

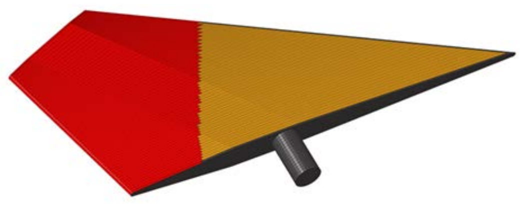
2. Numerical example

2.1. A typical all-movable rudder

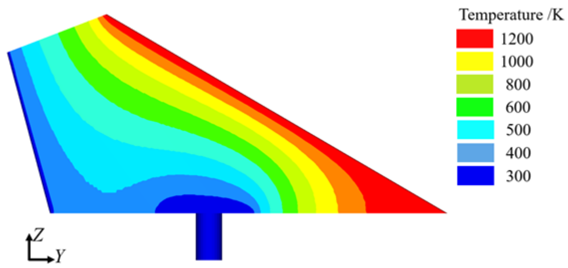
An all-movable rudder is designed to further verify the effectiveness and superiority of the proposed method. Its geometric model and dimensions is illustrated in Fig. 1(a).The finite element model of the all-movable rudder structure is established and illustrated in Fig. 1(b).



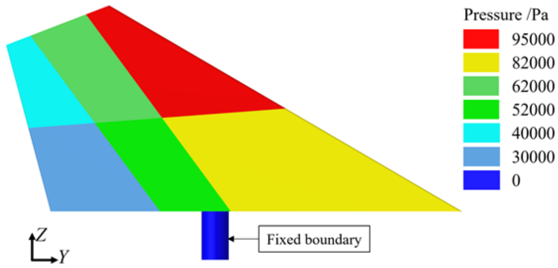
(a) Geometric model and dimensions



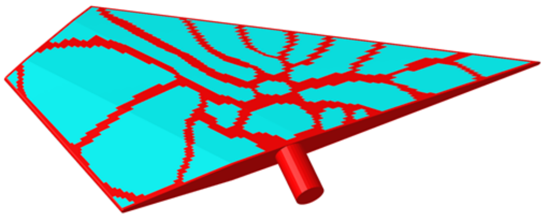
(b) Finite element model



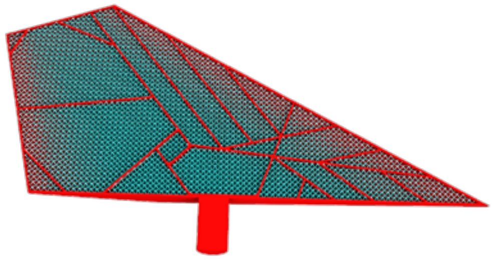
(c) Temperature field



(d) Distribution of aerodynamic pressure



(e) Temperature field



(f) Temperature field

Fig 1. The all-movable rudder

The temperature field and the aerodynamic pressure load on the rudder structure is illustrated in Fig. 1(c) and Fig. 1(d). The optimized configuration and the reconstructed model of the all-movable rudder structure with lattices and stiffeners is illustrated in Fig. 1(e) and Fig. 1(f), respectively.