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Guidance Law Based on the Sliding Mode Control with Impact Angle Constrained

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Abstract

Traditional guidance laws aimed at indirectly intercepting or collide the target are not effective in destroying strategic targets. In this paper, a novel three-dimensional guidance law is proposed based on the sliding mode control with impact angle constraints, which guarantees to intercept the target at a desired angle. In order to meet the realities of combat, the guidance law is based on finite time convergence. The stability issue of the guidance law is theoretically analysed to intercept the manoeuvring target with unknown acceleration at desired impact angle. The stability is also analysed by Lyapunov theory, which ensures that the guidance law can converge to zero in finite time. In order to demonstrate the generality of the proposed method, experiments are conducted in this paper for non-manoeuvring targets, manoeuvring targets with constant acceleration, and weaving manoeuvring targets. Simulation results demonstrate the proposed approach effectively and robustly intercepts the target with impact angle constrained.

Keywords: *Guidance law, sliding mode control, impact angle constrain, finite time convergence.*

Nomenclature

Latin

V_m – velocity of the missile, m/s

V_t – velocity of the target, m/s

r – relative distance between missile and target, m

a_{ym} – acceleration in yaw directions of the missile, m/s^2

$a_{ym}^{\max}$ – upper bound of the missile acceleration in the yaw direction, m/s^2

a_{zm} – acceleration in pitch directions of the missile, m/s^2

$a_{zm}^{\max}$ – upper bound of the missile acceleration in the pitch direction, m/s^2

a_{yt} – acceleration in yaw directions of the target, m/s^2

a_{zt} – acceleration in pitch directions of the target, m/s^2

Greek

θ_L – elevation angle of the line-of-sight, deg

θ_{Lf} – desired elevation angle of the line-of-sight, deg

φ_L – azimuth angle of the line-of-sight, deg

φ_{Lf} – desired azimuth angle of the line-of-sight, deg

θ_m – angle between V_m and the missile, deg

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φ_m – angle between V_m and the line-of-sight, deg
θ_t – angle between V_t and the target, deg
φ_t – angle between V_t and the line-of-sight, deg

Subscripts
m – missile
t – target

1. Introduction

The proportional navigation guidance (PNG) [1] has been widely used in the tactical guided weapon systems since the 1970s. PNG is a guidance law in which the angular velocity of rotation of the missile's velocity vector is proportional to the angular velocity of rotation of the target's line of sight during the flight. PNG is not only easy to implement with fewer parameters required, but also has a curved trajectory in the first half of the trajectory and a straight trajectory in the second half of the trajectory, which is favourable for attacking the target. However, it has the disadvantage that hit-point missiles need to use normal overload to be affected by the speed and direction of attack of the missile, and is not suitable for attacking manoeuvring targets.

In the recent past, scientists and researchers had done many works on guidance laws such as biased PNG [2, 3], optimal control [4, 5], sliding mode control (SMC) [6, 7] and so on. In [8], a closed-form solution was proposed for the navigation gain selection of the two-stage proportional navigation guidance method in order to realize all possible impact angles without violating the field-of-view constraints. Namhoon Cho and Youdan Kim [9] developed a pure proportional navigation guidance law for stationary targets. It is globally accurate for all directions of flight relative to the target under fully nonlinear engagement kinematics conditions. The authors in [10] presented a new approach to optimal error dynamics based on the optimal control. The method is unique in that it can be applied to any combat target as long as the tracking error for the missile guidance problem is properly defined. Similar to [10], the optimal guidance command proposed by the CK Ryoo et al. [11] was represented by a linear combination of the slope and the step response of the lateral acceleration of the missile. The optimal guidance laws in the form of state feedback for lag-free and first-order lag systems are derived. In [12], the authors proposed a new guidance law with both impact time and impact angle constraints. They first simplified the missile dynamics under a small heading error approximation and derived an optimal guidance law with impact angle constrained for stationary targets.

SMC has been proved to against system uncertainties and external disturbances with strong robustness to parametric uncertainties and disturbances [13, 14]. A new impact time guidance law with field-of-view (FOV) constraints was proposed in [15]. The authors designed a terminal sliding-mode (TSM) controller, which was used to ensure that the switching surface convergence on finite time before interception. Zhang X et al. [16] analysed the impact time control guidance (ITCG) law of missile intercepting into the stationary targets. The ITCG law is derived by using SMG and Lyapunov stability theorem. SMC has the advantages of fast response, insensitivity to parameter variations and perturbations.

In [17], Zhang S et al. developed two cooperative guidance schemes for multiple missiles intercepting the target. The proposed approach can effectively suppress chattering and ensure fast convergence in finite time. An impact time and angle control guidance (ITACG) law for a stationary target based on the non-singular terminal sliding mode control (NTSMC) theory was investigated in [18]. The simulation results show that the proposed guidance law has a good performance even though the missile has a constant acceleration. Kumar S R and Ghose D [19] investigated a guidance scheme, which is based on switching between ITCG and impact angle control guidance (IACG) laws. The proposed approach considered the curvature of the trajectory due to requirement of impact angle. In [20], a 3-D nonlinear ITACG law was developed for intercepting a stationary target. The guidance law is divided into two stages IACG and PNG. J. Zhu et al. in [21] proposed a guidance law that can meet the terminal latitude, longitude, time and angle constraints. The manoeuvring parameters can be directly solved online without time-to-go prediction. The technique is applied to fulfil the terminal constraints, in which a specific TVSM surface is constructed with two unknown coefficients. The study in provided a time-varying sliding mode (TVSM) against stationary and moving targets, which is [22] constructed with two unknown coefficients. A fixed-time guidance law was proposed in [23] to guarantee that the line-of-sight (LOS) angular rates can be steered to zero before the terminal time.

The majority of the guidance laws mentioned above are based on two-dimensional space, whereas real-world environments need to be considered in three-dimensional space. Most of the papers analysed

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guidance laws for stationary targets and does not consider intercepting guidance attacks on manoeuvring targets.

In this paper, a novel IACG law is presented for the missile intercepting manoeuvring targets with a desired impact angle. The main contribution of this paper is as follows:

- A novel three-dimensional guidance law is proposed based on the sliding mode control with impact angle constraints, which guarantees the missile to intercept the target at a desired angle
- The switching surface is proposed to the sliding mode ensure convergence in finite time before intercepting into the target.
- To demonstrate the effectiveness and robustness of the proposed method, experiments are conducted in different scenarios.

The rest of this paper is organized as follows. Section II describes the problem formulation of three-dimensional engagement geometry. Section III presents the proposed IACG law in the presence of an unknown target manoeuvring disturbances. Lyapunov stability is also demonstrated in this section. Section III demonstrates effectiveness of the proposed guidance law by simulating various engagement scenarios. The conclusions and future work are given in Section IV.

2. Problem formulation

The following general assumptions should be stated before the proposed guidance law.

- Assumption 1. The missile dynamics is assumed to be ideal, i.e., no autopilot lag [24].
- Assumption 2. The gravity is ignored during the design of guidance law.
- Assumption 3. The missile and target are considered as a point mass with constant velocities, and the missile has a speed advantage over the target.

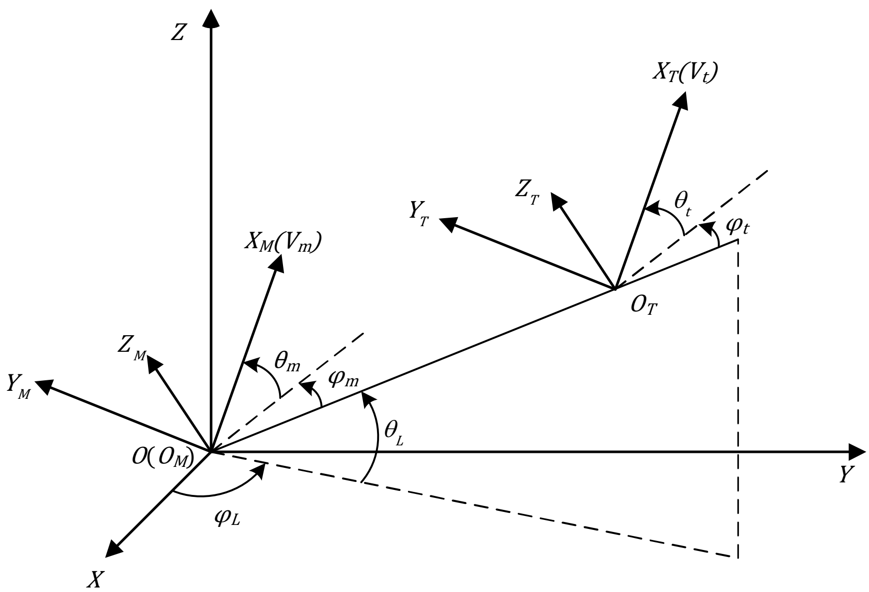

Fig 1. Missile-target interception geometry in three-dimensional space

The missile-target interception geometry in three-dimensional space is shown in Fig 1. $OXYZ$, $O_MX_MY_MZ_M$ and $O_TX_TY_TZ_T$ represents the reference coordinate frame, missile body coordinate frame and target body coordinate frame, respectively. V_m and V_t denote the velocity of the missile and target. $\rho = V_t/V_m$ denotes the speed ratio between the missile and target. r denotes the relative distance between missile and target. θ_L denotes the elevation angle of the LOS. φ_L denotes the azimuth angle of the LOS. θ_m is the angle between V_m and the missile. φ_m is the angle between V_m and the LOS. θ_t is the angle between V_t and the target. φ_t is the angle between V_t and the LOS. a_{ym} and a_{zm} denote the acceleration in yaw and pitch directions of the missile, respectively. a_{yt} and a_{zt} represent the

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acceleration in yaw and pitch directions of the target, respectively. The three-dimensional engagement dynamics can be expressed as follows [23]:

$$\dot{r} = V_t\cos\theta_t\cos\varphi_t - V_m\cos\theta_m\cos\varphi_m \tag{1}$$

$$\dot{\theta}_L = (V_t\sin\theta_t - V_m\sin\theta_m V_m)/r \tag{2}$$

$$\dot{\varphi}_L = (\rho\cos\theta_t\sin\varphi_t - \cos\theta_m\sin\varphi_m V_m)/(r\cos\theta_L) \tag{3}$$

The motion kinematics of missiles and targets can be described as

$$\dot{\boldsymbol{x}}_m = \boldsymbol{C}_L^I[V_m\cos\theta_m\cos\varphi_m, V_m\cos\theta_m\sin\varphi_m, V_m\sin\theta_m]^{\mathrm{T}} \tag{4}$$

$$\dot{\theta}_m = \frac{a_{zm}}{V_m} - \dot{\varphi}_L\sin\theta_L\sin\varphi_m - \dot{\theta}_L\cos\varphi_m \tag{5}$$

$$\dot{\varphi}_m = \frac{a_{ym}}{V_m\cos\,\theta_m} + \dot{\varphi}_L\sin\theta_L\cos\varphi_m\tan\theta_m - \dot{\theta}_L\sin\varphi_m\tan\theta_m - \dot{\varphi}_L\cos\theta_L \tag{6}$$

$$\dot{\boldsymbol{x}}_t = \boldsymbol{C}_L^I[V_t\cos\theta_t\cos\varphi_t, V_t\cos\theta_t\sin\varphi_t, V_m\sin\theta_t]^{\mathrm{T}} \tag{7}$$

$$\dot{\theta}_t = \frac{a_{zt}}{\rho V_m} - \dot{\varphi}_L\sin\theta_L\sin\varphi_t - \dot{\theta}_L\cos\varphi_t \tag{8}$$

$$\dot{\varphi}_t = \frac{a_{yt}}{\rho V_m\cos\,\theta_t} + \dot{\varphi}_L\sin\theta_L\cos\phi_t\tan\theta_t - \dot{\theta}_L\sin\varphi_t\tan\theta_t - \dot{\varphi}_L\cos\theta_L \tag{9}$$

$$\boldsymbol{C}_L^I = \begin{bmatrix} \cos\varphi_t\cos\theta_L & -\sin\varphi_L & -\cos\varphi_L\sin\theta_L \\ \sin\varphi_L\cos\theta_L & \cos\varphi_L & -\sin\varphi_L\sin\theta_L \\ \sin\theta_L & 0 & \cos\theta_L \end{bmatrix} \tag{10}$$

where $\boldsymbol{C}_L^I$ is the transformation matrix from the LOS frame to the inertial reference coordinate.

Differentiating Eq. 1 and Eq. 2 with respect to time yields

$$\ddot{\theta}_L = \frac{\cos\theta_L}{r}a_{zt} - \frac{\cos\theta_m}{r}a_{zm} - \varphi_L^2\cos\theta_L\sin\theta_L - \frac{2\dot{r}\dot{\theta}_L}{r} \tag{11}$$

$$\ddot{\varphi}_L = \frac{\cos\varphi_L}{r\cos\theta_L}a_{yt} - \frac{\mathrm{si}\quad{}_t\sin\varphi_t}{r\cos\theta_L}a_{zt} + \frac{\sin\theta_m\sin\quad{}_m}{r\cos\theta_L}a_{zm} - \frac{\cos\varphi_m}{r\cos\theta_L}a_{ym} + 2\dot{\varphi}_L\dot{\theta}_L\,\tan\theta_L - \frac{2\dot{r}\dot{\varphi}_L}{r} \tag{12}$$

It can be assumed that θ_m, $\varphi_m \neq \pm\pi/2$, $|a_{ym}| \leq a_{ym}^{\max}$, $|a_{zm}| \leq a_{zm}^{\max}$. $a_{ym}^{\max}$ and $a_{zm}^{\max}$ are the upper bound of the missile acceleration in the yaw and pitch directions, respectively. In order to make the missile to intercept the target with the desired impact angles, let θ_{Lf} and φ_{Lf} be the desired elevation angle and azimuth angle of the LOS, respectively. From the nonlinear control point of view, for the homing missile with the desired impact angle, we can define the state variables as $\boldsymbol{x}_1 = [x_{11}, x_{12}]^{\mathrm{T}} = [\theta_L - \theta_{Lf}, \varphi_L - \varphi_{Lf}]^{\mathrm{T}}$ and $\boldsymbol{x}_2 = [x_{21}, x_{22}]^{\mathrm{T}} = [\dot{\theta}_L - \dot{\theta}_{Lf}, \dot{\varphi}_L - \dot{\varphi}_{Lf}]^{\mathrm{T}} = [\dot{\theta}_L, \dot{\varphi}_L]^{\mathrm{T}}$. Based on Eq. 11 and Eq. 12, the dynamics of the system can also be rewritten as the follows

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2 \tag{13}$$

$$\dot{\boldsymbol{x}}_2 = \boldsymbol{M} + \boldsymbol{N} + \boldsymbol{B}\boldsymbol{u} \tag{14}$$

$$\boldsymbol{M} = [\frac{\cos\theta_L}{r}a_{zt}, \frac{\cos\varphi_L}{r\cos\theta_L}a_{yt} - \frac{\sin\theta_t\sin\varphi_t}{r\cos\quad{}_L}a_{zt}]^{\mathrm{T}} \tag{15}$$

$$\boldsymbol{N} = [-\varphi_L^2\cos\theta_L\sin\theta_L - \frac{2\dot{r}\dot{\theta}_L}{r}, 2\dot{\varphi}_L\dot{\theta}_L\,\tan\theta_L - \frac{2\dot{r}\dot{\varphi}_L}{r}]^{\mathrm{T}} \tag{16}$$

$$\boldsymbol{B} = \begin{bmatrix} -\frac{\cos\theta_m}{r} & 0 \\ \frac{\sin\theta_m\sin\varphi_m}{r\cos\theta_L} & -\frac{\cos\varphi_m}{r\cos\theta_L} \end{bmatrix}^{\mathrm{T}} \tag{17}$$

$$\boldsymbol{u} = [a_{zm}, a_{ym}]^{\mathrm{T}} \tag{18}$$

where $\boldsymbol{M}$ is the unknown disturbances caused by target maneuvering.

In this paper, the proposed guidance law enables the missile to intercept the target with desired elevation angle and azimuth angle of the LOS at terminal phase. By designing an appropriate guidance law $\boldsymbol{u}$ such that the $\boldsymbol{x}_2$ converges to zero in a finite time, in the presence of an unknown disturbances $\boldsymbol{M}$.

3. Guidance law design

Before investigating the guidance law proposed in this paper, the following lemma should be stated for convenience:

Lemma [25]: Considering the nonlinear system $\dot{x} = f(x,t),\ x \in R^n$. Suppose that it exists a continuous and positive definite function $V(x)$ is given as

$$\dot{V}(x) \le -\mu V(x) - \lambda V^{\delta}(x) \tag{19}$$

where μ, $\lambda > 0$ and $0 < \delta < 1$ are positive constants. $x(t_0) = x_0$, and t_0 is the initial state. Then, the time of system states arriving at the equilibrium point, T, satisfies the following equation

$$T \le \frac{1}{\mu(1-\delta)} \ln \frac{\mu V^{1-\delta}(x_0) + \lambda}{\lambda} \tag{20}$$

That is, system states are finite-time convergent.

Sliding mode control is also known as variable structure control, is essentially a special class of nonlinear control. The nonlinearity manifests itself as a discontinuity in the control, and unlike other control methods, the structure of the sliding mode control is specifically characterized by an unfixed. The reason for applying SMC to the guidance law in this paper is that SMC has the advantages of fast response, insensitivity to parameter variations and perturbations, no need for online identification of the system, and simple physical implementation. Since the acceleration information of maneuvering targets cannot be fully measured and predicted, the advantages of sliding mode control allow the missile to intercept into the target with unknown maneuvering information while flying with high speeds.

To achieve desired impact angle with desired impact angle and acceptable miss distance, we select the following sliding mode surfaces,

$$\boldsymbol{S} = [s_1, s_2]^{\mathrm{T}} = \dot{\boldsymbol{x}}_1 + k_1 \boldsymbol{x}_1 + k_2 \boldsymbol{f}(\boldsymbol{x}_1) \tag{21}$$

$$\boldsymbol{f}(\boldsymbol{x}_1) = [f(x_{11}), f(x_{12})]^{\mathrm{T}} \tag{22}$$

$$f(x_{1i}) = \begin{cases} R_1 x_{1i} + R_2 x_{1i}^2 \mathrm{sign}(x_{1i}) & |x_{1i}| < \eta \\ |x_{1i}|^R \mathrm{sign}(x_{1i}) & \text{otherwise} \end{cases} \quad i = 1,2 \tag{23}$$

$$R_1 = (2-R)\eta^{R-1} \tag{24}$$

$$R_2 = (R-1)\eta^{R-2} \tag{25}$$

The time derivative of $\boldsymbol{S}$ is given as

$$\dot{\boldsymbol{S}} = \ddot{\boldsymbol{x}}_1 + k_1 \dot{\boldsymbol{x}}_1 + k_2 \dot{\boldsymbol{f}}(\boldsymbol{x}_1) = \boldsymbol{M} + \boldsymbol{N} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{G} \tag{26}$$

$$\boldsymbol{G} = k_1 \boldsymbol{x}_2 + k_2 \dot{\boldsymbol{f}}(\boldsymbol{x}_1) \tag{27}$$

Inspiring from the reaching law in [26], the signum function causes the discontinuous chattering phenomenon. To avoid this, this paper modifies the reaching law to reduce the chattering of sliding mode control yields

$$\dot{\boldsymbol{S}} = -h_1 \tanh(\gamma \boldsymbol{S})^{\alpha} - h_2 \tanh(\gamma \boldsymbol{S})^{\beta} - h_3 \boldsymbol{S} = -h_1 \begin{bmatrix} |s_1|^{\alpha} \tanh(\gamma s_1) \\ |s_2|^{\alpha} \tanh(\gamma s_2) \end{bmatrix} - h_2 \begin{bmatrix} |s_1|^{\beta} \tanh(\gamma s_1) \\ |s_2|^{\beta} \tanh(\gamma s_2) \end{bmatrix} - h_3 \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \tag{28}$$

Based on Eq. 21 and Eq. 28, the following guidance law command can be obtained:

$$\boldsymbol{u} = \left[a_{zm}, a_{ym}\right]^{\mathrm{T}} = -\boldsymbol{B}^{-1}(\boldsymbol{N} + \boldsymbol{G} + h_1 \tanh(\gamma \boldsymbol{S})^{\alpha} + h_2 \tanh(\gamma \boldsymbol{S})^{\beta} + h_3 \boldsymbol{S}) \tag{29}$$

where h_1, h_2 and h_3 are controller gains. α, β and γ are positive constants.

It can be observed from Eq. 29 that the guidance command proposed in this paper does not contain the unknown disturbances $\boldsymbol{M}$ of target acceleration. Therefore, guidance law command $\boldsymbol{u}$ enables the missile to intercept the target with desired impact angle, in the presence of an unknown disturbances $\boldsymbol{M}$.

To verify whether the proposed method can intercept targets under the required constraints, stability analysis of the guidance law is required. Let's consider Lyapunov candidate functions as

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$$V = \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S} \tag{30}$$

$$\begin{aligned} \dot{V} &= \boldsymbol{S}^{\mathrm{T}}\dot{\boldsymbol{S}} \\ &= \boldsymbol{S}^{\mathrm{T}}(\boldsymbol{M}+\boldsymbol{N}+\boldsymbol{Bu}+\boldsymbol{G}) \\ &= \boldsymbol{S}^{\mathrm{T}}\left(-h_1\tanh(\gamma\boldsymbol{S})^{\alpha} - h_2\tanh(\gamma\boldsymbol{S})^{\beta} - h_3\boldsymbol{S}\right) \\ &= -h_1\boldsymbol{S}^{\mathrm{T}}\tanh(\gamma\boldsymbol{S})^{\alpha} - h_2\boldsymbol{S}^{\mathrm{T}}\tanh(\gamma\boldsymbol{S})^{\beta} - h_3\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S} \end{aligned} \tag{31}$$

According to Lemma 1, the Eq. 31 can be modified as follow

$$\begin{aligned} \dot{V} &\le -h_2\boldsymbol{S}^{\mathrm{T}}\tanh(\gamma\boldsymbol{S})^{\beta} - h_3\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S} \\ &\le 2h_3V - 2^{\frac{1+\beta}{2}}h_2V^{\frac{1+\beta}{2}} \end{aligned} \tag{32}$$

when $0 < \beta \le 1$, the system $\boldsymbol{S}$ is finite-time convergent. Therefore, it can be concluded that V converges to zero in finite time. Through the guidance law $\boldsymbol{u}$ in Eq. 29, the missile is able to intercept the target in finite time and meet the kinematic constraints during flight. with desired impact angle to achieve the maximum damage to the target.

4. Simulation results

In this section, to evaluate the effectiveness, superiority and robustness of proposed guidance law, the performance analysis is carried out through three different scenarios. The scenarios are the missile against a constant-velocity target, maneuvering target with constant acceleration and maneuvering target with weaving acceleration.

Table 1. Simulation parameters

Parameters	Initial Conditions		
Missile position $\boldsymbol{x}_m$ = $(x_m(0), y_m(0), z_m(0))$, m	(-10392, 0, -6000)		
Target position $\boldsymbol{x}_t$ = $(x_t(0), y_t(0), z_t(0))$, m	(0, 0, 0)		
Relative distance between missile and target r, m	12000		
Missile speed V_m, m/s	900		
Target speed V_t, m/s	400		
Elevation angle of the LOS θ_L, deg	30		
Desired elevation angle of the LOS θ_{Lf}, deg	35		
Azimuth angle of the LOS φ_L, deg	5		
Desired azimuth angle of the LOS φ_{Lf}, deg	0		
Angle between V_m and the missile θ_m, deg	10		
Angle between V_m and the LOS φ_m, deg	10		
Angle between V_t and the target θ_t, deg	20		
Angle between V_t and the LOS φ_t, deg	180		
Max missile acceleration $a_{ym}^{\max}$ and $a_{zm}^{\max}$, m/s^2	25*g, g=9.8		
Target acceleration a_{yt} and a_{zt}, m/s^2	scenario 1	scenario 2	scenario 3
	0	2g	2g*sin(2t)
Proposed guidance law	$h_1 = 0.2$, $h_2 = 0.4$, $h_3 = 1$, $\alpha = 1.5$, $\beta = 0.5$, $\gamma = 10$, $k_1 = 0.1$, $k_2 = 1$, $R = 0.2$, $\eta = 0.25$		

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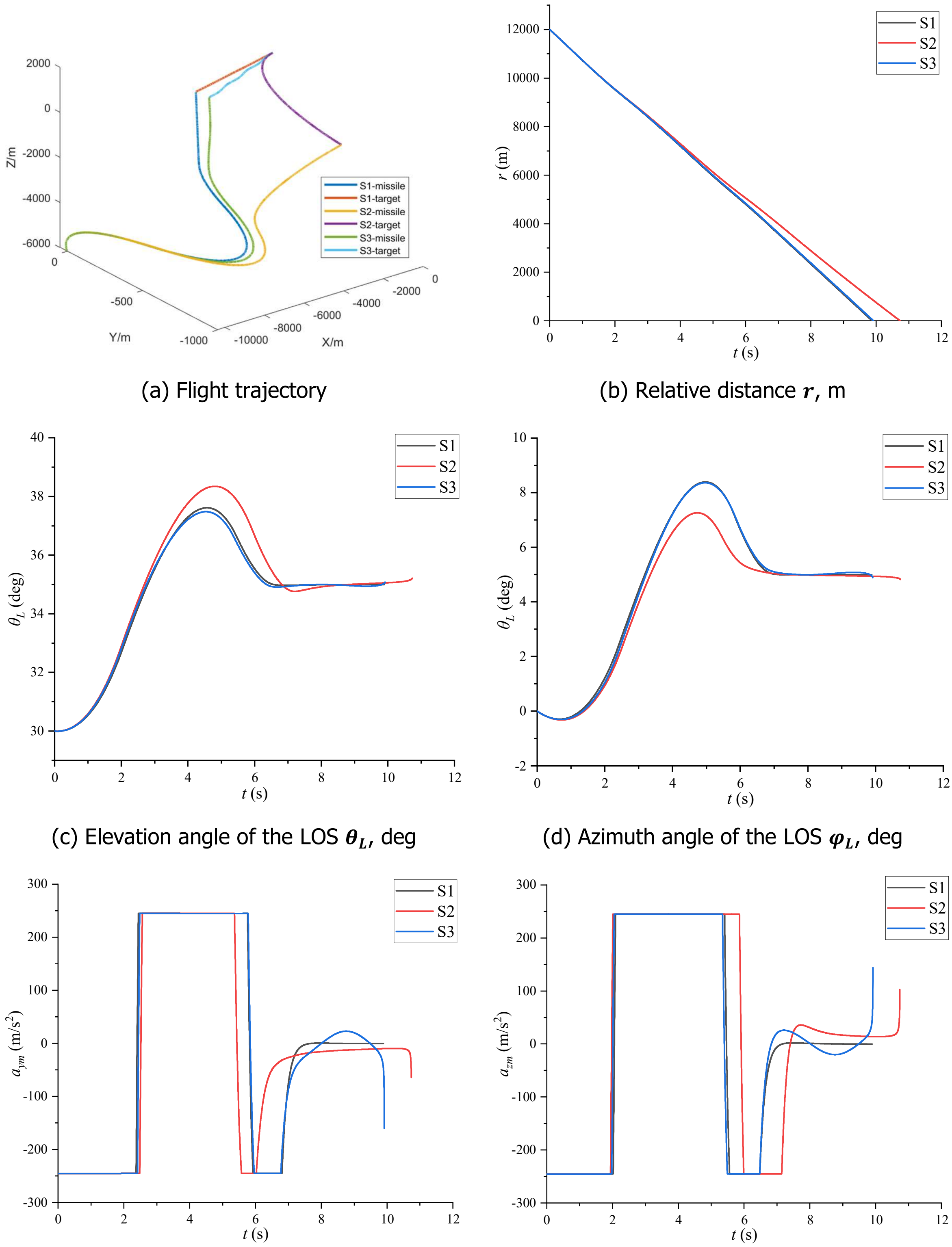

(a) Flight trajectory

(b) Relative distance $\boldsymbol{r}$, m

(c) Elevation angle of the LOS $\boldsymbol{\theta_L}$, deg

(d) Azimuth angle of the LOS $\boldsymbol{\varphi_L}$, deg

(e) Yaw acceleration $\boldsymbol{a_{ym}}$, m/s^2

(f) Pitch acceleration $\boldsymbol{a_{zm}}$, m/s^2

Fig 2. Simulation results for different scenarios under the proposed guidance law

The simulation parameters are listed in Table 1. Note that the for different scenarios, the proposed approach use the same guidance gains for each missile-target engagement. All the engagements are terminated when the relative distance $\dot{r} < 0$ and $r < 1$ m. The simulation are conducted on a notebook with the following configurations: 13th Gen Intel(R) Core(TM) i7-13700H 2.40 GHz, 16.00 GB of RAM, and an NVIDIA GeForce RTX 4060 GPU.

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The simulation results are shown in Fig. 2, which depicts flight trajectories of missile and target, relative distance between missile and target, elevation angle of the LOS, azimuth angle of the LOS, yaw and pitch acceleration of the missile. The legends S1, S2 and S3 represent scenario 1, 2 and 3 respectively. It can be observed from Fig. 2a that the method proposed in this paper is able to intercept the target within a miss distance in all three scenarios. Also, the relative distance between the missile and the target decreases linearly with time until it intercepts the target, as shown in Fig. 2b. Under initial conditions, the missile is able to accurately intercept the target in less than 11 s, which ensured timeliness. Moreover, the missile achieves the desired elevation and azimuth LOS angle in both directions are illustrated in Fig. 2c and Fig 2d, respectively. Elevation angle of the LOS $\boldsymbol{\theta_L}$ and Azimuth angle of the LOS $\boldsymbol{\varphi_L}$ increase during an initial period of 0-5 s and then decrease rapidly. $\boldsymbol{\theta_L}$ and $\boldsymbol{\varphi_L}$ reach the desired impact angle at the moment of about 7 s, then change in a small rang around desired impact angle. This proves that the guidance law proposed in this paper enables the missile to intercept the target with the desired impact angle in a finite time, and the error of impact angle is within acceptable limits. To intercept the target, it is evident from Fig. 2e and 2f that the missile Yaw acceleration $\boldsymbol{a_{ym}}$ and pitch acceleration $\boldsymbol{a_{zm}}$ demand higher accelerations in the initial phase, then the acceleration rapidly converges to zero. The reason for the rapid increase in acceleration command at the last moment is to allow the missile to intercept the target at the desired impact angle. As can be seen from the above simulation and figures, the guidance law proposed in this paper enables the missile to intercept a target with unknown acceleration at a desired impact angle in a finite time. The miss distance and error of impact angle are within acceptable limits.

Table 2. Guidance performance

Statistics	Scenario 1	Scenario 2	Scenario 3
Miss distance, m	0.58	0.52	0.20
Impact time, s	9.88	10.73	9.92
Error of elevation LOS angle $\lvert\theta_L - \theta_{Lf}\rvert$, deg	0.21	0.01	0.08
Error of azimuth LOS angle $\lvert\varphi_L - \varphi_{Lf}\rvert$, deg	0.18	0.01	0.11
Energy consumption $\mathrm{sum}\lvert a_{ym}\rvert$, m/s^2	1.57e+06	1.68+06	1.72e+06
Energy consumption $\mathrm{sum}\lvert a_{zm}\rvert$, m/s^2	1.82e+06	1.60e+06	1.64e+06

Further, in order to verify the accuracy and validity of the proposed guidance law from a more comprehensive perspective, the miss distance, impact time, error of elevation LOS angle $\lvert\theta_L - \theta_{Lf}\rvert$, error of azimuth LOS angle $\lvert\varphi_L - \varphi_{Lf}\rvert$, energy consumption $\mathrm{sum}\lvert a_{ym}\rvert$ and energy consumption $\mathrm{sum}\lvert a_{zm}\rvert$ are record in Table 2. As can be seen from the first row in Table 2, the missile intercepts the target with miss distance less than 1m, which is acceptable in practical applications and meets the experimental settings. For both non-manoeuvring target moving at a constant speed and manoeuvring target with unknown acceleration, the guidance law proposed in this paper can accurately intercept the target. In all three scenarios, the missile is able to intercept the target in a short time and in less than 11 s, as shown in the second row in Table 2. From the error of elevation and azimuth LOS angle, the missile can intercept the target at the desired impact angle with an acceptable error. Moreover, this paper also counts the energy consumption of the missile during flight, and from the table it can be seen that the energy consumed by the missile in intercepting the manoeuvring target with constant acceleration is higher than that manoeuvring target with weaving acceleration.

5. Conclusions

In this paper, a novel three-dimensional guidance law is proposed based on the sliding mode control with impact angle constraints, which guarantees the missile to intercept various types of targets with unknown acceleration at a desired impact angle. The guidance command does not include target acceleration can make the missile to intercept the target with unknown manoeuvring. It has been demonstrated by three different simulation scenarios that the guidance law proposed in this paper enables the missile to intercept the target with miss less than 0.58 m, and the desired impact angle is less than 0.21 deg in all cases. The simulation results demonstrate the effectiveness, superiority and robustness of proposed guidance law. In our future work, the guidance law can be studied combined

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with control-guidance integration to take into account aerodynamic, mass, and velocity variations during missile flight.

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