HiSST: 3rd International Conference on High-Speed Vehicle Science Technology
14 -19 April 2024, Busan, Korea

Asymptotic analysis of heat and mass transfer performance of a microscale wavy wall

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Abstract

Real surface of hypersonic vehicles could be rough rather than smooth at different scales, and non-equilibrium heat and mass transfer to rough walls is a critical phenomenon that needs to be evaluated in design of the thermal protection system. Take the wavy wall as a typical example, this paper analyzes the heat and mass transfer performance of a surface with microscale roughness by using the asymptotic methods. Analytical results for the local heat flux and chemical reaction rate increments are derived for different non-equilibrium degrees and wave steepnesses, and the corresponding scaling laws are discussed and validate by data from the direct simulation Monte Carlo method. This theoretical study provides a convenient and practical method to evaluate and correct the chemical reaction–diffusion performance of rough walls, and is potentially useful for development of new numerical and experimental techniques in dealing with the complex boundaries in engineering problems.

Keywords: *rough wall, microscale transport, non-equilibrium reaction, rarefied gas effects, DSMC*

1. Background

New-generation hypersonic vehicles flying at high altitudes face a long-duration, high-temperature and high-speed flow environment, even the so-called non-ablative surface could be covered by microscale rough structures such as wrinkles and holes due to slight ablation or corrosion [1, 2], as shown in Fig. 1. The microscale roughness could also result from the imperfect manufacture precision. The size of the roughness elements is on the order of submicron or micrometers which is sufficiently smaller than the thickness of boundary layer [3]. Therefore, the microscopic roughness has an almost negligible effect on the macroscopic flow, and what is noteworthy is its effects on the heat and mass transfer performance near the wall. For example, the roughness will lead to local heat flux peak, resulting in non-uniform ablation and then further variation in wall morphology; the roughness will also increase the active area of gas-surface interaction, which is equivalent to change the rates of surface reaction rate and mass loss. Since the scale of the roughness can be comparable to the mean free path of the ambient gas molecules, the rarefied gas effects should be evaluated in studying the corresponding reaction-diffusion process. The related heat and mass transfer problem is challenging because of its multi-scale and multi-physics nature, as well as the complex boundary, and by using the current techniques, it is extremely costly, if not impossible, to either numerically or experimentally capture the detailed features in the microstructure. As a result, it is urgent to carry out a theoretical modeling analysis based on appropriate physical simplification and reasonable mathematical approximation. By revealing the mechanism of the microscopic flow in roughness elements, we could ascertain the

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corresponding aerodynamic heating and gas-surface reaction features and further provide a reference for the engineering design.

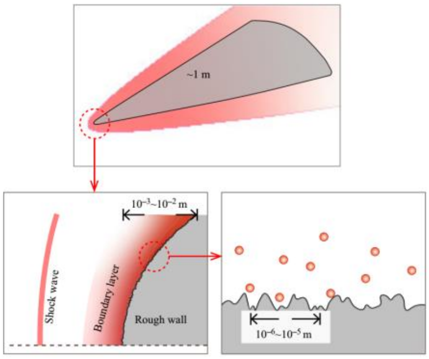

Fig 1. The microscale roughness on the surface of a hypersonic vehicle.

2. Idea, Method and Results

Since the roughness scale is much smaller than the characteristic scale of the macroscopic flow field, the two-dimensional and quasi-one-dimensional simplified models will be effective to describe the heat and mass transfer performance of microscopic rough walls, and even the flow velocity could be ignored near the wall. Following our preliminary work [4] recently on the rectangular and triangular roughness structures, this paper considers a sinusoidal wavy wall which is more general in mathematics but also more complex in analysis, as shown in Fig. 2. The heat is transferred from the high temperature flow field to the cold wall, and the wall surface is chemically active and induces a reaction-diffusion process. The heat and mass transfer problem is solved by using several asymptotic techniques for some typical and practical situations.

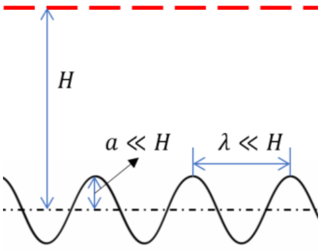

Fig 2. Geometric model of a wavy wall

First, when the steepness of the wavy wall is small, the perturbation method is used to solve the temperature field and concentration field, which leads to the heat flux distribution along the wall, the local peak heat flux, the average heat flux and the effective chemical reaction rate. These results are compared to the counterparts in a corresponding smooth wall case.

Secondly, when the steepness of the wavy wall is large or moderate, the reaction-diffusion process is divided into two parts, the first outside the roughness elements and the second inside. While the external process is one-dimensional and can be solved analytically, the internal process is equivalently described by a quasi-one-dimensional model, and the asymptotic solutions are obtained under several conditions. The equivalent reaction rate of the rough wall surface is further deduced and the rarefied gas effects on the internal reaction-diffusion feature are discussed. It is also found from the results that the chemical performance of a rough wall could be homogeneously equivalent to a smooth wall with an effective reaction rate, and the correction factor obeys different scaling laws under different non-equilibrium degrees and geometric conditions.

Finally, the direct simulation Monte Carlo (DSMC) method [5] is also employed to conduct a series of simulation cases to validate the theoretical modeling results. Fig. 3 shows the theoretical prediction of the reaction rate multiplying factor $\boldsymbol{\Phi}$ and its comparison with the DSMC results under various non-equilibrium degrees ($\boldsymbol{Th}$) and wave steepnesses ($\boldsymbol{\delta}$). A good agreement can be observed between the theoretical and numerical results in a large range of parameters.

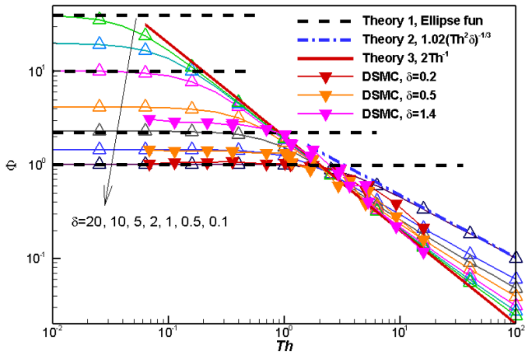

Fig 3. Comparison between the theoretical and the numerical results of multiplying factor under various non-equilibrium degrees ($\boldsymbol{Th}$) and wave steepnesses ($\boldsymbol{\delta}$).

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