Trajectory Optimization Method for Dual Mode Scramjet Engine Vehicle Using a Unstart Informed Combustion Model

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Abstract

Hypersonic air-breathing vehicles, which implement ramjet and scramjet propulsion systems, demonstrate greater specific thrust in comparison to rocket engines. Additionally, these vehicles do not require oxidizers and are thus poised to be the future of hypersonic transportation. The trajectory of hypersonic aircraft plays a crucial function in enhancing flight efficiency because of the inherent connection between combustion efficiency and aerodynamic performance. Therefore, multiple studies have aimed to optimize the trajectory of hypersonic vehicles. Unfortunately, previous studies have not successfully addressed the possibility of disastrous operational breakdowns in the propulsion system, including unstart. In this research, we suggest a trajectory optimization method for hypersonic aircraft that presents a solution for preventing operational failure. To this end, a comprehensive combustion analysis model was developed and combined with the sequential convex programming to obtain optimal trajectory.

Keywords: Dual-mode scramjet, Trajectory optimization, Sequential Convex Programming, Combustion.

Nomenclature (Tahoma 11 pt, bold)

α – angle of attack
γ – flight path angle
γ_{heat} – ratio of specific heats
θ – pitch angle
ρ – density
D – drag
$\mathrm{G_Q}$ – function including forcing terms in differential equation for flow quantity Q
g – gravity of earth
h – altitude
$\mathrm{I_{yy}}$ – Moment of Inertia
L – Lift
M – Mach number
M_c – Mach number at combustor entrance
$\mathrm{M_i}$ – Mach number at isolator entrance
$\mathrm{M_o}$ – Moment
m – mass
$\mathrm{m_f}$ – fuel flow rate
p – pressure
p_c – pressure at combustor entrance
$\mathrm{p_i}$ – pressure at isolator entrance
Q – Physical quantity
q – angular velocity
r – downrange
T – temperature
$\mathrm{T_h}$ – thrust
u – velocity of internal flow
V – flight velocity
x – axial coordinate

1. Introduction

Hypersonic air-breathing propulsion systems, such as ramjets or scramjets, offer superior specific impulse compared to rocket engines and eliminate the need for oxidizer. These characteristics allow hypersonic vehicles to achieve remarkable efficiency in long-range flight. Consequently, extensive research efforts have been devoted to advancing hypersonic air-breathing engine technology over the past few decades [1].

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To address the significant wave drag associated with high Mach numbers, researchers typically define a high-altitude operating envelope. However, this strategy is challenged by the low static pressure at high altitudes, which simultaneously reduces combustion efficiency and thrust. As a result, the trajectory must be tailored to the specific vehicle configuration and fuel type to achieve optimal performance.

As a result, many studies have been conducted to optimize the trajectory of hypersonic aircraft[2,3]. However, current trajectory optimization methods do not account for catastrophic engine failures such as unstart and blow-out. If these failures are included in the trajectory, they can cause sudden loss of thrust or combustion instability, so it is necessary to derive a trajectory that can exclude them.

In this study, a new trajectory optimization methodology for hypersonic air-breathing engine is proposed to overcome the limitations of the presented trajectory optimization problems. A comprehensive combustion and aerodynamic analysis model was developed. An additional methodology that predicts the ram-scram transition and unstart was implemented. Subsequently, to perform trajectory optimization considering unstart exclusion, we introduced a novel surrogate modeling technique. Finally, the optimal trajectory was obtained using the sequential convex optimization method with the developed combustion model.

2. Methodology

2.1. Dual mode scramjet vehicle model

For this study, the aircraft model used was based on a geometry similar to that of the X-51A, a scramjet aircraft from the United States. Fig. 1 illustrates the model. The model has a total length of 4.6 m and a total weight and fuel mass of 671 kg and 120 kg, respectively[1]. The X-51A aircraft uses hydrocarbon fuel, but for simplicity of combustion analysis, hydrogen fuel was used in this study.

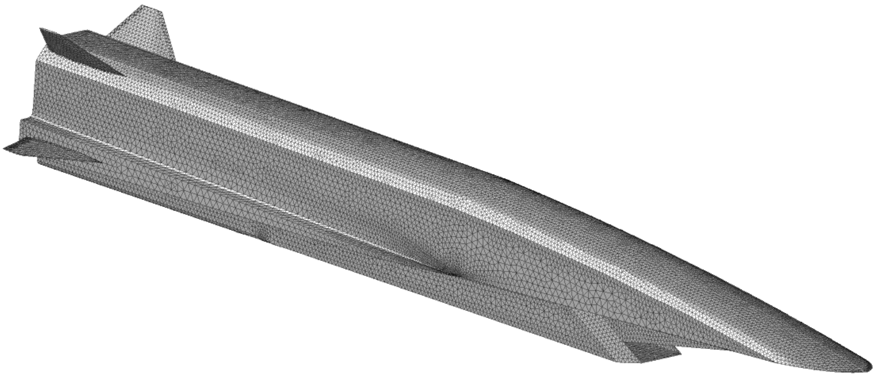

Fig 1. Dual-mode scramjet vehicle model

2.2. Combustion analysis

In order to analyze the combustion phenomena, which include reactions and fuel mixing, a quasi-one-dimensional analysis that allows rapid computation is used in this study. In this study, robust ordinary differential equations developed by Cha[4] are used. The ram-scram combustion phenomenon can be reliably analyzed by transforming the governing equation with chemical species fractions into the form shown in Eq. 1. The Radau method, a technique for solving ordinary differential equations, was used to solve this equation.

$$\frac{1}{Q}\frac{dQ}{dx} = \frac{G_Q}{1-M^2}, Q = u, T, \rho, p, M \tag{1}$$

Fig. 2 illustrates the validation outcomes of the present combustion model compared to Boyce's experimental setup [5]. The experiment involved a rectangular duct equipped with plates featuring an expansion angle of 1.72° at both the top and bottom. The duct had an initial height of 47 mm and a consistent width of 100 mm. Hydrogen fuel was introduced via a centrally positioned strut injector under flow conditions of Mach 2.47, temperature of 1025 K, and static pressure of 59 kPa. The investigation encompassed two equivalence ratios, namely 0.38 and 0.58. For both equivalence ratios, we can see that the results of our model follow the trends of the experiment well. Note that our model predicts a slightly higher pressure than the experiment. This is due to the quasi-one-dimensional nature of the analysis. In the experiment, the fuel is injected into the center of the channel, so combustion is dominant in this region. Therefore, the pressure in the center is higher than at the wall where the pressure is measured. However, for this model, the pressure is assumed to be uniform throughout the combustion chamber. Because of this assumption, the measured pressure in the model is higher than in the experiment.

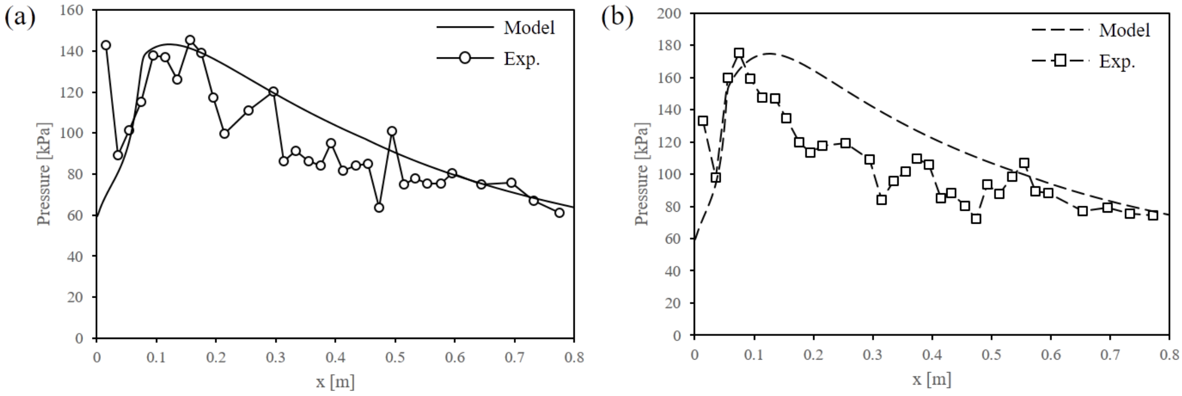

Fig 2. Validation results for combustion model (a) ϕ=0.38, (b) ϕ=0.58

The approach of Heiser and Pratt was used to consider the effects of the pre-combustion shock train within the isolator that occurs in ram mode[6]. Using adiabatic assumptions and conservation of momentum, the rate of pressure rise in the isolator is calculated. The calculation is performed using equation (2) below.

$$\frac{p_c}{p_i} = 1 + \gamma_{heat}^2 M_i^2 - \gamma_{heat} M_i M_c \sqrt{\frac{1+\left[\frac{\gamma_{heat}-1}{2}\right] M_i^2}{1+\left[\frac{\gamma_{heat}-1}{2}\right] M_c^2}} \tag{2}$$

Combining the above isolator analysis method with the combustion analysis method, a dual-mode scramjet operating mode classification method is proposed. The proposed operating mode classification procedure is shown in Fig 3. Through this process, the operating modes are categorized into the following four: Unstart, Ram, Scram, and Blowout. First, the combustion analysis is performed assuming the scram mode, and then the ram mode analysis is performed when the sonic point occurs. In the scram mode analysis, if the fuel is not burned, it is classified as blow out, and if the fuel is burned, it is classified as scram. Whether the fuel is burned or not is determined by whether the mass fraction of water in the final combustion products exceeds a certain value. If a subsonic solution occurs under all conditions during the ram mode analysis, it is classified as unstart. Otherwise, it is ram mode. Of the above four modes, blowout and unstart are operational failures and should be avoided during operation.

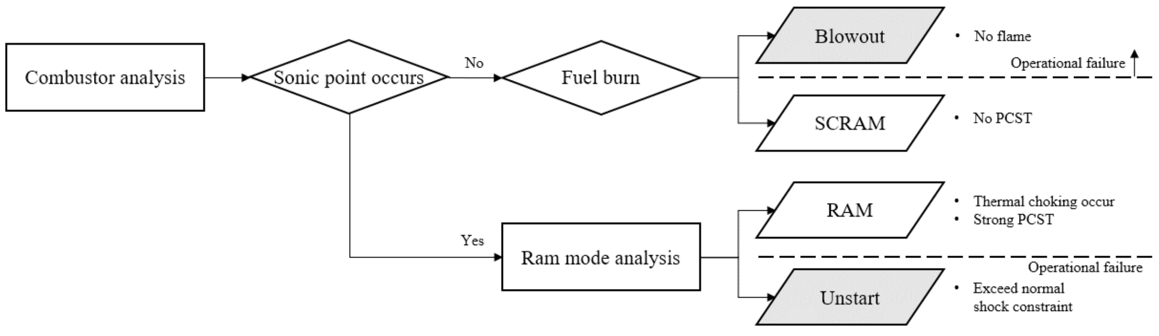

Fig 3. Operational mode classification process

Combustion analyses were performed to classify the ram-scram transition, unstart, and blow-out modes. Fig. 4 depicts a classification of operating modes based on altitude, Mach number, and fuel consumption. The classification shows that the operating mode can be determined by the amount of fuel when other conditions are fixed. In this study, the unstart and blow-out modes were constrained with the upper and lower limits of the fuel quantity.

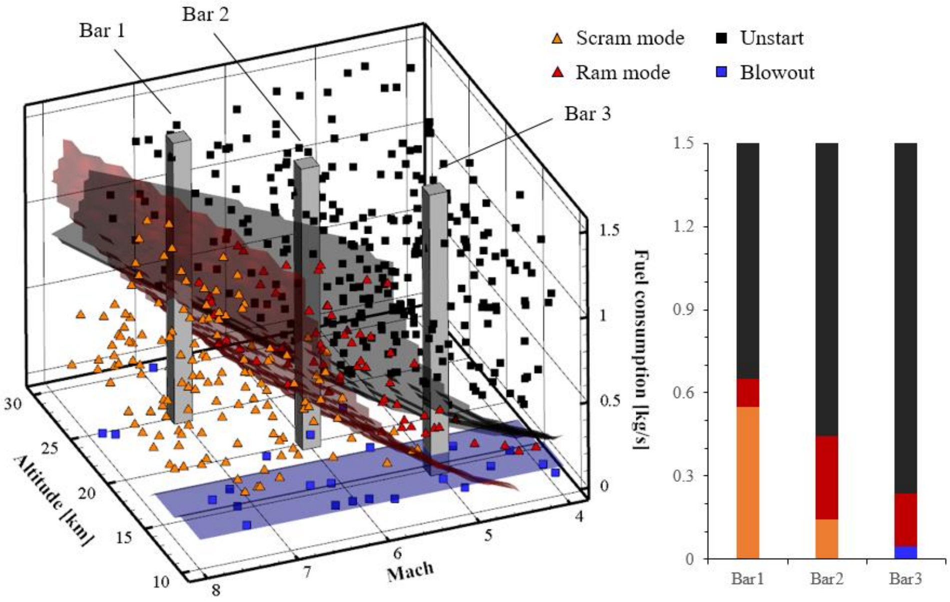

Fig 4. Scramjet engine operation mode classification

2.3. Aerodynamic analysis

In a hypersonic air intake engine, the flow into the combustion chamber is compressed and transmitted by a shock wave at the inlet. To calculate the combustion chamber inlet flow under operating conditions, the intensity of the shock wave generated by a two-dimensional ramp was calculated using the reduced-order model introduced by Dalle[7]. The thrust generated by the nozzle was calculated using the control volume method for air intake engines.

In addition, aerodynamic forces acting on the aircraft are required for trajectory optimization. The local surface inclination Method[8] was used to analyze this. Using this method, lift, drag, and moments were calculated and tabulated.

2.4. Inlet/Nozzle analysis

This study focuses on analyzing inlet and nozzle performance in a two-dimensional geometry. The interaction between ramp shock waves and the cowl can lead to significant boundary layer formation, potentially causing inlet unstart[9]. To avoid this, the vehicle operates under conditions where shock waves don't interfere, assuming no interference between inlet ramp shock waves. Other reflected shock waves are disregarded for simplicity. Inlet performance is evaluated using the oblique shock wave equation to sequentially calculate flow changes at each shock wave and determine the quantity of flow delivered to the isolator.

The nozzle expands the flow to produce thrust, which is determined using a control volume approach. This method necessitates computing the mass flow rate and velocity at the nozzle exit. In this study, a control volume-based thrust calculation method was employed, incorporating a correction factor to account for incomplete expansion. This correction adjusts the gross thrust term in the equation, reflecting the ratio of the thrust coefficient of a truncated nozzle, similar in length to the one studied, to that of a fully expanded nozzle.

Determining thrust force values based on various operating conditions involves a considerable computational burden. Rapid calculations for the multitude of conditions are impractical. Hence, this study adopts Gaussian process modeling to represent both thrust and aerodynamic forces[10]. Gaussian process modeling is ideal for deterministic computer modeling. To efficiently sample calculating points across the range, Latin hypercube sampling, a multivariate modeling sampling method, is employed[11].

2.5. Trajectory optimization procedure

In this paper, we used the sequential convex programming approach for trajectory optimization of the dual mode scramjet vehicle model. Sequential convex programming is an optimization technique used to solve nonlinear optimization problems by breaking them down into a series of smaller convex subproblems using convexification and discretization [12].

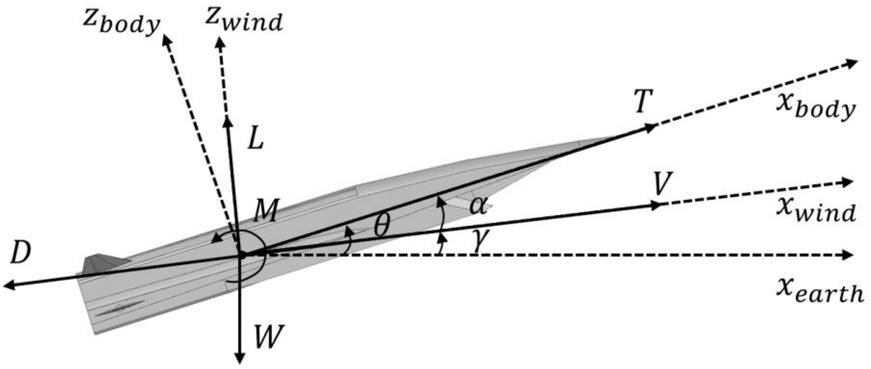

Fig 5. Free body diagram of 3-DOF dynamics

3-DOF dynamics with 7 states(Mach number, altitude, angle of attack, angular velocity, pitch angle, mass and downrange) and 2 inputs(control surfaces deflection angle and thrust) was employed at the trajectory optimization and it was discretized using trapezoidal rule. Fig. 5 shows a free body diagram of 3-DOF dynamics.

$$\dot{\mathrm{V}} = \frac{(T cos\alpha - D)}{m} - g sin\gamma, \qquad \dot{\mathrm{h}} = \mathrm{V}\sin\gamma, \qquad \dot{\alpha} = q + \frac{q\cos\gamma}{V} - \frac{L + T_h \sin\ \alpha}{mV}$$

$$\dot{q} = \frac{M_o}{I_{yy}}, \quad \dot{\theta} = q, \quad \dot{m} = -m_f, \quad \dot{r} = \mathrm{V}\cos\gamma \tag{2}$$

The nonlinear optimization problem has been formulated to identify the operational limits and optimize the mission trajectory for the vehicle model. The constraints of the optimization problem, including the dynamics (Eq. 2) and fuel flow limits (Fig. 2), consist of the upper and lower bounds of state and input variables along with the trust region constraint for each iteration of the SCP. In addition, for performance analysis of the vehicle model, trajectory optimization was conducted for minimum time and maximum payload missions. The objective functions for these problems are as follows.

$$\mathrm{J} = \begin{cases} \min \quad t_f & for\ minimum\ time\ problem \\ \min -m(t_f) & for\ maximum\ payload\ problem \end{cases} \tag{3}$$

While the original problem may have a feasible solution that meets all constraints, the discretized convex problem may occasionally fail to do so[13]. Creating a reference trajectory that satisfies constraints in the initial iteration can be especially challenging. To tackle this, a two-stage solution-finding procedure is utilized. In the first stage, a trajectory that fulfills all constraints is generated. Subsequently, in the second stage, the solution trajectory is derived by refining the trajectory generated in the first stage, using it as the initial trajectory.

3. Results and discussions

In order to verify the application of the performance analysis methodology and the two-step robust trajectory optimization methodology presented in this study, we checked the converging history of the trajectory optimization process, as shown in Fig. 6. The example problem is set to a final range of 600 km, initial and final altitudes of 20 km, and an initial Mach number of 5. The trajectory is sought to minimize the flight time. The first step of exploring physically feasible trajectories results in the blue trajectory. The scramjet aircraft is operated at high altitude to avoid high wave drag. However, this can result in poor combustion performance. The aircraft used in this study uses a two-ramp inlet geometry, which means it cannot gain enough thrust at high altitude and enters at low altitude. Once it has sufficient Mach number, it climbs back to high altitude. In the second step, which is to find the time-minimizing trajectory, the trajectory is more ascending than the physical trajectory represented by the blue line. This trajectory initially uses a large amount of fuel to accelerate. This results in a higher velocity at an earlier point in time and a lower final arrival time.

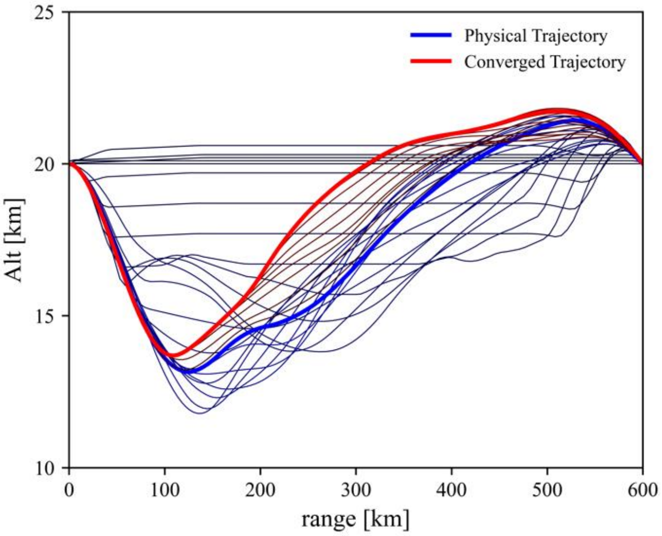

Fig 6. Converging history of trajectory optimization

Fig. 7 shows the two trajectories optimized for minimum time and maximum payload with a range of 1200 km. The initial Mach number and altitude are 5 and 15 km respectively, with a final altitude of 25 km. The duration of flight for the minimum time-optimized trajectory was 518 seconds, whereas the maximum payload-optimized trajectory's flight time was 544 seconds. Both trajectories were cruised at high altitude to avoid the huge wave drag. The time-optimised trajectory consumed more fuel in initial stage to enable faster acceleration.

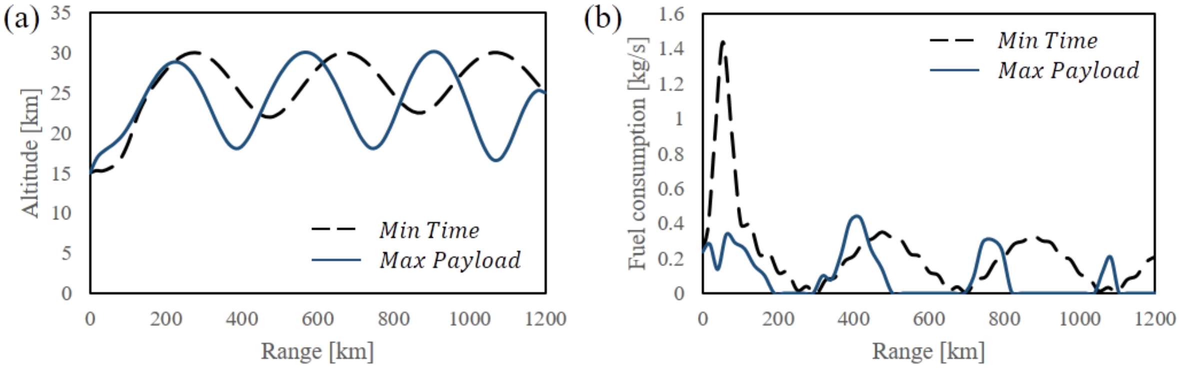

Fig 7. Optimized trajectory for two objectives (a) Altitude, (b) Fuel consumption

4. Conclusion

This study demonstrates the effective integration of combustion analysis, aerodynamic analysis, and SCP in the trajectory optimization of hypersonic air-breathing vehicles. The application of Gaussian process classifiers for combustion mode discrimination emerges as a powerful tool in navigating the complex operational range of these vehicles. Recognizing and incorporating unstart and blowout as constraints is vital, and our methodology supports this inclusion. The optimization results consistently emphasize the significance of higher altitudes, primarily driven by aerodynamic considerations, for achieving objectives of time or fuel mass minimization. These findings contribute to advancing the field of hypersonic flight trajectory optimization and offer valuable insights for future research and practical applications in the development of hypersonic air-breathing vehicles.

Acknowledgments

This work was supported by the Scramjet Combined Propulsion System Specialized Research Laboratory (No. 16-106-501-035) of Korea.

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