



Local surface inclination method calculations for the HEXAFly-INT vehicle

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Abstract

To improve the preliminary design of high-speed vehicles, the usage of analytical tools could avoid the necessity of CFD simulations in the early stages, decreasing the time necessary for design iterations, as long as the results have a reasonable agreement with data from CFD or experiments. In this work, results obtained from analytical calculations based on the local surface inclination method and tangent-wedge method, allied to the Eckert formulation to estimate the friction in a high-velocity turbulent boundary layer, are compared with CFD data related to the HEXAFly-INT geometry. The analytical lift coefficient, c_L , fits well with CFD data. The analytical drag coefficient, c_D , is underestimated if only the pressure influence on the drag is considered, disregarding the viscous effects. Considering the wall shear stress by means of estimating the friction coefficient in a high-velocity turbulent boundary layer improves the estimative of the analytical drag coefficient and, consequently, the L/D ratio. The L/D ratio behaves within the maximum and minimum limits presented by ESA.

Keywords: *lift, drag, waverider, semi-analytical calculations*

Nomenclature

Latin

A – Area

AoA – Angle of Attack

c_p – Pressure coefficient

c_D – Drag coefficient

c_L – Lift coefficient

F – Total force vector

M – Mach number

N – Number of triangles in the STL file

\hat{n} – Normal vector

p – Static pressure

q – Dynamic pressure

V – Flow velocity

Greek

β – Shock wave angle

γ – Ratio of specific heats

σ – Normal stress

τ – Wall shear stress

θ – Flow deflection angle

Subscripts

i – i -th triangle from STL file list

∞ – Freestream flow properties

1. Introduction

Simulating hypersonic flows is a costly computational task due to high-temperature flow phenomena, such as excitation of the vibrational mode in diatomic molecules, dissociation, and ionisation. Several Navier-Stokes solvers are available for this kind of calculation, such as LAURA [1], VULCAN [2], FUN3D

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[3], ANSYS Fluent [4], DLR TAU-code [5], among others. Such solvers are computationally expensive, requiring high computational power.

Relying only on CFD (computational fluid dynamics) solvers to develop aircraft geometry is very time-consuming, demanding high CPU/GPU power with geometry configurations particularly when established during preliminary design that may not satisfy all mission requirements. Also, the MacLeamy curve shows us how important it is to choose a good design since late changes have a great cost impact [6]. Low-order methods can be used for rapid estimates of pressure distribution, providing a reliable alternative way to calculate aerodynamic coefficients such as lift and drag. The use of CFD may hence be limited to further investigations with only a select few geometries that meet predefined criteria.

Some authors use a linearized theory, such as the thin-airfoil theory or slender-body theory, assuming that the flow is linear and inviscid [7]. These methods can provide a good approximation of the aerodynamic coefficients for certain geometries in a limited range of Mach numbers.

There are a few different low-order panel methods that can be used for supersonic and hypersonic flow. For instance, the vortex lattice method (VLM) is a panel method that uses a lattice of vortex filaments to represent the flow around an aircraft or rocket [8]. This method can provide a good approximation of the aerodynamic forces and pressure distribution, but it doesn't account for viscous effects. VLM is typically used for low to moderate Mach numbers, in the range of $M < 5$. This is because the method is based on linear theory, and it breaks down at high Mach numbers when the flow becomes highly nonlinear. At very high Mach numbers, shock waves and other compressibility effects can have a significant impact on the aerodynamic forces, and the vortex lattice method may not be accurate enough to capture these effects.

Rapid estimates of pressure distribution can be obtained using local surface inclination methods [7] with different approaches, such as Newtonian and modified Newtonian methods, mostly used for blunted body geometries which generate normal shock waves, and tangent-wedge method for slender geometries producing plane oblique shockwaves.

In this investigation, a local surface inclination method associated with a tangent-wedge method is applied to evaluate c_L , c_D , the lift-to-drag relation, and the position of the center of pressure for a hypersonic vehicle. The external surface is discretized with a triangular mesh, and the aerodynamic force acting in each triangle is estimated without calculating the flow field around the vehicle [9]. The Eckert formulation [10] is used to estimate the friction coefficient in a high-velocity turbulent boundary layer, improving the estimative of the drag coefficient and, consequently, the L/D ratio.

The results are compared to literature data from the HEXAFLY-INT at Mach numbers 7.5, 6, 4, and 2, and 32 km altitude [11, 12, 13], allowing to assess the accuracy of the approach and propose engineering corrections for future applications.

2. Methodology

Using the tangent-wedge method [6, 8], the pressure over the i -th triangle is estimated based on the oblique shock wave theory [14]:

$$\frac{p_i}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1} [(M_\infty \beta_i)^2 - 1] \quad (1)$$

$$\tan \theta_i = 2 \cot \beta_i \left[\frac{(M_\infty \sin \beta_i)^2 - 1}{M_\infty^2 (\gamma + \cos 2\beta_i) + 2} \right] \quad (2)$$

where θ_i , β_i , γ , M , and p are, respectively, the local element inclination concerning the freestream flow field, shock wave angle for a given element inclination (θ_i), the specific heat ratio, Mach number, and static pressure. The subscript ∞ refers to freestream flow properties and i refers to the i -th triangle. The pressure is considered zero in the expansion region, which is the worst-case scenario for pressure drag. The assumption adopted in this study is that air behaves as an ideal gas with a constant specific heat ratio.

The local surface inclination method considers only the normal stress, σ , acting on the surface (pressure). To obtain a better assessment of drag, the wall shear stress, τ , which includes the viscous effects, is considered by using the Eckert formulation to estimate the friction in a high-velocity turbulent boundary layer [10].

Therefore, the i -th pressure and friction coefficients are given by

$$c_{p,i} = \frac{p_i / p_\infty - 1}{0.5 \gamma M_\infty^2} \quad (3)$$

$$c_{f,i} = \frac{T_{e,i}}{T^*} 0.451 / \ln^2 \left[0.056 Re_i \left(\frac{T_{e,i}}{T^*} \right)^{\kappa-1} \right] \quad (4)$$

where κ is a constant equals 0.67 [10], Re_i is the Reynolds number based on the distance s_i of i -th triangle centroid to the vehicle leading-edge and the airflow properties at the boundary-layer edge, which has a subscript e . T^* is the Eckert reference temperature [10].

$$T^*_i = T_{e,i} \left(0.5 + 0.044 r M_{e,i}^2 + 0.5 \frac{T_w}{T_{e,i}} \right) \quad (5)$$

$$Re_i = \rho_{e,i} V_{e,i} s_i / \mu(T_{e,i}) \quad (6)$$

$$\mu(T) = \mu_0 \frac{110.4 + T_s}{110.4 + T} \left(\frac{T}{T_s} \right)^{1.5} \quad (7)$$

Dynamic viscosity μ is estimated by Sutherland's law, and the airflow properties at the boundary-layer edge are estimated using the oblique shock wave theory [14] for the incident region, $\theta > 0$, or the Prandtl-Meyer relation [14] for the expansion, $\theta < 0$ (**Fig. 1**).

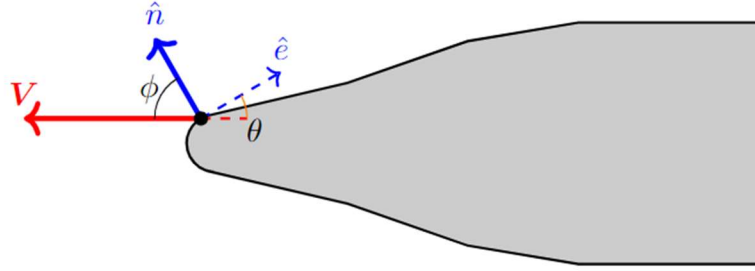


Fig 1. Scheme for calculation of flow deflection angle.

$$\phi = \cos^{-1} \left(\frac{v \cdot \hat{n}}{V_\infty} \right) \quad (8)$$

$$\theta = \frac{\pi}{2} - \phi \quad (9)$$

where v and V_∞ are the vehicle velocity vector and magnitude, respectively. \hat{n}_i is the vector normal to the surface of the i -th triangle, and θ is the flow deflection angle.

The total force acting over the vehicle is given by

$$F = -q_0 \sum_{i=1}^N (c_{p,i} \hat{n}_i A_i + c_{f,i} \hat{e}_i A_i) \quad (8)$$

$$\hat{e}_i = (\text{proj}_{\hat{n}_i} v - v) / |\text{proj}_{\hat{n}_i} v - v| \quad (9)$$

$$q_0 = 0.5 \rho_\infty V_\infty^2 \quad (10)$$

where $\text{proj}_{\hat{n}_i} v$ is the projection of v onto \hat{n}_i , q_0 is the freestream dynamic pressure, ρ_∞ is the undisturbed flow density. A_i is the i -th triangle area, and \hat{e}_i is the shear stress direction.

The pressure force-weighted average of centroid positions relative to the vehicle leading-edge, x_i , is used to estimate the center of pressure, X_{cp} .

$$X_{cp} = \frac{\sum x_i p_i A_i}{\sum p_i A_i} \quad (11)$$

Once the total force has been calculated, the step-by-step procedure to estimate the lift and drag coefficients for a given angle of attack and sideslip angle is presented in detail in Rolim et al. [9].

The local surface inclination method based HipeX tool [9] is used with the HEXAFLY-INT geometry (**Fig. 2**) [11, 12] with zero control surface deflection angle on both sides to estimate the aerodynamic coefficients for an altitude of 32km and flight Mach numbers of 7.5, 6, 4, and 2. Results calculated with this analytical tool are compared with previously published Hexafly-INT higher order CFD calculations from CIRA and the European Space Agency (ESA) [13]. Standard HipeX implementation and an expanded one that includes viscous effects estimation are analysed.

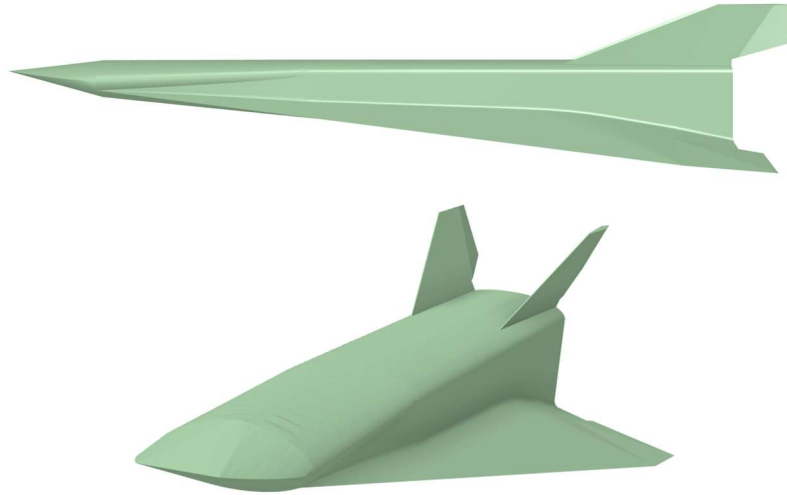


Fig 2. HEXAFLY-INT geometry [11, 12].

The sideslip angle was set equal to zero degrees and the angle of attack (AoA) varied from minus five to fifteen degrees.

3. Results

The results from the analytical tool (HipeX) to estimate the lift and drag coefficients applying the tangent-wedge method [9] are compared with the CFD results [13] for flight Mach number 7.5 (**Fig. 3**). The analytical method presents a well-fit lift coefficient. The drag coefficients show a similar behavior for both methods, increasing when the angle of attack is incremented. However, the drag coefficient is underestimated by the analytical method if only the pressure component is considered. Including the viscous effects allows a much better representation of the drag coefficient, **Fig. 3a**. Consequently, the Lift-to-Drag ratio presented better quantitative results by accounting for viscous effects, staying within the maximum and minimum limits observed in the CFD reference curves (**Fig. 3b**). Additionally, the variation of the analytically estimated L/D with angle of attack is similar to the CFD reference.

The simplified estimate of the longitudinal position for the center of pressure presented a relative error of less than 10% compared with the CFD, for the case of flight Mach number 7.5 (**Fig. 4**).

Comparing the results for flight Mach numbers 2, 4, and 6 (**Fig. 5**), drag and lift coefficients are close to CFD data for hypersonic Mach number ($M_\infty = 6$) (**Fig. 5a**) and high supersonic Mach number ($M_\infty = 4$) (**Fig. 5b**). For low supersonic ($M_\infty = 2$) the results show a greater difference concerning CFD (**Fig. 5c**).

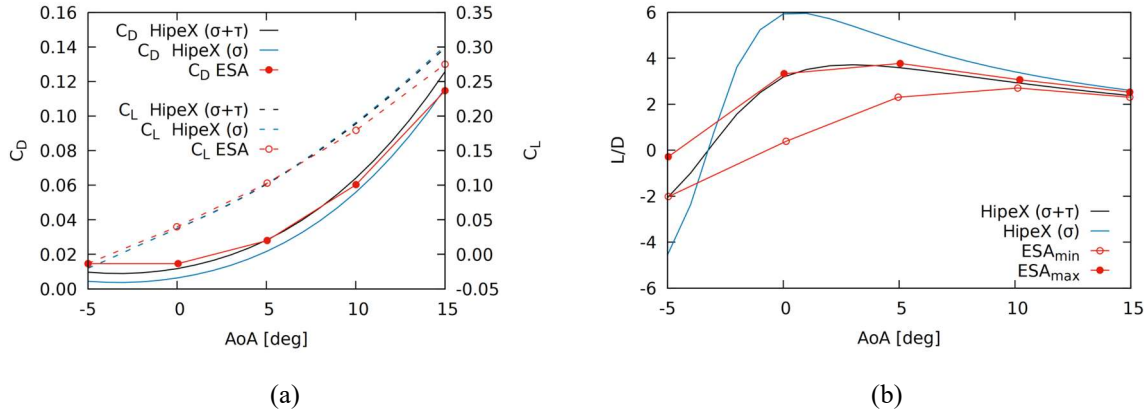


Fig 3. (a) Drag and lift coefficients for HEXAFly-INT; and (b) Analytical and CFD (HEXAFly-INT) comparison of L/D Ratio.

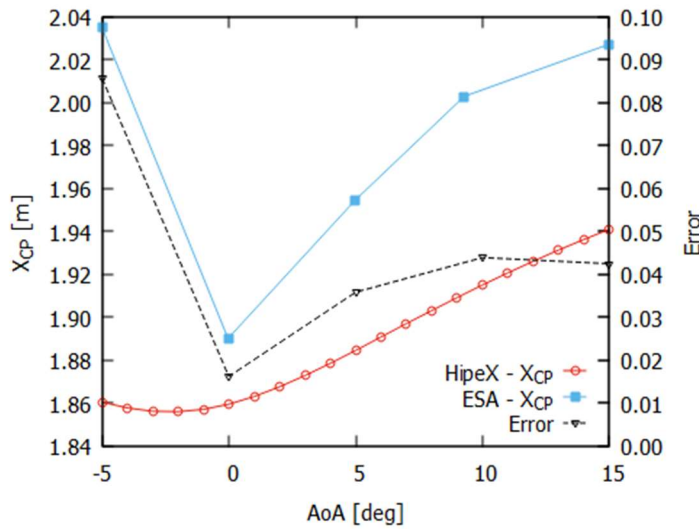


Fig 4. x-coordinate for the center of pressure, from the leading-edge, and the relative error $\left(\frac{|X_{CP_{ESA}} - X_{CP_{HiPeX}}|}{X_{CP_{ESA}}}\right)$.

The aerodynamic efficiency (Lift-to-Drag Ratio) behaves within ESA limits for Mach number 6, close to them for Mach number 4, and off-limits for Mach number 2 (**Fig. 6**). Although the L/D ratio for Mach number 2 is worse compared with high supersonic and hypersonic Mach numbers ($M_\infty = 4$ and $M_\infty = 6$ - **Fig. 6**; and $M_\infty = 7.5$ - **Fig. 3b**), it reasonably estimates the coefficients for lowest angles of attack (**Fig. 5**).

The worst behavior for the lowest supersonics is expected as the bow shock will change the entire flow field around the vehicle. The simplified method considers that each triangle will be directly affected by the undisturbed airflow, missing the upstream flow field effect, which is regarding in CFD analysis.

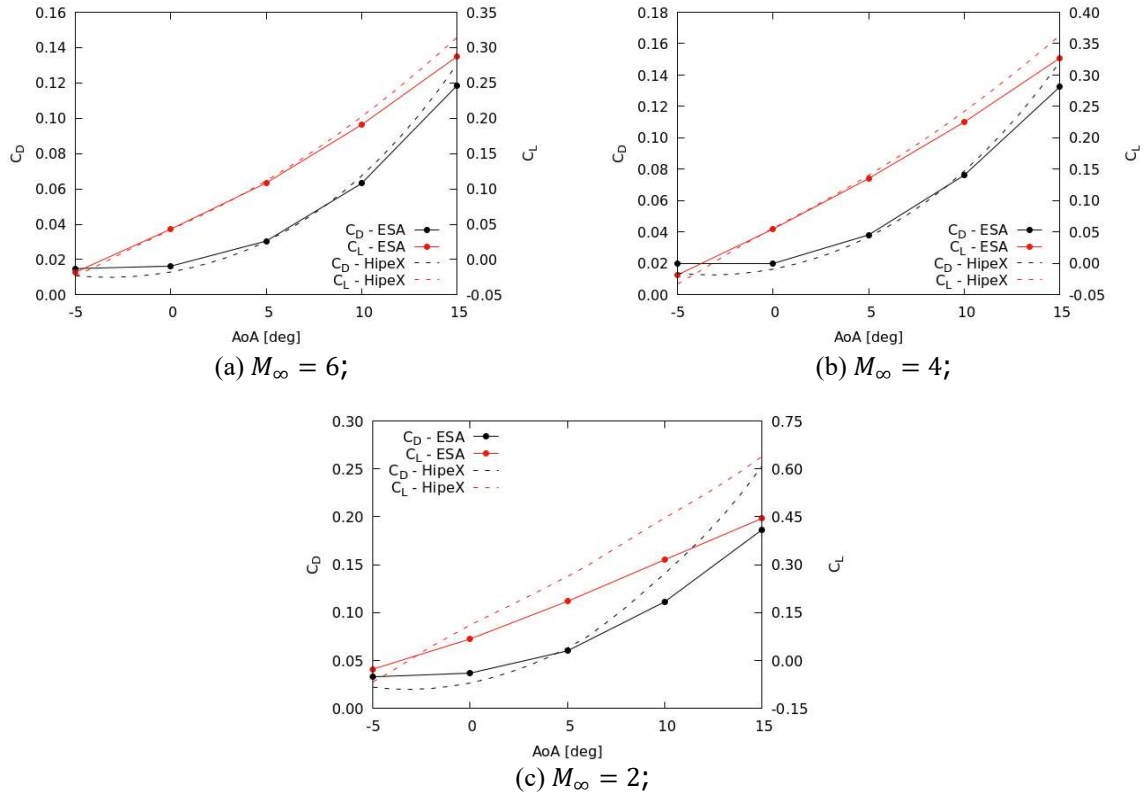


Fig 5. Drag and lift coefficients for HEXAFly-INT.

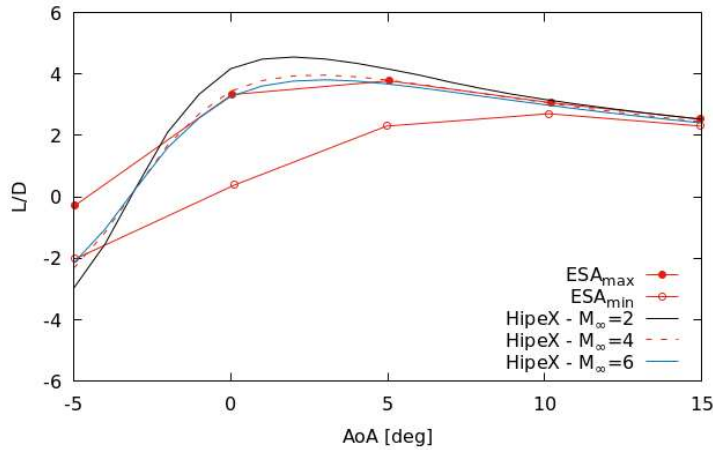


Fig 6. Analytical and CFD (HEXAFly-INT) comparison of Lift-to-Drag ratio for flight Mach numbers 2, 4, and 6.

Although the Lift-to-Drag ratio presented better results for the highest Mach numbers, the longitudinal computed position of the center of pressure deviates less from CFD results for the lowest Mach numbers (**Fig. 7**). Using the local surface inclination method to estimate the X_{CP} seems to be a conservative approach once the longitudinal position of the center of pressure is closer to the vehicle's leading edge which is the worst-case scenario for the control analysis.

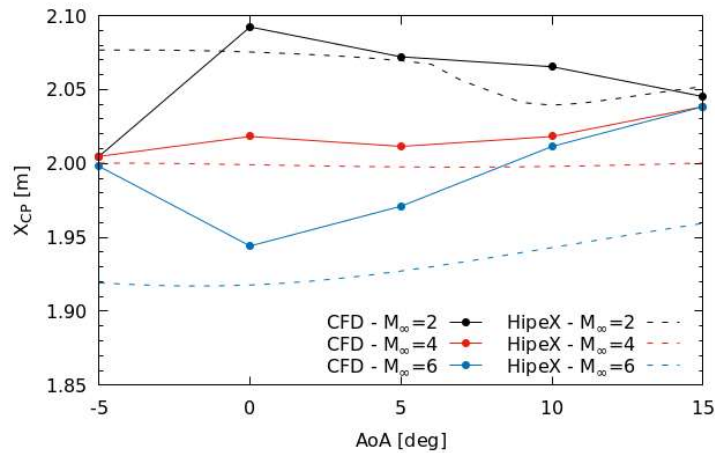


Fig 7. Analytical and CFD (HEXAFLY-INT) comparison of the longitudinal center of pressure for flight Mach numbers 2, 4, and 6.

4. Conclusions

Two analytical approaches to estimate the drag and lift were compared with CFD data for the HEXAFLY-INT geometry. For Mach number 7.5 at 32km altitude, the analytical lift coefficient fits well with CFD data, while the drag coefficient is underestimated if only the pressure influence on the drag is considered, disregarding the viscous effects. Considering the wall shear stress by estimating the friction coefficient in a high-velocity turbulent boundary layer improves the estimation of the analytical drag coefficient and, consequently, improves the lift-to-drag ratio estimation. The L/D ratio varies within the maximum and minimum limits observed in the CFD reference and presents a similar trend regarding the angle of attack. The simplified method to estimate the center of pressure presented a relative error of less than 10% compared with the CFD data for Mach number 7.5. Drag and lift coefficients for Mach number 2 presented the worst results, particularly for AoA greater than 5, but the center of pressure estimative is closer to the CFD for the lowest Mach numbers, e.g., two and four. Therefore, this approach could be used to estimate the aerodynamic coefficients for a preliminary hypersonic vehicle design, since the results show a good agreement with the CFD data.

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References

1. Cheatwood, F.M.; Gnoffo, P.A. User's manual for the Langley aerothermodynamic upwind relaxation algorithm (LAURA). NASA Langley Research Center, 1996. Accessed Sep 27 2023

2. White, J.A.; Morrison, J.H. A pseudo-temporal multi-grid relaxation scheme for solving the parabolized Navier–Stokes equations. In: *14th Computational Fluid Dynamics Conference*. Norfolk, 1999. <https://doi.org/10.2514/6.1999-3360>
3. Anderson, W.K.; Bonhaus, D.L. An implicit upwind algorithm for computing turbulent flows on unstructured grids. *Computers & Fluids*, 23(1):1-21, 1994. [https://doi.org/10.1016/0045-7930\(94\)90023-X](https://doi.org/10.1016/0045-7930(94)90023-X)
4. ANSYS. *Fluent User’s Guide*, 2019.
5. Schwamborn, D.; Gerhold, T.; Heinrich, R. The DLR TAU-code: Recent applications in research and industry. In: *European Conference on Computational Fluid Dynamics*. Netherlands, 2006.
6. CURT. *Collaboration, Integrated Information and the Project Lifecycle in Building Design, Construction and Operation*, 2004.
7. Anderson J.D. *Hypersonic and High Temperature Gas Dynamics*. McGraw-Hill, New York, 1989.
8. NASA. NASA SP-405: Vortex-Lattice Utilization. NASA Langley Research Center, 1976.
9. Rolim, T. C.; Cintra, S. C.; Pellegrini, M. M. da C. Development and Application of Computational Tool Using Local Surface Inclination Methods for Preliminary Analysis of Hypersonic Vehicles. *Journal of Aerospace Technology and Management*, 12, e2120, 2020. <https://doi.org/10.5028/jatm.v12.1123>
10. Eckert, E. R. G. Engineering Relations for Heat Transfer and Friction in High Velocity Laminar and Turbulent Boundary Layer Flow Over Surfaces With Constant Pressure and Temperature. *Journal of the Aeronautical Sciences*, Vol. 22, 1955, PP. 565-587.
11. J. Steelant et al., “Flight Testing Designs in HEXAFLY-INT for High-Speed Transportation,” in 1st International Conference on High-Speed Vehicle Science and Technology (HiSST), Moscow, Russia, Nov. 2018.
12. S. Di Benedetto et al., “The high-speed experimental flight test vehicle of HEXAFLY-INT: a multidisciplinary design,” *CEAS Space Journal*, vol. 13, no. 2, pp. 291–316, Apr. 2021, doi: 10.1007/s12567-020-00341-5.
13. Schettino A., Pezzella G., Maini M., Di Benedetto S., Fernandez-Villace V., Steelant J., Choudhury R., Gubanov A. and Voevodenko N., ‘Aerodynamic Database of the HEXAFLY-INT Hypersonic Glider’, *CEAS Space Journal*, Feb. 2020, doi.org/10.1007/s12567-020-00299-4.
14. Anderson J.D. *Modern compressible flow - with historical perspective*. McGraw-Hill, New York, 1990.