



# **Design of Gravity-Assist Trajectories for Thrust Pinch in an Integrated Aircraft-Propulsion System**

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# **Abstract**

The transition thrust pinch occurs in an over-under Turbine Based Combined Cycle (TBCC) engine when the parallel operation of the turbojet and ramjet is not possible, under which the vehicle cannot continue accelerating because of the poor net thrust. Instead of turning to developing newly revolutionary propulsion systems, this paper proposes an alternative strategy to solve this problem via trajectory optimization. A loosely coupled integrated vehicle powered by a ramjet engine is considered, while the turbojet is reasonably simplified into the various initial conditions to represent mode transition. By applying Gauss Pseudospectral Method (GPM) to the integrated aircraft/propulsion system model, optimal trajectories under different constraints are calculated. The result shows that this strategy is to some extent beneficial for the transition thrust pinch situation. In addition, both the dynamic pressure and the combustor temperature constraints will largely affect the scale of improvement so that those constraints should be taken into consideration to make this strategy productive. Although the total energy of the vehicle decreases because of the cost to reallocate the balance between the kinetic and potential energies, this strategy is reasonable and effective with propulsion systems at hand.

**Keywords**: TBCC; Aircraft/Propulsion Integration; Transition Thrust Pinch; Trajectory Optimization; GPM

### **Nomenclature**

E – specific energy g – acceleration due to gravity h – altitude Ma – Mach number q – dynamic pressure t – time V – velocity

### **1. Introduction**

The Turbine Based Combined Cycle (TBCC) engine [1] is a promising candidate for the hypersonic airbreathing propulsion system because of its capability of maintaining acceptable thrust and fuel consumption over the entire flight spectrum. Compared with the development degree of turbojets and ramjets/scramjets, the transition from the former to the latter is still a relatively critical issue that has not been completely solved yet.

From the perspective of engine designers, one of the challenges in mode transition is that there are still two kinds of gaps between gas turbine engines and scramjets. One is so-called Mach gap [2] and the other one is called thrust gap [3], or described as transition thrust pinch [4]. Methods to this problem can be concluded into two categories. One is using auxiliary power units or developing revolutionary engines [4] as used in Mach gap situation, the other is completing mode transition while accelerating, which needs wider operation ranges for both engines and seems difficult under technological level available nowadays. Those two kinds of method existed to improve the thrust pinch situation all rely on new propulsion systems, at least not ones at hand. Successful mode transition should avoid any of the gaps mentioned above. The solutions that have been mentioned yet all seem to be improving engines, which is not that easy and takes a lot. Instead of turning to developing engines, we are looking forward to an approach to solving this problem under technology available at hand in

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this paper. Looking back to all those solutions above which are basically developed on propulsion system itself, turning to hypersonic vehicle of which propulsion system is the subsystem will provide us with a wider view.

When aircraft/propulsion integration [5] is taken into consideration, thrust level as well as acceleration ability has a close connect to aircraft performance, while the research object changes from TBCC engine to the whole aircraft. The general aircraft performance problem is kind of a way to find the balance exists between the potential and kinetic energy change of the aircraft, the energy dissipated against the drag, and the energy derived from the fuel [6]. From this point of view, the transition thrust pinch equals to the kinetic energy pinch.

This work aims to offer a possible plan to improve transition thrust pinch situation through trajectory optimization, without bothering to develop new revolutionary engines. Firstly, transition thrust pinch is explained and a brief introduction to a general optimal control problem is given. Then, the analysis models including the engine model and the aircraft model are mathematically shown as well as the total energy concept for analysis use in this paper and the numerical solution. Finally, several trajectories are optimized based on different constraints.

# **2. Materials and Methods**

Without loss of generality, a ramjet is selected as the high-speed engine in a combined cycle engine in this paper to simplify the problem. Physically modeling the engine is a significant part to not only distinguish the existing researches in which the ramjet is described as thrust coefficient, but also involve aircraft/propulsion integration into consideration so that the potential and kinetic energies can be taken into consideration.

As the focus is trajectory and the relevant transformation between kinetic energy and potential energy, the change of gravitational acceleration, sideslip and unsteady aerodynamic effects are neglected in this paper for this little influence. Therefore, a point-mass aircraft [7] is chosen as the vehicle.

For a general hypersonic vehicle, the sum of its kinetic energy and potential energy makes up the total energy of the vehicle. Because the weight change of the vehicle along the relative short trajectory we concerned about (only 1% of the vehicle) in this paper is small enough to be neglected, the specific energy [6] instead of total energy is used in this paper in order to make the result more visual, as formulated in Eq. 1.

$$
E = h + V^2/g \tag{1}
$$

As mentioned in the Introduction, the specific trajectory optimization problem is solved by using Gauss pseudospectral method. The optimization was implemented with first-order derivatives for both the constraint Jacobian and the gradient of the objective function. In particular, orthogonal collocation of the dynamics is then performed at the Legendre-Gauss (LG) points. For trajectory optimization problem in this paper, 16 nodes (i.e., 14 LG points) is used to approximate the trajectory. Note that the node amount is chosen to get relatively accurate approximation of the solution as well as maintaining a reasonably sized NLP, and could be adjusted according to balance between precision and cost. Furthermore, the optimal solutions in this paper are obtained with a tolerance of  $10^{-6}$ .

### **3. Results**

### **3.1. Existence of Solution to Speed Increment**

[Fig. 1](#page-2-0) shows that this method is effective to the transition thrust pinch situation and that speed increment could be gotten by the optimal trajectory instead of flying along the constant dynamic pressure path. What's more, for the same vehicle, one with wider allowable turns to be longer and narrower. The wider operational dynamic pressure zone allowable, the higher terminal speed the vehicle would reach to, which is obviously advantageous at least for the staging of combined cycle engines. What's more, the optimal results also show that the terminal energy state of every single optimal trajectory seems to be almost the same level as the gravitational potential energy the vehicle at the altitude of 30km. However, the trajectory with widest dynamic pressure zone ends up with lowest total energy while obtaining fastest speed, as shown in [Fig. 1](#page-2-0) (lower left) with denser contours of constant specific energy. In general, the source of the energy for propulsion system is the power-plant fuel. Now that the additional chemical energy of fuel into the system for all optimal trajectories is the same, the difference in terminal total energy could also show that the total amount of kinetic energy and potential energy is not conserved, and that gaining speed increment would pay the price. The reason is obvious that several non-conservative forces exist in this system and cause dissipation.



**Fig. 1 Optimal flight envelope with respect to dynamic pressure**

# <span id="page-2-0"></span>**3.2. Solution Under Temperature Constraint**

Under different combustor stagnation temperature, the optimal trajectories with maximum terminal speed display different shapes under the same dynamic pressure limitation temperature, as shown in [Fig. 2a](#page-2-1)).







### **Fig. 2 Optimal results under different combustor stagnation temperature**

<span id="page-2-1"></span>All trajectories firstly get to the minimum dynamic pressure and end at the maximum dynamic pressure. The optimal results show that the higher permitted combustor stagnation temperature is, the faster terminal speed can reach to through trajectory optimization. What's more, in this case, only trajectories with combustor stagnation temperature higher than 3000K could get speed increment through trajectory optimization, the rest will not improve the transition pinch and the total energy even decreases a lot along the optimal trajectory. This phenomenon can be explained as follows. The equivalent ratio to some extent means the magnitude of energy injection into the system. If this magnitude is too small to make up for the loss caused by the vehicle drag, the total energy of the system will definitely decay. In addition, the trajectory with lowest permitted combustor stagnation temperature (1500K in this case) shows an odd trend, along which the vehicle turns to have some fluctuations. What needs to be pointed out is that the temperature limitation ranges from 1500K, under which the corresponding equivalent ratio is little and reflects the extreme lean oil situation, to 3500K, above which the material and active cooling technology cannot stand any more.

To figure out the reason why that odd trend mentioned above happens, the simulation about acceleration along different trajectories is performed, as shown in [Fig. 2b](#page-2-1)). It is noted that although the initial states and flight condition of all trajectories are the same, the accelerations are different because of different angles of attack. The transition thrust pinch is obvious here for the negative initial accelerations. From the result, we can find the trough of wave in [Fig. 2a](#page-2-1)) corresponds to the point that the wave crest in [Fig. 2b](#page-2-1)) (upper right), which means that the vehicle for the first time comes nearest to zero. However, this trial does not get acceleration for the vehicle, so that the vehicle gets to climb and gets ready to dive to accelerate again. In addition, we can get another primary conclusion that the vehicle with the transition thrust pinch gets speed increment by diving through trajectory optimization. If the energy injected into the system is not enough, the vehicle may fly along a trajectory that has some fluctuations, which may not help to actually gain speed increment. It should be pointed out that all trajectories in [Fig. 2b](#page-2-1)) seem to get acceleration in the end, even for those do not get speed increment in [Fig. 2a](#page-2-1)). However, the terminal states of those optimal trajectories are limited by the fixed fuel amount, which means vehicles along those trajectories fail to continue operation for running out of fuel. This setting is realistic because the fuel supplied to the vehicle cannot be infinite. What's more, although all trajectories finally manage to get positive accelerations, those with low combustor stagnation temperature allowable cannot get speed increment, compared to the initial flight speed in this case, and the speed will only fade away.

### **4. Conclusion**

The goal of this study is to find possible solutions for the transition thrust pinch in an over-under TBCC engine without developing revolutionary propulsion systems. Trajectory optimization via GPM is adopted to try to gain speed increment through adjusting the balance between the kinetic and potential energies of the ramjet-powered vehicle, while initial net thrust of the vehicle is barely positive. The results and analyses show that the trajectory optimization is indeed helpful to have speed increment and beneficial for the transition thrust pinch. Trajectories with wider dynamic pressure zones allowable could get more speed increment while the rest of conditions remained the same. With combustor temperature limitation considered, trajectories that are able to withstand higher temperature will get more speed increment, while ones with lower temperature allowable even could not improve the thrust pinch situation, although the trajectories might have several fluctuations in order to make the acceleration as near to zero as possible.

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