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Effect of reacting flow on flutter at hypersonic flight speed

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Abstract

In the flutter analysis of flight vehicle structures moving at moderate supersonic speeds, the air is usually considered as perfect gas, and the unsteady aerodynamic pressure can be calculated through linearised aerodynamic theory. At high supersonic and hypersonic speeds, two effects become significant. First, aerodynamic nonlinearity essentially affects limit cycle amplitudes and should be taken into account. Second, in the hypersonic flight the flow at high temperatures produced by the bow shock is a reacting mixture and cannot be considered as a perfect gas. While the aerodynamic nonlinearity effect was excessively studied in literature, the reacting flow effect with respect to flutter was not studied previously. In this paper we analyse the changes in the flutter theory produced by the effect of chemical reactions. As an example, we analyse the flutter boundary of a flat skin panel in the reacting flow.

Keywords: *flutter, hypersonic flow, reacting flow*

1. Introduction

Flight vehicles moving at high speeds can experience flutter of various structures, such as wings, control surfaces, and skin panels. For theoretical analysis of the flutter boundary, the stability of coupled linearised structural and aerodynamic models is studied. In the structure is unstable, the danger of flutter can be estimated by the limit cycle analysis, which requires nonlinear structural model, whereas the aerodynamic nonlinearity in most cases can be neglected [1, 2]. At high supersonic speeds, however, aerodynamic nonlinearity becomes significant and affects the limit cycle amplitude (but not the flutter boundary). Also, at hypersonic speeds the air after the bow shock, consisting at normal conditions of O_2 and N_2 molecules (the presence of other components is minor and is neglected in this study), experiences ionisation and becomes a mixture of various species, namely, O, N, NO, O_2 and N_2, whose composition, being in general non-equilibrium, spatially evolves.

The first effect of the hypersonic flow, aerodynamic nonlinearity, has been studied in literature by many authors [3, 4, 5, 6, 7, 8]; this effect can yield larger limit cycle amplitude and change of the bifurcation type from supercritical to subcritical Hopf bifurcation. However, the second effect, non-equilibrium state of the air, to our knowledge, has not been studied previously, while it can affect not only the limit cycle amplitude, but also flutter boundary. In this study we investigate the impact of the non-equilibrium reacting air flow on flutter boundary.

2. Effect of reacting flow on unsteady aerodynamics

2.1. General considerations

It is known that at large supersonic speeds, $M \to \infty$, plane section law is valid, which implies that in the coordinate system connected to the gas (in which the gas is at rest), during the motion of a slender body the flow particles move normal to the direction of the body motion, whereas velocity component co-directed with the body is negligible (fig. 1). This means that the body acts in each plane occupied by

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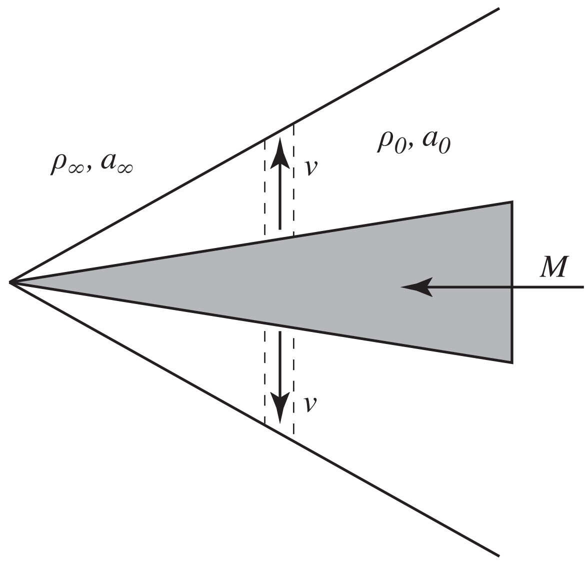

Fig 1. Motion of the gas perpendicular to the flight direction.

the gas as a two-dimensional piston. Taking closed-form solution for the piston, a pressure perturbation expressed through the body velocity is obtained:

$$p = p_\infty \left(1 + \frac{\gamma - 1}{2}\frac{v}{a_\infty}\right)^{\frac{2\gamma}{\gamma-1}}.$$

In linearised form, the pressure perturbation takes the form

$$p = p_\infty \left(1 + \gamma \frac{v}{a_\infty}\right).$$

Expressing the body velocity v through the deflection w in the coordinate system connected to the body, the well-known piston theory [10, 9, 3] for pressure perturbation is obtained:

$$\Delta p = \frac{\gamma p_\infty}{a_\infty}\left(U\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right). \tag{1}$$

At high speeds the second term, $\partial w/\partial t$, which expresses aerodynamic damping, can be neglected comparing to the first term, aerodynamic stiffness, which yields the quasi-static expression

$$\Delta p = \gamma p_\infty M_\infty \frac{\partial w}{\partial x}. \tag{2}$$

This formula shows a good correlation with the exact pressure calculations for Mach numbers $3 - 5$ and larger, but can be improved to include Mach number range $2 < M < 3$ [11] by the following modification:

$$\Delta p = \frac{\gamma p_\infty M_\infty^2}{\sqrt{M_\infty^2 - 1}}\frac{\partial w}{\partial x}. \tag{3}$$

In this form it coincides with Ackeret's formula for pressure acting on slender body in supersonic flow. As Ackeret's formula is the exact solution of linearised flow equations, this is the 'best' quasi-static expression and is taken in this study as the basis for the analysis of the reacting flow effect.

It can be traced that in the case of reacting mixture instead of a perfect gas, the plane section law stays valid, as its derivation [10] uses only momentum equations, but not thermodynamics. We will assume that at each spatial location, the mixture can be considered as being in local equilibrium, and the expression (3) stays valid, with γ, p_∞, and M_∞ substituted by its local values (fig. 1). While the use of local steady pressure and Much number values is obvious and common in aeroelasticity, the change of the specific heat ratio is a purely hypersonic flow effect. Let us consider this is more details.

When the flow particle crosses the bow shock, its temperature increases, and the mixture primarily consisting of molecular oxygen and nitrogen, O_2 and N_2, becomes chemically non-equilbrium, because the equilibrium air state corresponding the temperature behind the shock, consists also of atomic oxygen O and nitrogen N, as well as nitric oxide NO (other species and ionisation effects are neglected in this study for simplicity). Hence, oxygen and nitrogen molecules start to dissociate, yielding the appearance of significant amount of other species, which results in flow with a system of chemical reactions that tends downstream to the equilibrium state corresponding to the current temperature. On the other hand, during the motion of the particle, temperature is also changing downstream, so that the composition of the mixture at each point is the solution of the coupled system consisting of the equations of motion and equations of chemical reactions.

We will assume that for the calculation of pressure perturbation for flutter analysis purposes, the flow can be considered as locally equilibrium at each point. Then the effect of the hypersonic flow consists in two points. First, in the quasi-static piston theory,

$$\Delta p = K(x,y)\frac{\partial w}{\partial x}, \quad K(x,y) = \frac{\gamma p M^2}{\sqrt{M^2-1}}, \tag{4}$$

the coefficient K a function of the spatial location. For the case of a sufficiently short skin panel in non-reacting flow, K is constant along the panel, which yields classical coupled-mode flutter mechanism [3, 12, 11]. However, if the flow is essentially non-equilibrium, chemical reactions yield significant change of K, primarily because of the change of the mixture composition along the panel.

Second, the flow composition is changed because of the deflection of the surface. This change yields the change of the flow pressure p_∞ of the order of the surface deflection, i.e., it should be taken into account in the linearized approximation for the pressure perturbation. However, in the example considered below we will show that this component of the pressure perturbation is much smaller than due to regular piston theory (4), that is why it can be neglected in most circumstances.

2.2. Conditions at which the reacting flow effect is the most pronounced

We can now deduce that two conditions should be satisfied to make the reacting flow effect more pronounced from the flutter point of view. First, the temperature downstream of the bow shock should be higher to increase the difference of the air equilibrium compositions before and after the shock. The further the mixture behind the shock from the equilibrium, the longer it will change its composition downstream, which will result in more significant change of K over the body.

Second, the flow speed behind the shock should be smaller, in order to make visible the change of the mixture composition along the body. Indeed, if the flow speed is too high, the composition of the mixture will not change much during the motion of the flow particles along the structure, but if it moves slower, this change will be more pronounced, which will result in essentially non-constant piston theory coefficient K.

3. Example

3.1. One-dimensional flow

To validate our conclusions, we consider an example of a one-dimensional air flow. At the inlet, which was assumed to be the point just behind the shock (Fig. 2), we specify Mach number $M_{in} = 2$, temperature $T_{in} = 6\,000$ K, pressure $p_{in} = 100\,000$ Pa, and mixture composition consisting of 21 % O_2 and 79 % N_2. These parameters correspond to the flow over a wedge behind the shock wave with free-stream Mach number $M_\infty = 15$, altitude 34 km, and half of the wedge angle $\alpha = 37.5°$. The flow parameters

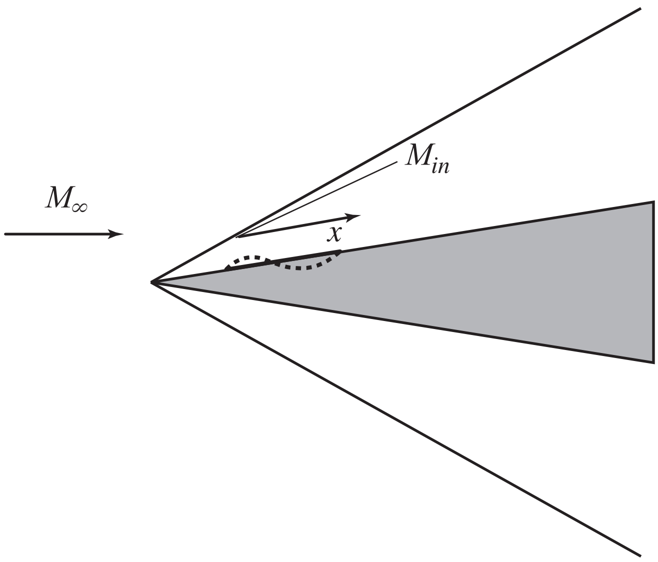

Fig 2. Reacting one-dimensional flow over the wedge behind the shock. Solid and dotted bold curves show the panel under consideration in its unperturbed and perturbed states.

and its composition were calculated downstream ($x > 0$) by solving Navier-Stokes equations for the mixture coupled with system of 17 chemical reactions [13] not involving ionisation.

Figure 3a shows the evolution of calculated mass fractions of air components. It is seen that fractions of molecular oxygen and nitrogen drop, whereas O and NO appear (fraction of atomic nitrogen N stays negligible at this temperature). As a result of reactions, specific heat rate of air is changing downstream (Fig. 3b); also, the flow temperature essentially drops (Fig. 3c), because a portion of the internal energy is used for the dissociation of O_2 and N_2 molecules. Due to the decrease of the temperature, flow pressure also decreases (Fig. 3d), and the flow accelerates (Fig. 3e). Finally, the piston theory coefficient K (4) essentially changes (Fig. 3f), being at the distance of 2 m downstream from the inlet 19 % lower than at the inlet.

3.2. Comparison of unsteady pressure with two-dimensional calculation

Let us now validate the use of Ackeret's formula (4) for the pressure perturbation due to the wall deflection in the reacting flow. Consider a skin panel of $L = 0.3$ m length located at the surface of a wedge (Fig. 2) that occupies the range $x_0 = 0.01 < x < 0.31$ m. Assuming simply supported boundary conditions at the panel leading and trailing edges, its first natural mode is

$$W(x) = A \sin(\pi(x - x_0)/L). \tag{5}$$

Let us consider steady two-dimensional flow domain shown in Fig. 4. At the inlet we specify the same parameters as in one-dimensional flow analysis. At the top boundary of the simulation domain, free-slip wall condition is specified (i.e., normal velocity and shear stress are zero). At the bottom boundary, deflection of the wall in the shape (5) is set. In this study we neglect the boundary layer effect, so that free-slip wall condition is specified over the deformed wall. As well as in one-dimensional study, Navier-Stokes equations with 17 reactions of the system [13] are solved.

For the amplitude $A = 0.0002$ m, Fig. 5 shows calculated distribution of pressure for cases of reacting and non-reacting flows. It is seen that while for non-reacting flow the deviation of the flow pressure from zero is caused by the panel deflection, for the reacting flow there is also a bulk pressure decrease due to chemical reactions. Fig. 6a shows the distribution of $K(x)$ along the panel from one-dimensional analysis (zoomed view of Fig. 3f); in the reacting flow K drops by 10 % along the panel compared to its

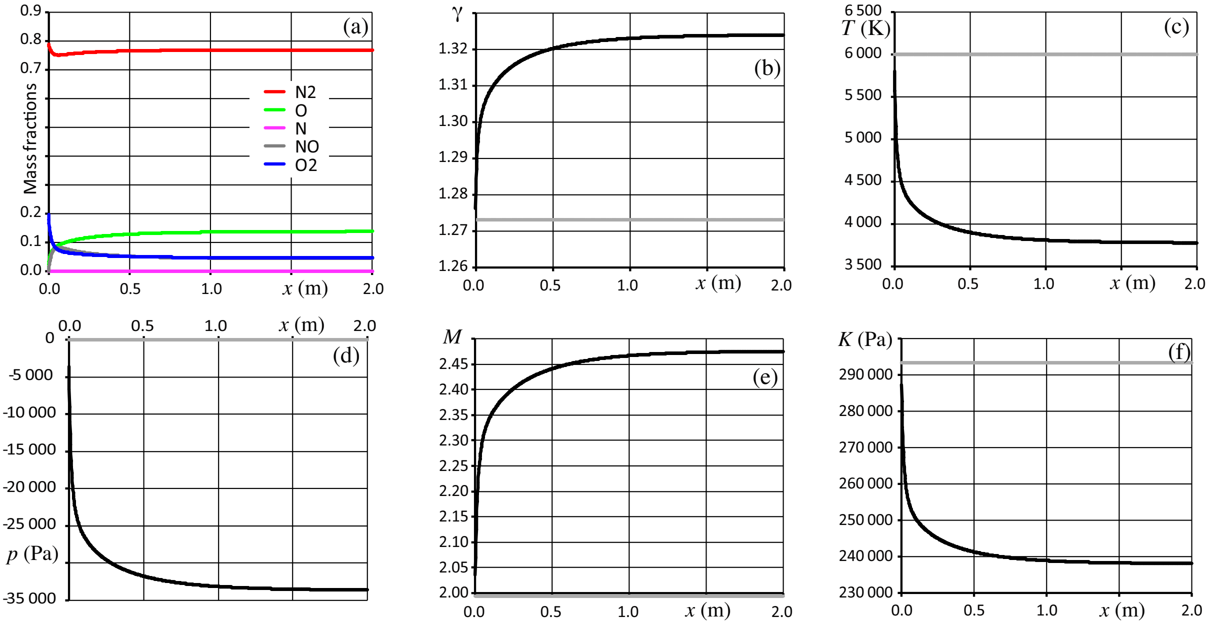

Fig 3. Distribution of mass fractions (a), specific heat rate (b), temperature (c), pressure drop $p - p_{in}$ (d), Mach number (e), piston theory coefficient K (f) for reacting (black) and non-reacting (grey) flows.

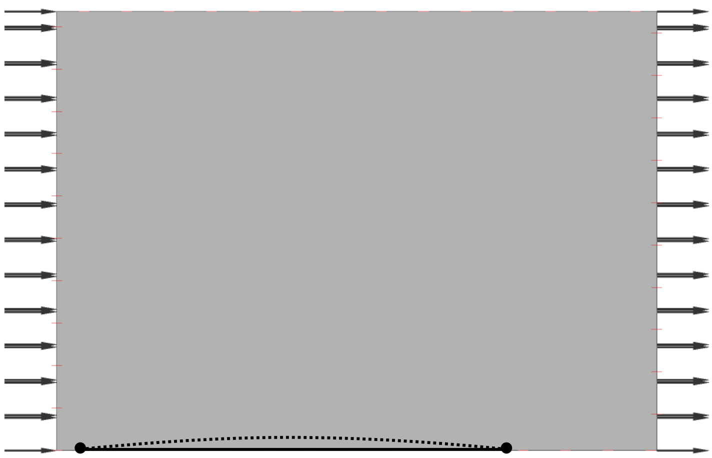

Fig 4. Geometry of two-dimensional flow of a bent panel. Leading and trailing edge locations are shown by circles. Dotted curve shows scaled shape of the panel deflection (5).

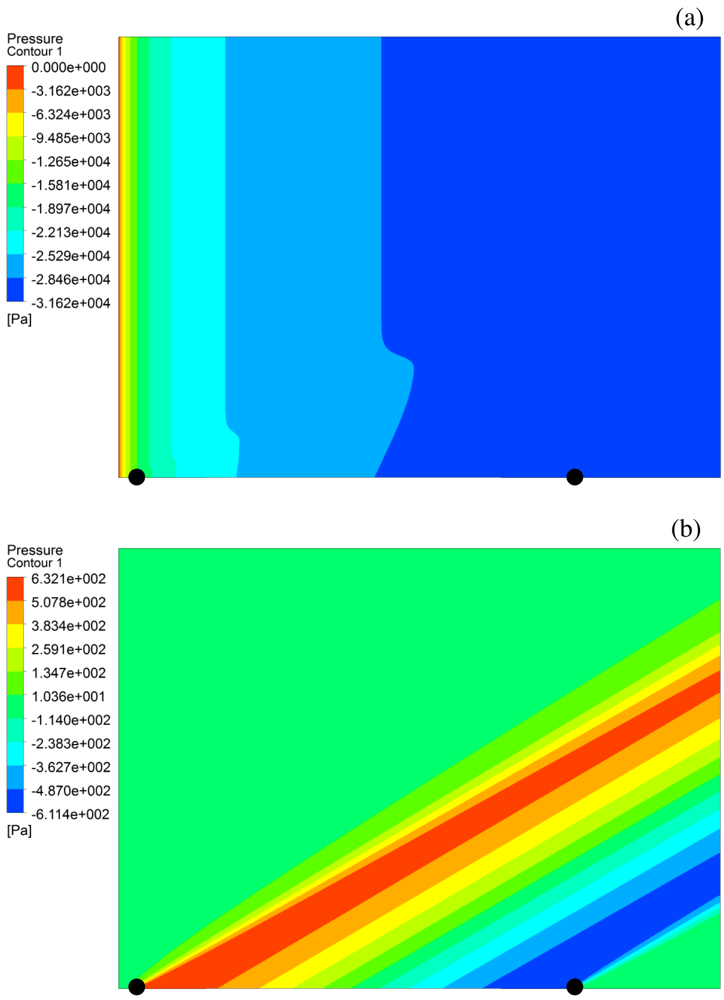

Fig 5. Distribution of pressure for reacting (a) and non-reacting (b) flows.

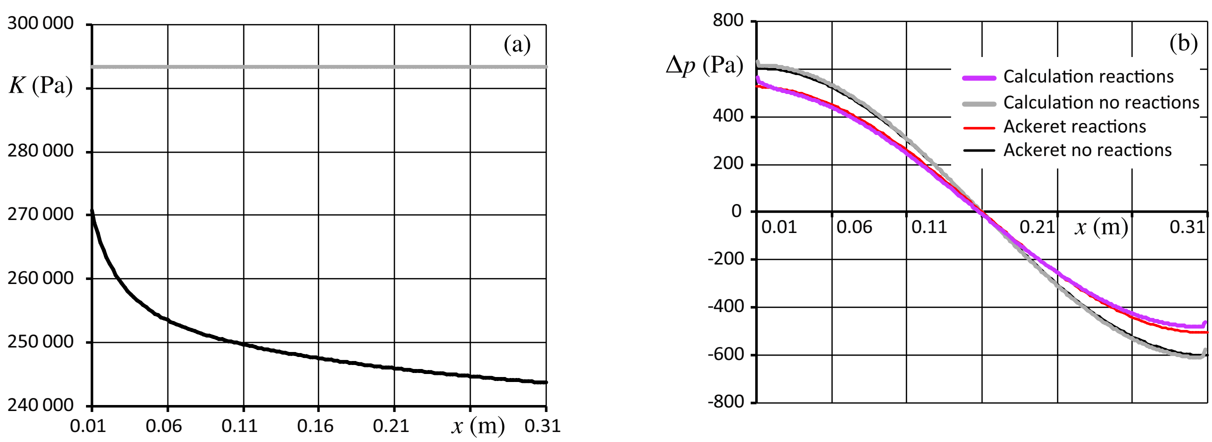

Fig 6. Distribution $K(x)$ for reacting (black) and non-reacting (gray) flow over the panel (a), pressure perturbation according to two-dimensional calculation and formula (4) (b).

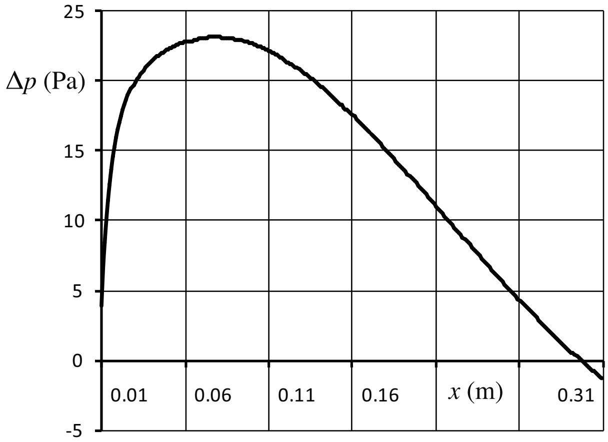

Fig 7. Perturbation of the flow pressure due mixture composition change occured due to the surface deflection.

value at the leading edge of the panel, which should result in the increase of the critical Mach number of a panel. Fig. 6b shows the resulting pressure perturbation for the amplitude from the two-dimensional calculations and obtained by (4) for reacting and non-reacting flows, respectively. It is seen that in both cases the two-dimensional pressure perturbation and the one calculations through (4) are in a good agreement. We conclude that, as expected, formula (4) can be used in the case of reacting flow, and the effect of reactions consists in the non-constant $K(x)$ distribution.

3.3. Contribution of the perturbation of the mixture composition to the pressure perturbation

To estimate the effect of the flow composition change due to the surface deflection, the corresponding pressure perturbation component was calculated in the following manner. Pressure of the mixture is a sum of partial pressures produced by each of 5 components of the mixture:

$$p = \sum_{i=1}^{5} \rho_i \kappa_i \frac{R}{m_i} T,$$

where ρ_i, κ_i, and m_i are the density, mass fraction, and molar mass of i-th component, R and T are the gas constant and temperature. Then, to retain only the effect of the mixture composition change, we assume that the temperature of the mixture is not changed due to deflection of the surface (its change is actually taken into account by the piston theory component of the pressure perturbation), and the pressure perturbation due to composition change is given by

$$\Delta p = \sum_{i=1}^{5} \rho_i \Delta\kappa_i \frac{R}{m_i} T,$$

where $\Delta\kappa_i$ is the perturbation of the mixture composition due to the deflection of the surface.

The calculation result is shown in Fig. 7. It is clearly seen that the pressure perturbation generated by the mixture composition change due to the surface deflection is less than 5% of the pressure perturbation due to the surface deflection calculated without taking the composition change into account (Fig. 6b). That is why we conclude that the change of the mixture composition can be neglected when calculating the unsteady pressure.

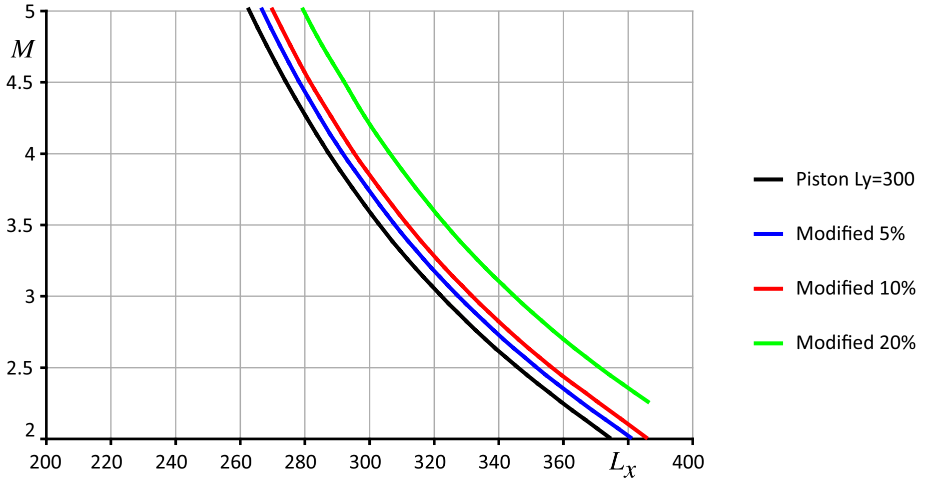

Fig 8. Panel flutter boundary in the $L_x - M$ plane: non-reacting flow, and modifications of the piston theory coefficient (drop of K along the panel) due to reactions by 5, 10, 20 %.

3.4. Impact of the reacting flow on flutter boundary

Now consider the dynamics of a skin panel (Fig. 2). Its linear equation of motion has the form

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho_m h \frac{\partial^2 w}{\partial t^2} + \Delta p = 0,$$

where D and h are its bending stiffness and thickness, ρ_m is the panel material density. Substitution of the quasi-static expression (4) yields the coupled aeroelastic equation of motion

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho_m h \frac{\partial^2 w}{\partial t^2} + K(x)\frac{\partial w}{\partial x} = 0, \tag{6}$$

Its stability is analysed numerically by the Bubnov-Galerkin method. The computational code and its validation are described in [11] and are not discussed here for the sake of brevity.

To analyse the effect of non-constant piston theory coefficient $K(x)$ on the flutter boundary, we have considered a simply supported rectangular steel panel of 1 mm thickness, 0.3 m width (spanwise) and different lengths (chordwise) and calculated stability boundary for the equation (6). The following $K(x)$ distributions are considered, modelling different drop amounts compared to the inlet value:

- $K(x) = K(x_0)(1 + 0.22(\exp(-2x/L_x) - 1))$, which models the drop of K by 20%.
- $K(x) = K(x_0)(1 + 0.16(\exp(-x/L_x) - 1))$, which models the drop of K by 10%.
- $K(x) = K(x_0)(1 + 0.09(\exp(-x/L_x) - 1))$, which models the drop of K by 5%.

Calculated flutter boundaries are shown in Fig. 8, where L_x and L_y are dimensionless panel length and width (rated to its thickness). It is seen that for the panel length $L_x = 0.3$ m the drop of K by 10 % (as, for example, in Fig. 6a) results in the change of critical Mach number M_{cr} by ~ 0.2; higher drop of K, by 20 %, changes M_{cr} by 0.6. For shorter panels the effect of the same amount of the drop of K becomes larger.

4. Conclusions

We have shown that non-equilibrium chemical reactions in hypersonic flow and corresponding change of local specific heat rate and temperature along the body surface result in the non-constant, decreasing downstream piston theory coefficient, whereas the piston theory itself stays valid. This results in the increase of the critical Mach number M_{cr} compared to non-reacting flow. Considered examples show that, depending on specific flight conditions, this can yield the increase of M_{cr} by the order of 0.2 for panel of 0.3 m length, which is of the order of 10% of M_{cr} calculated without taking reactions into account.

The effect of the change of the air composition due to the surface deflection on the pressure perturbation is the order lower than pressure perturbation calculated without taking the composition change into account. In other words, the perturbation of the composition of the mixture can be neglected in most cases.

In this study we did not take into account catalytic processes on the body surface, however, their effect can be estimated in a similar manner. The results obtained can be useful in the design of light-weight and robust hypersonic flight vehicles.

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