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Optimal Climbing Trajectories of Hypersonic Aircraft based on Direct Methods of Flight Dynamics

M.A. Kiselev[1], *A.Yu. Chekin*[2], *Ya.V. Moroshkin*[3]

Abstract

This paper shows the results of research for finding optimal climbing for reaching cruise flight of hypersonic aircraft. The problem to find optimal control has two cases. In the first case the optimization criterion is fuel consumption mass. In the second one it's a time to achieve the cruise speed. The research object is a hypersonic aircraft with 300 tones take-off weight and maximum speed of 6 Machs. A propulsion system consists of 6 turbo ramjets. As fuel the engines use kerosene and hydrogen. The problem of optimal control is solved by modifying the direct variational method, which has been developed by one of the authors. The method provides high accuracy of the trajectory using cubic splines interpolation. The amount of splines is determined in each case on the basis of the convergence of the solution. In addition, the method provides the limitations of the magnitude and rate of vertical g-force and roll.

Keywords: *direct methods, flight dynamics, trajectory optimization, hypersonic aircraft.*

In 1932 Konstantin Tsiolkovsky developed the theory of flying rocket-powered aircraft into the stratosphere and made drawings of hypersonic aircrafts. However so far it has not been created any serial hypersonic passenger aircraft. Nowadays leading industrialized countries carry out programs of development of aerospace aircraft and aerospace systems. These programs will be able to lead to technologies, which provide ability of creating a hypersonic passenger aircraft. TSAGI and other leading research centers are carrying out the researches on hypersonic passenger aircraft. This paper shows results of estimates of time and fuel costs to reach the cruise flight.

For research task it is chosen a tailless aircraft which has a long fuselage. A low wing has a thin diamond-shaped profile in section and complex shape in the plan with a variable sweep along the leading edge, the Leading-edge extension has a large sweep angle, and the cantilever part of the wing has a moderate sweep angle. Turboramjet engines (which is a combination of turbojet and ramjet engines) with parallel bypasses are located in the ventral nacelle. The lower forward part of the fuselage, like the braking surface, is integrated with the flat air intake. The lower surface of the tail section of the fuselage is shaped as a nozzle of the engine. Fuel (kerosene and hydrogen) is located in the fuselage tanks. Kerosene is used in the main combustion chamber of a turbojet engine. Hydrogen is used in the afterburner combustion chamber of the turbojet engine and in the combustion chamber of the ramjet engine. The basic digits of the aircraft are presented in the table 1.

The research tool is mathematical modeling of aircraft motion. The problem of optimal control is solved by modifying the direct variational method, which has been developed by one of the authors. The method provides high accuracy of the trajectory using cubic splines interpolation. The amount of splines is determined in each case on the basis of the convergence of the solution. In addition, the method provides the limitations of the magnitude and rate of vertical load factor and roll. The global extremum of the functional is determined using a combination of the statistical method based on the Ψ-transformation and the method of coordinate descent.

The solution of the problem is (Table 2) that the optimal climbing for reaching cruise flight ($H = 30$ km, $M = 6$) is characterized by a long duration (22.3 min), fuel consumption (up to 64% of the total

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[1] *GosNIIAS, Moscow, Viktorenko street, 7, makiselev@2100.gosniias.ru*

[2] *GosNIIAS, Moscow, Viktorenko street, 7, aychekin@2100.gosniias.ru*

[3] *GosNIIAS, Moscow, Viktorenko street, 7, yvmoroshkin@2100.gosniias.ru*

reserve) and distance (up to 8% of the total range). Time gain of 7%, obtained on the trajectory formed in solving the problem by the criterion minimum of time, in comparison with the trajectory formed in the solution of the problem by the criterion minimum of fuel mass, is achieved by an additional fuel consumption of 14%. The saved 9.4 tons of fuel because of the minimum fuel mass program can increase the length of the cruising part of the flight to about 1500 km. The horizontal distances of climbing and reaching flying speed of the minimum of time program and the minimum fuel mass program differ slightly.

Table 1. – The principal digits of the aircraft

Parameters	Values
Take-off mass	300 000 kg
Fuel mass	120 000 kg
Number of engines	6
Wing area	1571 m^2
Aircraft length	104 m
Maximum of Mach number	6
Maximum of vertical g-force	2.5
Maximum of impact air pressure	75 kPa
Stalling lift coefficient	0.65
Static ceiling	35 km

Table 2. – The solution of the problem

Parameters	Variances		Comparison	
	the criterion minimum of time	**the criterion minimum of fuel mass**	**difference**	**relative difference, %**
Time, seconds (minutes)	1252 (20,9)	1338 (22,3)	86 (1,4)	7
Range, km	904,3	911	6,7	0.7
Fuel mass, т	77,3	67,9	9,4	14

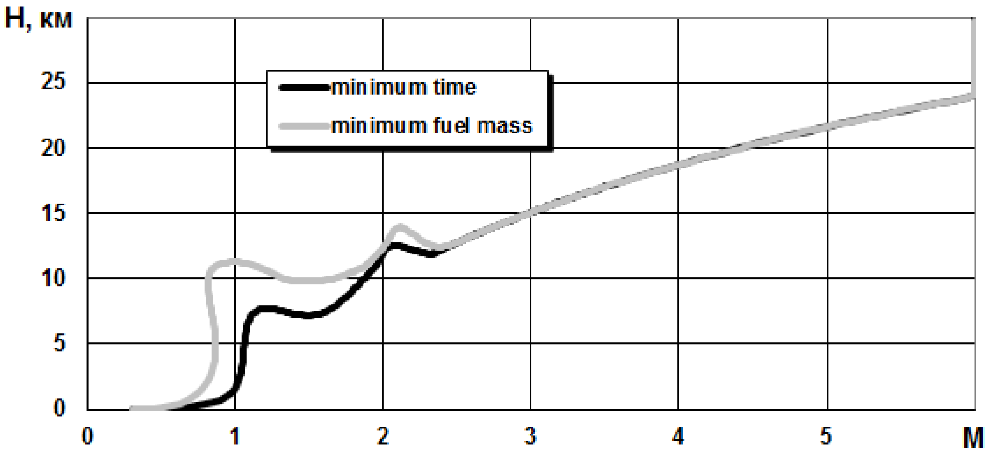

Fig 1.

The flight of climbing and reaching cruise speed (Figure 1, Table 3) has two stages:

- First part – increasing the speed from the initial speed to the speed which corresponds to the maximum of impact air pressure at 75 kPa, and at the same time climbing from 0 km to 12 km by the minimum of time program or from 0 km to 12.5 km by the minimum fuel mass program;

- Second part – climbing at maximum speed which corresponds to the maximum of impact air pressure at 75 kPa.

Table 3. – Data comparison

Parameters	**Flight stages**			
	1		**2**	
	minimum time	**minimum fuel mass**	**minimum time**	**minimum fuel mass**
H_0, km	0	0	12	12.5
t, s	632	779	620	559
L, km	300	345	604,3	566
m_f, tons	42,8	37,1	34,5	30,8

In the first stage, time and fuel costs are 51% of the time and 55% of the fuel at the minimum time program, 58% of the time and 55% of the fuel at the minimum fuel mass program. The second stage has less time (49% for the program minimum time and 42% for the minimum fuel mass program), and fuel (45% for both programs), but this is a longer stage (67% and 62% for the programs minimum time and minimum fuel mass respectively). At the same time, both programs of climbing and reaching cruise speed in axes H (M) at the second stage of flight nearly coincide. This coincidence is explained by the fact that the maximum values of the excess specific power and the ratio of the excess specific power and fuel-flow rate at constant energy height and at altitudes higher than 12 km and 12,5 km respectively are reached to maximum speeds for these altitudes. Therefore the integral parameters of the optimal programs for climbing and reaching speed of the hypersonic passenger aircraft for both programs are affected by the structural capabilities of airframe, which determine the magnitude of maximum impact air pressure, and the mechanical properties of materials of this airframe and the efficiency of the cooling system that defines the maximum Mach number. Thus, for example, a decrease of 7% in of maximum impact air pressure (from 75 kPa to 70 kPa), assuming that the rest parameters are unchanged, leads to an increase of 19% the duration of the last stage at the minimum time program (from 559 to 667 s), and an increase of 15% distance (from 566 to 651 km), and an increase of 12% the fuel consumption (from 20.8 to 34.6 tons). At the same time, an increase of maximum impact air pressure from 75 kPa to 80 kPa leads to a decrease by 13% the flight time, by 10% range and by 8% fuel consumption of the same stage. These results prove the importance of the issues of matching the characteristics of the airframe and the engine when creating the hypersonic passenger aircraft.

The first stage determines more the differences in the integral parameters of the two programs.

Figure 2 shows altitude – Mach number dependencies.

- curve 1 is a program of climbing and reaching cruise speed for a minimum time;
- curve 2 is the program of climbing and reaching cruise speed for the minimum time calculated by the Ostoslavskii-Lebedev method with the assumption that vertical load factor is 1, the line curve represents the instantaneous transition region;
- curve 3 is a program of climbing and reaching cruise speed with a minimum fuel consumption;
- curve 4 is the program of climbing and reaching cruise speed with a minimum fuel consumption, calculated by the Ostoslavsky-Lebedev method, assuming that vertical load factor is 1, the dashed line represents the instantaneous transition region.

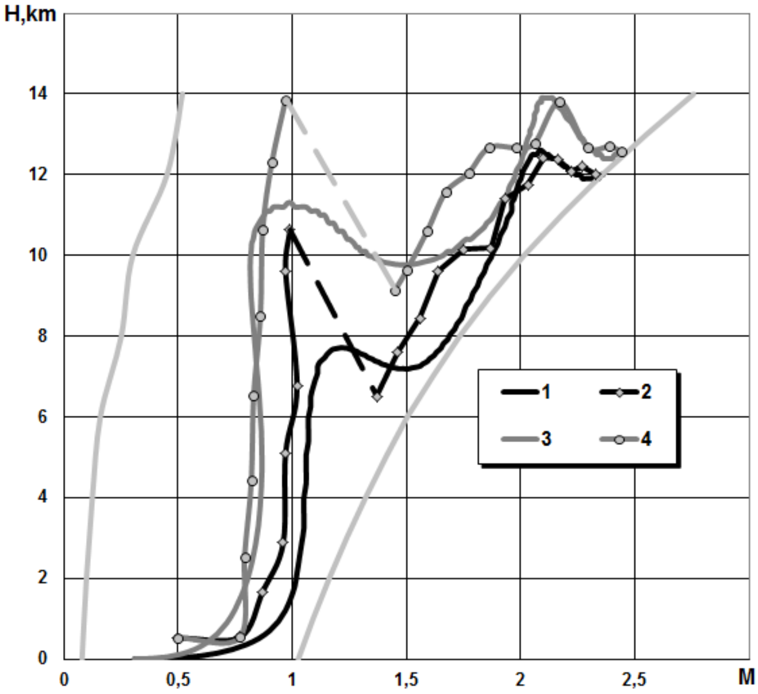

Fig 2.

As follows from these dependencies, the optimal trajectories are close to the trajectories calculated by the energy method.

The first stage of the flight (Figure 2) has four sections:

- the first section – acceleration above the earth's surface;
- the second section – climbing with a constant Mach number;
- the third section – acceleration with the descending (curve goes down);
- the fourth section - acceleration to the speed corresponding to impact air pressure with the climbing.

The first three sections are a typical climb and speed program for most modern aircraft. Programs of this type are also considered for the hypersonic passenger aircraft. The question is whether or not optimal programs can be simplified, for example, by eliminating a fairly unconventional zoom climb in the fourth section? And how will the other parameters be changed? To answer this question, the fourth section of the program for climbing and reaching speed for a minimum time has been replaced by moving at maximum impact air pressure at the altitude range from 7.5 km to 12 km, and a similar section of the minimum time program was replaced by two parts: the first is the increasing Mach number from 1.66 to the maximum value at the constant altitude 10 km; the second part is flying at the maximum impact air pressure in the altitude range from 10 km to 12.5 km. This simplification results in a very slight increase in the time and distance of the flight for both programs (4 s and 7 km for the minimum time program and 10 s and 14 km for the minimum fuel consumption program), but leads to an increase in fuel consumption by 2.1 tons for the minimum time program and 4.5 t for the minimum fuel consumption program. This is due to a slight decrease in the excess specific power and a significant decrease in the ratio of the excess specific power and fuel-flow rate (by about 40%) when hypersonic passenger aircraft moves along simplified trajectories.

Thus, the results prove the importance and necessity of studying the trajectories of the hypersonic passenger aircraft before the stages of formation of the hypersonic passenger aircraft appearance. Only a multidisciplinary approach, combining issues related to aerodynamic design, engine construction, flight dynamics research, strength and thermal protection, will create a successful hypersonic passenger aircraft in the future, providing a drastic reduction of time of transcontinental flight.