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Integrated optimization of aerospace vehicles based on the maximum principle

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Abstract

The technique of integrated optimization of aerospace vehicle parameters and trajectories by the united criterion of target efficiency is described. The technique uses a decomposition of a multidisciplinary problem into single-disciplinary subtasks of flight dynamics and control, aerodynamics, propulsion and structures. The sensitivity functions of the target criterion with respect to design parameters are objectively calculated on the basis of solution of the trajectory optimization problem using the Pontryagin maximum principle. The technique has the special advantages in the optimization of multiregime high-speed vehicles, when an influence of layout parameters on the target efficiency is essentially depend on flight conditions. Some applications of the technique for the integrated optimization of space launchers are demonstrated. The qualitatively new optimal solutions in comparison with traditional ones are revealed due to the proposed approach.

Keywords: *multidisciplinary optimization, maximum principle, distributed criteria*

1. Introduction

Now the trend to use more comprehensive computational methods at the early aircraft design stages is observed. The multidisciplinary optimization (MDO) techniques are developed to combine diverse and possibly remote programs and databases intended for advanced single-discipline investigations into the unified framework [1].

Development of computational resources and simulation methods allows increasing the accuracy of analysis in separate disciplines. However, in order to simplify the interaction of specialists and optimization complexity, the coupling data (that are used to link the several disciplines) tend to be minimized. It results in solutions with the reserves in aircraft performance, which are latent on the boundary of the aerospace disciplines, remaining unused.

A peculiarity of aircraft as a designed object consists in the fact it's characteristics change depending on flight regimes. Selection of the trajectory can influence significantly both on the target performance and on the optimum parameters of aircraft layout. The methods of so-called multipoint aerodynamic shape optimization are developed, which are based on searching for a compromise in improvement of aircraft characteristics on several established flight regimes simultaneously [2]. Since the results of such researches rely on the relative "weight" of each regime, it seems important to develop a technique for objective formation of the specific influence functions of optimized parameters along all regimes, taking into account the continuity of their alteration.

It is shown in [3, 4] how such distributed influence functions can be obtained relying on Pontryagin maximum principle [5]. The objective criteria for the separate disciplines, termed as local distributed criteria (LDC), are naturally formed in the frame of trajectory optimization on the basis of the adjoint equation set solution and the definition of Lagrange multipliers.

The importance of elaborate coupling of trajectory and other design parameters is particularly substantial for problems in which an influence of the trajectory on aircraft performance is strong, or cruising segments are not dominating. The examples of such aircraft are supersonic airplane and space launchers [6-9].

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2. The integrated optimization technique

The problem of integrated optimization of aircraft parameters and trajectories is solved. The criterion of optimality is the functional

$$\Phi(\mathbf{x},\mathbf{p}) \Rightarrow \max_{\mathbf{u},\mathbf{p}} , \tag{1}$$

where $\mathbf{x}(t)\in\mathbb{R}^n$ is the state vector, $\mathbf{u}(t)\in\mathbb{R}^m$ is the control vector, t is the time, $\mathbf{p}\in\mathbb{R}^p$ is the vector of optimized parameters. Aircraft motion is defined by the vector differential equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p},t) , \quad t_i \le t \le t_f . \tag{2}$$

The constraints can be imposed on the state vector and on the control vector:

$$\mathbf{X}(\mathbf{x},\mathbf{p},t) = 0, \mathbf{X} \in \mathbb{R}^N , \tag{3}$$

$$\mathbf{U}(\mathbf{x},\mathbf{u},\mathbf{p},t) \le 0, \mathbf{U} \in \mathbb{R}^M . \tag{4}$$

To search the optimum solution

$$\{\mathbf{u}, \mathbf{p}\}_{opt} = \arg\max \Phi$$

optimization of parameters is conducted

$$\mathbf{p}_{opt} = \arg\max \Phi|_{\mathbf{u}=\mathbf{u}_{opt}}, \tag{5}$$

where the optimum control $\mathbf{u}_{opt}$ should be obtained at the solution of the enclosed problem of trajectory optimization:

$$\mathbf{u}_{opt} = \arg\max \Phi|_{\mathbf{p}=fix} . \tag{6}$$

To solve the problem (6) the Pontryagin maximum principle [5] is used with Hamiltonian H

$$H = \boldsymbol{\psi}^T \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p},t) + \boldsymbol{\lambda}^T \mathbf{U}(\mathbf{x},\mathbf{u},\mathbf{p},t),$$

where $\boldsymbol{\psi} \in\mathbb{R}^n$ is the adjoint vector, $\boldsymbol{\lambda} \in\mathbb{R}^M$ is the vector of Lagrange multiplicities. The optimum control is determined from the condition

$$\mathbf{u}_{opt} = \arg\max_{\mathbf{u}} H .$$

The regular procedure of solution of the multipoint boundary value problem for the state and adjoint differential equations is fulfilled in the ASTER [10] package. The result of trajectory optimization is the optimal control $\mathbf{u}(t)$, state $\mathbf{x}(t)$ and adjoint $\boldsymbol{\psi}(t)$ variables, Lagrange multipliers $\boldsymbol{\nu}$, $\boldsymbol{\lambda}(t)$, which correspond to the optimal solution at the nonperturbed value $\mathbf{p}$. The Bliss formula [11] determines the variation of the functional (1) through variation of $\mathbf{p}$:

$$\delta\Phi = \delta_p\Phi + \boldsymbol{\nu}^T \delta_p\mathbf{X} + \int_{t_i}^{t_f} \left(\boldsymbol{\psi}^T \delta_p\mathbf{f} + \boldsymbol{\lambda}^T \delta_p\mathbf{U}\right)dt = \nabla_p \Phi\, \delta\mathbf{p} \Rightarrow \max \tag{7}$$

where $\delta_p(.)$ is the variation of a function (.) caused by a variation $\delta\mathbf{p}$. The variation (7) has sense of local distributed criteria (LDC) for the parameter $\mathbf{p}$. The right member $\mathbf{f}$ of Eq. (2), boundary conditions and constraints (3), (4) depend on the vector of aircraft characteristics $\mathbf{C}(\mathbf{x}, \mathbf{p})$. The variation (7) can be presented as:

$$\delta\Phi = \frac{\partial\Phi}{\partial\mathbf{p}}\delta\mathbf{p} + \boldsymbol{\nu}^T \frac{\partial\mathbf{X}}{\partial\mathbf{C}}\frac{\partial\mathbf{C}}{\partial\mathbf{p}}\delta\mathbf{p} + \int_{t_i}^{t_f}\left(\boldsymbol{\psi}^T \frac{\partial\mathbf{f}}{\partial\mathbf{C}}\frac{\partial\mathbf{C}}{\partial\mathbf{p}}\delta\mathbf{p} + \boldsymbol{\lambda}^T \frac{\partial\mathbf{U}}{\partial\mathbf{C}}\frac{\partial\mathbf{C}}{\partial\mathbf{p}}\delta\mathbf{p}\right)dt. \tag{8}$$

The multipliers in front of $\partial\mathbf{C}/\partial\mathbf{p}$ in (8) are calculated in the process of solving of trajectory optimization problem and have sense of distributed influence functions of $\mathbf{C}$ on Φ along the trajectory. They are transmitted to program complexes of other disciplines, for example aerodynamics and strength, and can be used in internal problems of optimization to calculate the effect of objective variables on functional Φ pursuant to (8). The functions $\mathbf{X}(\mathbf{C})$, $\mathbf{f}(\mathbf{C})$, $\mathbf{U}(\mathbf{C})$, as well as matrixes $\partial\mathbf{X}/\partial\mathbf{C}$, formulae in most cases. Thus, they lie in the subject domain of "Flight dynamics". The determination of $\partial\Phi/\partial\mathbf{p}$ and $\partial\mathbf{C}/\partial\mathbf{p}$ lies in the field of other disciplines, for example, "Aerodynamics" and/or "Structures".

The optimization of design parameters **p** by the criterion (1) is reached as a result of iterations containing calculations of the characteristics in Eq. (8) within the framework of separate disciplines. The use of (8) in single-discipline analysis (in aerodynamics, propulsion, structures etc.) allows to optimize design parameters (an aerodynamic layout, a structural scheme etc.) by global criterion (1) without to overstep the limits of these disciplines.

The main advantages of the LDC-technique are:

- it takes into account the specific effect of each elementary trajectory section on the target functional (1);
- control structure is modified automatically at a variation of aircraft parameters;
- the natural subordination of all single-discipline variables and parameters to one general problem of aircraft performance optimization (1) is achieved;
- LDC-technique does not require to simplify the single-discipline analysis and makes it possible to use advanced research methods.

Advantages of the LDC-technique are especially pronounced in the integrated optimization of trajectory and parameters of multiregime aircraft when an arrangement of priorities between different flight regimes can be difficult or impossible. For example, using the technique in an application to STS it is necessary to take into account the following features:

- a considerable changing of vehicle characteristics during the flight;
- an absence of a cruise flight mode which would largely determine the performance.

3. Trajectory optimization and it's effect on launcher performance

Let us consider the peculiarities of launcher trajectory optimization in more detail. The problem (5) is to find the admissible control **u,** which allows to put the launcher into the given orbit with the minimum propellant consumption, it corresponds to the maximization of injected (terminal) vehicle mass [10]. The motion of the launcher mass centre is presented by Eq. (2), where

$$\mathbf{f} = \{\mathbf{V},\ \mathbf{T}/m + \mathbf{A}/m + \mathbf{g} + \mathbf{\Omega},\ -\mu\}^{\mathrm{T}}, \tag{9}$$

V is the velocity vector, **T** is thrust vector, m is the launcher mass, **A** is the vector of aerodynamic forces, **g** is the vector of gravitational acceleration, Ω is the acceleration vector due to coordinate system noninertiality, μ is the mass flow rate.

The thrust T is constrained by the minimum and maximum values:

$$T_{\min} \le T \le T_{\max}.$$

The vector $\mathbf{u} = \{\mathbf{e}_\tau, \mathbf{T}\}^{\mathrm{T}}$ is considered as the control vector, where $\mathbf{e}_\tau$ is the unit vector directed along the vehicle's longitudinal axis.

The problem of optimal launcher injection into orbit has known analytical approximate solutions [12, 13]. The optimal program of a pitch angle tangent is a time-linear in the homogeneous gravitational field.

The current launchers use the simplified control law with a zero angle of attack in dense atmospheric layers, which allows to eliminate transversal loads. Such control is conventionally termed as the "gravitational turn" because the path bending happens due to gravitational forces only. Hereinafter a combination of a gravitational turn in dense atmospheric layers and the linear program of a pitch angle tangent on the subsequent sections will be termed as *the traditional* control law.

As has been shown [14], the strategy of the optimal control is determined by the correlation of the thrust (**T**), aerodynamic (**A**), inertial and gravitational (**G**) forces (Fig. 1). If the thrust dominates, the optimal control law is qualitatively in accordance with the traditional one [12, 13] that is obtained for uniform gravitational field under an assumption of the negligibility of aerodynamic forces. However, if $|\mathbf{T}| >> |\mathbf{A}|$ and $|\mathbf{G}| >> |\mathbf{A}|$, but

$$|\mathbf{T}+\mathbf{G}| \approx |\mathbf{A}|, \tag{10}$$

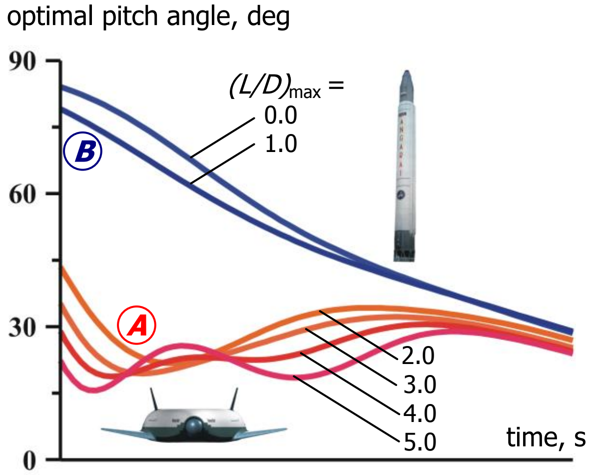

Fig. 1 Optimal time-programs of the pitch angle of the launcher at several $(L/D)_{max}$

the effect of aerodynamic forces can change the structure of the optimal control law and generate multiplicity of extremals.

According the classification given in [14] the extremals in the problem of optimization of launcher control in the atmosphere can be one of three types: *Ballistic, Aerodynamic and interMediate.*

***B*-type ("*Ballistic*") extremals:**

- provide the global optimum at low maximum lift-to-drag ratio $(L/D)_{max}$;
- the atmosphere is "perceived" only as a medium with some drag;
- the optimal pitch angle programs are quasi-linear to correspond the well-known "traditional" solutions [12, 13];
- the optimal start is nearly vertical.

A-type ("Aerodynamic") extremals:

- the optimal pitch angle program during the atmospheric flight has a pronounced oscillatory nature (Fig. 1);
- an inclined and quasihorizontal start is optimal;
- the atmosphere is perceived including as a medium that produces a lift, so the optimal trajectories pass into regions with higher dynamic pressures as compared with the *B*-type extremals;
- provide the global optimum at high lift-to-drag ratios.

M-type ("interMediate") extremals:

– do not provide a global optimum.

Due to existence of several types of local extremals even a small change in aerodynamic lift capabilities of the launcher can lead to a qualitative change in the optimal control laws and influence functions of launcher parameters on the functional.

4. Using the technique for integrated optimization of space launchers

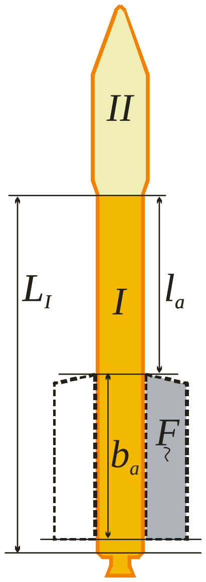

Fig. 2 Launcher scheme

Let us consider as an example using LCD-technique for multidisciplinary optimization of trajectory and parameters of two-stage launcher [15] by the criterion of the payload mass injected into an Earth orbit. Two trapezoidal airfoil consoles can be mounted on first stage booster (Fig. 2).

The vector of optimized design parameters $\mathbf{p} = \{F, b, l\}^T$ contains the following components (see Fig. 2):

1. F is ratio of airfoil console area to reference cross section area: $F \geq 0$;
2. $b = b_a/L_I$ is specific length of console aerodynamic chord: $0 \leq b \leq 1$;
3. $l = l_a/L_I$ is the specific distance from a plane of connection of first and second stage boosters up to a leading edge of the console: $0 \leq l \leq 1$, $l \leq 1 - b$.

The variation of **p** results in change of aerodynamic characteristics of the launcher, structural mass of first stage booster and the optimal injection trajectory. The payload is calculated as a difference of the final injected mass m_f and the structural mass of second stage booster m_s that is considered to be fixed:

$$m(\mathbf{p}) = m_f(\mathbf{p}) - m_s.$$

Accounting for the mutual influence of changes in the trajectory and launcher parameters in the optimization process made it possible to identify the qualitatively new optimal solutions in comparison with traditional ones. In particular, it has been obtained that the optimal launcher layout corresponds to $F = 3.1$, $b = 0.55$, $l = 0.15$, i.e. has a small empennage, the area of which is commensurable with the area of the midsection (Fig. 2). The examples of the distributed influence functions $f_{C_L^\alpha}$ of the aerodynamic coefficient C_L^α (the derivative of the aerodynamic lift to the angle of attack) on the relative payload mass under the variation of F at fixed l = 0.5, b = 0.5, which obtained on the basis of the maximum principle, are shown in Fig. 4.

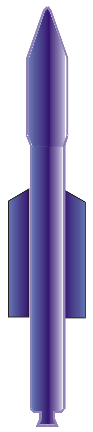

Fig. 3. The resulting optimal launcher layout

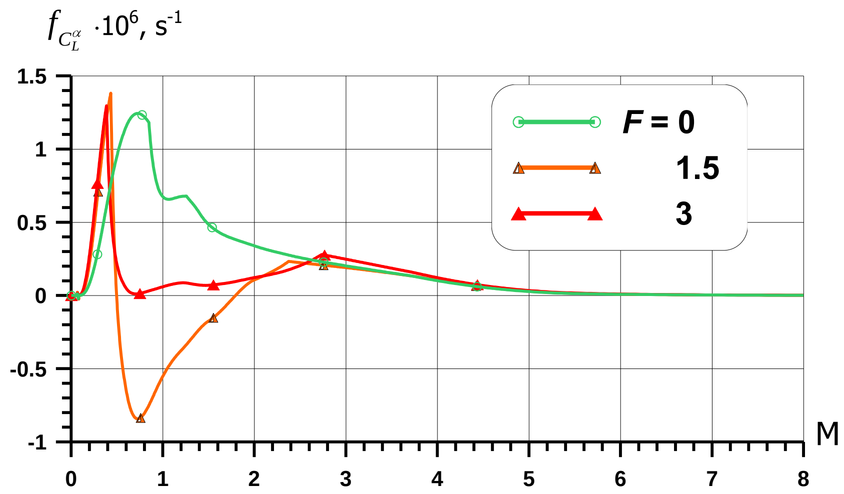

Fig. 4. Depending of influence function $f_{C_L^\alpha}$ on Mach number on the optimal injection trajectories under the variation of F

As we can see, the influence functions $f_{C_L^\alpha}$ are significantly rearranged, up to a sign change, even under a small variation in the lifting properties of the launcher (due to variation of F) with the corresponding rearrangement of the optimal control. This effect is explained by the fact that the technique automatically adjusts the optimal trajectory for the value of the parameter F (Fig.5).

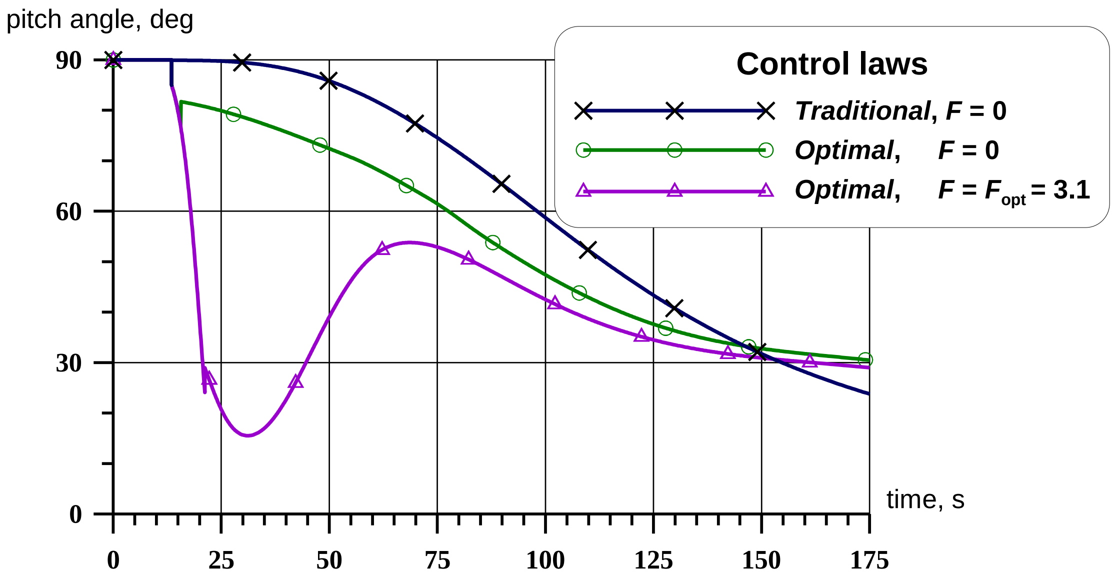

Рис. 5 Dependencies of pitch angle on time

The integrated optimization of parameters and trajectories of space launcher made it possible to increase the payload mass by 6.8% in comparison with payload mass of launcher without additional consoles and using traditional control law.

Conclusion

Thus, in the proposed approach to integrated optimization of aerospace vehicles, the following main advantages are revealed:

- the specific effect of each elementary trajectory section on the target functional is taking into account;
- the structure of control is automatically adapted at a variation of vehicle parameters;
- the natural subordination of all single-discipline variables and parameters to one general problem of vehicle performance optimization is achieved;
- simplification of numerical tools of single-discipline analysis is not required that makes it possible to use advanced research methods inside each discipline and, thereby, to increase the objectivity of the solutions obtained.

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