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Differential flatness-based finite time sliding mode control of hypersonic vehicle

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Abstract

The development of hypersonic flight control system is a challenging task, because of the tightly coupled, highly nonlinear and notoriously uncertain nature of hypersonic vehicle(HV) dynamics. Differential flatness method is applied to the linearization of the longitudinal model of HV. A finite time convergent controller is designed for the nominal linearized model. Then an integral sliding mode controller is added to deal with the uncertainty of the system. In addition, a discrete linear tracking differentiator(TD) is used to extract high order differential signal of the system, which can avoid the high order differential signal polluted in the numerical calculation. A case study is presented to illustrate the capability of the proposed control method.

Keywords: *hypersonic vehicle, differential flatness, finite time convergent, sliding mode control, discrete linear tracking differentiator*

Nomenclature

States
V – Velocity
h – Altitude
α– Angle of attack
γ – Flight path angle
q – Pitch rate
Force and moment
L– List
D – Drag
T – Thrust
M – Moment
Control input
δ_e– Elevator
β– Throttle rate
Aircraft parameters
m – Mass
I_{yy} – Pitching moment of inertia
Other parameters
ξ– Damping coefficient
ω_n – Natural frequency

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1. Introduction

With the rapid development of the demand for weapons, the hypersonic vehicles (HVs) have attracted more and more attention owing to the high-speed and prompt global response. The development of hypersonic flight control systems face the challenges stemming from the tightly coupled, highly nonlinear and notoriously uncertain nature of HSV dynamics. A suitable control system is needed to make HVs realize hypersonic stable flight.

There have been several approaches for the control of hypersonic vehicles[1-7]. Some of them proposed reference command tracking controllers for the linearized dynamics of hypersonic vehicles[1], and others designed controllers directly for the nonlinear dynamics of hypersonic vehicles, such as some adaptive sliding controllers[2][3], fuzzy controllers[4], etc. The controllers above had achieved good results.

In recent years, the theory of differential flatness[8] got a lot of attention due to its small amount of calculation and the characteristics of high efficiency. Domestic scholars apply it to the aircraft flying trajectory planning and control[9][10]. The advantage of differential flatness theory is that the nonlinear system states and control inputs can be expressed by the flat output and its limited order differential items. The physical meaning of the intermediate variables are more intuitive, and when a nonlinear system is differential flatted, all the states and control inputs will be seen after the trajectory planning, and the reference trajectory will be shown before the simulation.

In this work, differential flatness method is applied to the linearization of the model of HV. A finite time convergent controller is designed for the nominal linearized model. Then an integral sliding mode controller is added to deal with the uncertainty of the system. In addition, a discrete linear tracking differentiator(TD) is used to extract high order differential signal of the system, which can avoid the high order differential signal polluted in the numerical calculation.

2. Hypersonic Vehicle model

In this paper, the research object is longitudinal model of a generic air-breathing hypersonic vehicle. Thrust actuator second-order dynamic link is considered.

The longitudinal model is given in [11]:

$$\dot{V} = \frac{Tcos\alpha - D}{m} - \frac{\mu sin\gamma}{r^2} \tag{1}$$

$$\dot{\gamma} = \frac{Tsin\alpha + L}{mV} - \frac{(\mu - V^2 r)cos\gamma}{Vr^2} \tag{2}$$

$$\dot{h} = Vsin\gamma \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$

$$\dot{q} = M_{yy} / I_{yy} \tag{5}$$

$$\ddot{\beta} = -2\xi\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c \tag{6}$$

Where

$$L = \frac{1}{2}\rho V^2 SC_L(\alpha), D = \frac{1}{2}\rho V^2 SC_D(\alpha),\ T = \frac{1}{2}\rho V^2 SC_T(\beta), r = h + R_E$$

$$M_{yy} = \frac{1}{2}\rho V^2 S\bar{c}\left[C_{M,\alpha}(\alpha) + C_{M,\delta_e}(\alpha,\delta_e) + C_{M,q}(q,\alpha)\right]$$

$$C_L(\alpha) = 0.6203\alpha, C_{M,\delta_e}(\alpha,\delta_e) = c_e(\delta_e - \alpha)\quad C_D(\alpha) = 0.6450\alpha^2 + 0.0043378\alpha + 0.003772$$

$$C_T(\beta) = \begin{cases} 0.02576\beta & when\ \beta \le 1 \\ 0.0224 + 0.00336\beta & when\ \beta > 1 \end{cases},\quad C_{M,\alpha}(\alpha) = -0.035\alpha^2 + CM_\alpha + 5.3261\times 10^{-6}$$

$$C_{M,q}(q,\alpha) = (\bar{c}/2V)q\left(-6.796\alpha^2 + 0.3015\alpha - 0.2289\right),\ CM_\alpha = 0.036617\alpha,\ C_e = 0.0292$$

3. Controller Design

3.1. Linearization of hypersonic vehicle based on differential flatness

Definition 1: For some differential flat nonlinear dynamic system, by choosing appropriate flat output can make the linearization of nonlinear system. That is, if a set of system output can be found, when all state variables and input variables can be determined by this set of output and its limited order differential, the system is differential flat system. It can be expressed in mathematical form:

For nonlinear systems

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), \boldsymbol{u}(t)) \tag{7}$$

Where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ is the input vector, $\boldsymbol{f}$ is a continuous smooth function.

If there is a set of output $\boldsymbol{z}$ meet

$$z_i = h_i(\boldsymbol{x}, \boldsymbol{u}, \cdots, \boldsymbol{u}^{(r_i)}), (i = 1, \cdots, m) \tag{8}$$

And the system states and inputs can be expressed as follows:

$$\boldsymbol{x} = \varphi_0(z_1, \dot{z}_1, \cdots z_1^{(l_1)}, \cdots, z_m, \dot{z}_m, \cdots z_m^{(l_m)}) \tag{9}$$

$$\boldsymbol{u} = \varphi_1(z_1, \dot{z}_1, \cdots z_1^{(l_1)}, \cdots, z_m, \dot{z}_m, \cdots z_m^{(l_m)}) \tag{10}$$

Then the system is differential flat, $\boldsymbol{z}$ is flat output, $r_i (i = 1, \cdots, m)$ is the relative order of $\boldsymbol{z_i}$.

Consider the hypersonic vehicle model, V and r are chosen as flat output, that is,

$$z_1 = V \tag{11}$$

$$z_2 = r \tag{12}$$

Then we need to express state vector $[V, \gamma, h, q, \alpha]^T$ and control input vector $[\delta_e, \beta_c]^T$ by flat output $[z_1, z_2]^T$ and its limited derivatives.

From Eq.(3), we get

$$\gamma = arcsin(\frac{\dot{z}_2}{z_1}) \tag{13}$$

Then

$$\dot{\gamma} = \frac{\ddot{z}_2 z_1 - \dot{z}_1 \dot{z}_2}{z_1 \sqrt{z_1^2 - \dot{z}_2^2}} \tag{14}$$

In this paper, we consider the vehicle system characteristics in cruise state. The vehicle keep a small angle of attack. In order to get the analytical solution of α, here assuming that $sin\alpha \approx 0$ and $cos\alpha \approx 1$.

Eq.(1) and (2) can be written into

$$\dot{z}_1 = \frac{T - D}{m} - \frac{\mu \dot{z}_2}{z_1 z_2^2} \tag{15}$$

$$\frac{\ddot{z}_2 z_1 - \dot{z}_1 \dot{z}_2}{z_1 \sqrt{z_1^2 - \dot{z}_2^2}} = \frac{L}{m z_1} - \frac{(\mu - z_1^2 z_2) cos\{arcsin(\frac{\dot{z}_2}{z_1})\}}{z_1 z_2^2} \tag{16}$$

From Eq.(16), α is expressed as follows:

$$\alpha = \frac{2 m A_1}{0.6203 \rho z_1^2 S} \tag{17}$$

Where

$$A_1 = \frac{\ddot{z}_2 z_1 - \dot{z}_1 \dot{z}_2}{\sqrt{z_1^2 - \dot{z}_2^2}} + \frac{(\mu - z_1^2 z_2) cos\{arcsin(\frac{\dot{z}_2}{z_1})\}}{z_2^2} \tag{18}$$

Then

$$\dot{\alpha} = \frac{2m}{0.6203\rho S} \cdot \frac{z_1 \dot{A}_1 - 2\dot{z}_1 A_1}{z_1^3} \tag{19}$$

Substituting Eq.(14) and Eq.(19) into Eq.(4), we obtain the last state

$$q = \frac{\ddot{z}_2 z_1 - \dot{z}_1 \dot{z}_2}{z_1\sqrt{z_1^2 - \dot{z}_2^2}} + \frac{2m}{0.6203\rho S} \cdot \frac{z_1 \dot{A}_1 - 2\dot{z}_1 A_1}{z_1^3} \tag{20}$$

Where

$$\begin{aligned} \dot{A}_1 = & \frac{z_1 \dddot{z}_2 - \ddot{z}_1 \dot{z}_2}{\sqrt{z_1^2 - \dot{z}_2^2}} - \frac{(z_1 \ddot{z}_2 - \dot{z}_1 \dot{z}_2)(z_1 \dot{z}_1 - \dot{z}_2 \ddot{z}_2)}{(z_1^2 - \dot{z}_2^2)\sqrt{z_1^2 - \dot{z}_2^2}} - \frac{\left(2z_1 \dot{z}_1 z_2 + z_1^2 \dot{z}_2\right) cos\left[arcsin\left(\frac{\dot{z}_2}{z_1}\right)\right]}{z_2^2} \\ & + \frac{\left(\mu - z_1^2 z_2\right)\left(\frac{\dot{z}_2}{z_1}\right)\frac{\ddot{z}_2 z_1 - \dot{z}_1 \dot{z}_2}{z_1\sqrt{z_1^2 - \dot{z}_2^2}}}{z_2^2} - \frac{2\dot{z}_2(\mu - z_1^2 z_2) cos\left[arcsin(\frac{\dot{z}_2}{z_1})\right]}{z_2^3} \end{aligned} \tag{21}$$

So far, all the states are expressed by the flat outputs and their limited derivatives.
Then we need to express the control inputs by flat outputs.
Due to space limitations, the similar derivation process will not be repeated. Here gives the result directly:

$$\begin{aligned} \delta_e = & [\frac{2\dot{q} I_{yy}}{\rho z_1^2 S\bar{c}} + 0.035\alpha^2 - 0.036617\alpha + 0.0000053261 \\ & - \frac{\bar{c} q}{2 z_1}\left(-6.796\alpha^2 + 0.3015\alpha - 0.2289\right)] \cdot \frac{1}{c_e} + \alpha \end{aligned} \tag{22}$$

And

$$\beta = \begin{cases} \dfrac{2T}{0.02576\rho z_1^2 S} & when\ \beta \le 1 \\ \dfrac{1}{0.00336} \cdot (\dfrac{2T}{\rho z_1^2 S} - 0.0225) & when\ \beta > 1 \end{cases} \tag{23}$$

$$\dot{\beta} = \begin{cases} \dfrac{1}{0.01288\rho S}\left(\dfrac{\dot{T}}{z_1^2} - \dfrac{2T\dot{z}_1}{z_1^3}\right) & when\ \beta \le 1 \\ \dfrac{1}{0.00336} \cdot (\dfrac{2}{\rho S}\left(\dfrac{\dot{T}}{z_1^2} - \dfrac{2T\dot{z}_1}{z_1^3}\right) - 0.0225) & when\ \beta > 1 \end{cases} \tag{24}$$

$$\ddot{\beta} = \begin{cases} \dfrac{1}{0.01288\rho S}\left(\ddot{T} - \dfrac{2\dot{T}\dot{z}_1}{z_1} + \dfrac{2\left(\dot{T}\dot{z}_1 + T\ddot{z}_1\right)}{z_1^3} - \dfrac{6T\dot{z}_1^2}{z_1^4}\right) & when\ \beta \le 1 \\ \dfrac{1}{0.00336}(\dfrac{2}{\rho S}\left(\ddot{T} - \dfrac{2\dot{T}\dot{z}_1}{z_1} + \dfrac{2\left(\dot{T}\dot{z}_1 + T\ddot{z}_1\right)}{z_1^3} - \dfrac{6T\dot{z}_1^2}{z_1^4}\right) - 0.0225) & when\ \beta > 1 \end{cases} \tag{25}$$

Substituting Eq. (23), Eq. (24) and Eq. (25) into Eq. (6), we obtain the expression of β_c.
Through the derivation, all the states and control inputs are expressed as follows:

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{bmatrix} = \boldsymbol{f}(z_1, \dot{z}_1, \ddot{z}_1, \dddot{z}_1, z_2, \dot{z}_2, \ddot{z}_2, \dddot{z}_2, \ddddot{z}_2) \tag{26}$$

We got the states and control inputs expressed by flat outputs and their limited derivatives. It is proved that the system is differential flat.

If the model is differential flat, the linear feedback exist which can make the nonlinear model input-state linear. In this work, we set the height and speed of desired trajectory to be constant, for verifying the effectiveness of the differential flat theory. Because the tracking task is straightforward, concise state feedback is used to realize the trajectory tracking control. In final version, more challenging trajectory will be tracked, and controller design form may also be changed.

The linear model of system can be written into:

$$\begin{bmatrix} \dot{z}_1 \\ \ddot{z}_1 \\ \dddot{z}_1 \\ \dot{z}_2 \\ \ddot{z}_2 \\ \dddot{z}_2 \\ \ddddot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ \dot{z}_1 \\ \ddot{z}_1 \\ z_2 \\ \dot{z}_2 \\ \ddot{z}_2 \\ \dddot{z}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{27}$$

3.2. Finite time convergent controller design

Consider the r-th order nominal system:

$$\dot{e}_i = e_{i+1} \tag{28}$$

$$\dot{e}_r = v + d \tag{29}$$

Where *i=1,2,…,r-1,* $e_i = z^{(i-1)} - z_c^{(i-1)}$ *, d is the uncertainty of the system.*

First, when d=0, a finite time convergent controller under nominal state is designed as follows.

Theorem 1[12]: Let the positive constant $k_1, k_2, \cdots, k_r$ and $k_1', k_2', \cdots, k_r'$ be such that the polynomials $\lambda^r + k_r\lambda^{r-1} + \cdots + k_2\lambda^2 + k_1\lambda$ and $\lambda^r + k_r'\lambda^{r-1} + \cdots + k_2'\lambda^2 + k_1'\lambda$ are Hurwitz. There is $\varepsilon \in (0,1)$ such that, for every $\alpha \in (1-\varepsilon, 1)$, system (28)(29) is stabilized at the origin in finite time under the feedback:

$$v = -k_1 \operatorname{sgn}(e_1)|e_1|^{\alpha_1} - k_1' e_1 \cdots - k_r \operatorname{sgn}(e_r)|e_r|^{\alpha_r} - k_r' e_r \tag{30}$$

Where

$$\alpha_{i-1} = \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}, i = 2, \cdots, r$$
$$\alpha_r = \alpha, \alpha_{r+1} = 1 \tag{31}$$

We noticed that the controller need high order derivative signal z_i . In the actual computer numerical calculation, the signal will always be polluted by some noise. Therefore, extracting the each order differential signal reasonably becomes an important link in the simulation. In this paper, tracking differentiator is used to realize the precise calculating of each order differential signal. Tracking differentiator track the dynamic characteristic of the input signal quickly, and give the approximate differential signal.

For the first order and second order differential signal of z_1 in (28)(29), discrete linear TD is designed as follows:

$$\begin{cases} f^k = -r^3(x_1^k - w) - 3r^2 x_2^k - 3r x_3^k \\ x_1^{k+1} = x_1^k + p x_2^k \\ x_2^{k+1} = x_2^k + p x_3^k \\ x_3^{k+1} = x_3^k + p f^k \end{cases} \tag{32}$$

For the first order, second order and third order differential signal of z_2 in (28)(29), discrete linear TD is designed as follows:

$$\begin{cases} f^k = -r^4(x_1^k - w) - 4r^3 x_2^k - 6r^2 x_3^k - 4r x_4^k \\ x_1^{k+1} = x_1^k + p x_2^k \\ x_2^{k+1} = x_2^k + p x_3^k \\ x_3^{k+1} = x_3^k + p x_4^k \\ x_4^{k+1} = x_4^k + p f^k \end{cases} \tag{33}$$

Where k and $k+1$ are sampling times, w is the input, namely V_c and V , h_c and h . r is the speed factor, which for the four types of input can be respectively written as r_v and r_{vc} , r_h and r_{hc} . p is the integral step, which for the four types of input can be respectively written as p_v and p_{vc} , p_h and

p_{hc} . x_1, x_2, x_3, x_4 is respectively the original signal tracking, first order differential, second order differential and third-order differential. Tracking differentiator have played an important role in solving differential and instruction filtering.

3.3. Sliding mode controller design

Consider the integral sliding mode variable defined as follows:

$$s(e) = e_r(t) - \int_0^t v(\tau)\,d\tau \tag{34}$$

Theorem 2 Consider the perturbed system, if the sliding mode controller is designed as follows:

$$v' = v - K\,\mathrm{sgn}(s) \tag{35}$$

And the K satisfies

$$K > |d|_{\max} + \delta \tag{36}$$

Where $|d|_{\max}$ is the upper limit of uncertainty, and δ is a small constant.

The sliding mode is established for perturbed system in finite time.

Proof Defining the Lyapunov function $V = \frac{1}{2}s^2$.

Substituting the controller (35) into (34), we obtain:

$$\dot{s} = v' + d - v = d - K\,\mathrm{sgn}(s) \tag{37}$$

As $K > |d|_{\max} + \delta$,

$$\begin{aligned} \dot{V} = s\dot{s} &= \left(d - K\,\mathrm{sgn}(s)\right)s \\ &= ds - K|s| \\ &= ds - |d|_{\max}|s| - \delta|s| \\ &< -\delta|s| < 0 \end{aligned} \tag{38}$$

So the perturbed system (28)(29) can evolve on the sliding surface. This completes the proof.

The robust stability tracking of the longitudinal motion of HV is guaranteed with the controller (26)(30)(35), in which the uncertainties and external disturbance are rejected.

4. Simulation

In order to verify the applicability of the proposed method, a hypersonic vehicle velocity and attitude tracking control instance is presented here. In this work, three degrees of freedom nonlinear longitudinal dynamics model is considered. The vehicle flies at the initial speed of Mach number 15. The detailed initial conditions of the vehicle is given in table 1.

Table 1. The detailed initial conditions

Variable	Value	Variable	Value
V	6060 *ft/s*	S	3603 ft^2
h	110000 *ft*	c	80 *ft*
β	0.1517	ρ	$0.243*10^{-14}$ $slugs/ft^3$
m	9375 *slug*	μ	$1.39*10^{16}$ ft^3/s^2
q	0	α	0.0116 *rad*
γ	0	R	20903500 *ft*

In order to verify the robustness of the controller, parameter uncertainty is considered in the model, and the form of uncertainty is expressed in Table 2.

Table 2 Uncertainty parameters model

Parameter class	Distribution	Range of error
Aerodynamic coefficient	Normal distribution	30%
Aerodynamic moment coefficient	Normal distribution	30%
Atmospheric density	Normal distribution	30%

Set the desired state $V_c = 6160\, ft/s$ and $h_c = 112000\, ft$.

The parameters of the proposed controller are designed as follows:

$\alpha_{1v} = \frac{1}{2}$, $\alpha_{2v} = \frac{3}{5}$, $\alpha_{3v} = \frac{3}{4}$, $\alpha_{1h} = \frac{3}{7}$, $\alpha_{2h} = \frac{1}{2}$, $\alpha_{3h} = \frac{3}{5}$, $\alpha_{4h} = \frac{3}{4}$, $k_{1v} = 1$, $k_{2v} = 2$, $k_{3v} = 2$, $k'_{1v} = 150$, $k'_{2v} = 70$, $k'_{3v} = 15$, $k_{1h} = 1$, $k_{2h} = 2$, $k_{3h} = 3$, $k_{4h} = 2$, $k'_{1v} = 150$, $k'_{2v} = 70$, $k'_{3v} = 15$.

In addition, the parameters of the discrete linear TD are designed as $r_v = r_{vc} = r_h = r_{hc} = 1000$, and the integral step $p = steptime = 0.001s$. The comparative simulation results are shown in Fig. 1-Fig 3.

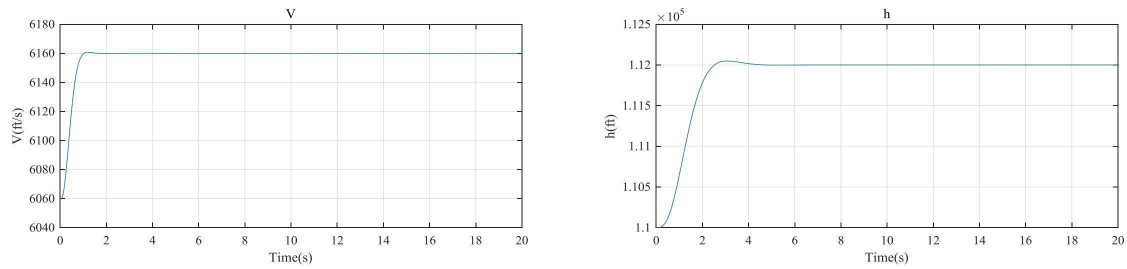

Fig. 1 The velocity and attitude tracking errors

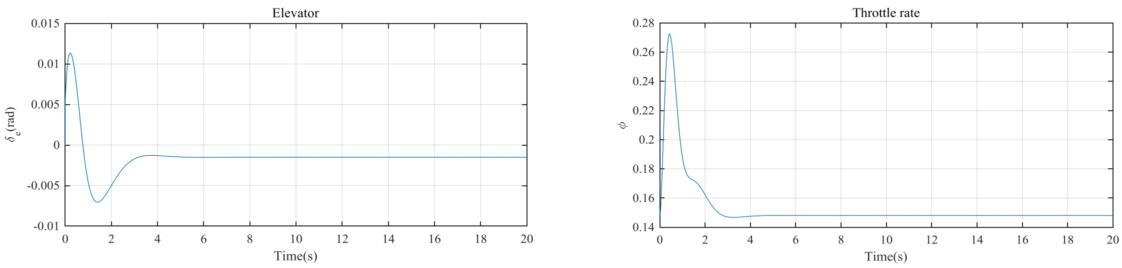

Fig. 2 The control quantities

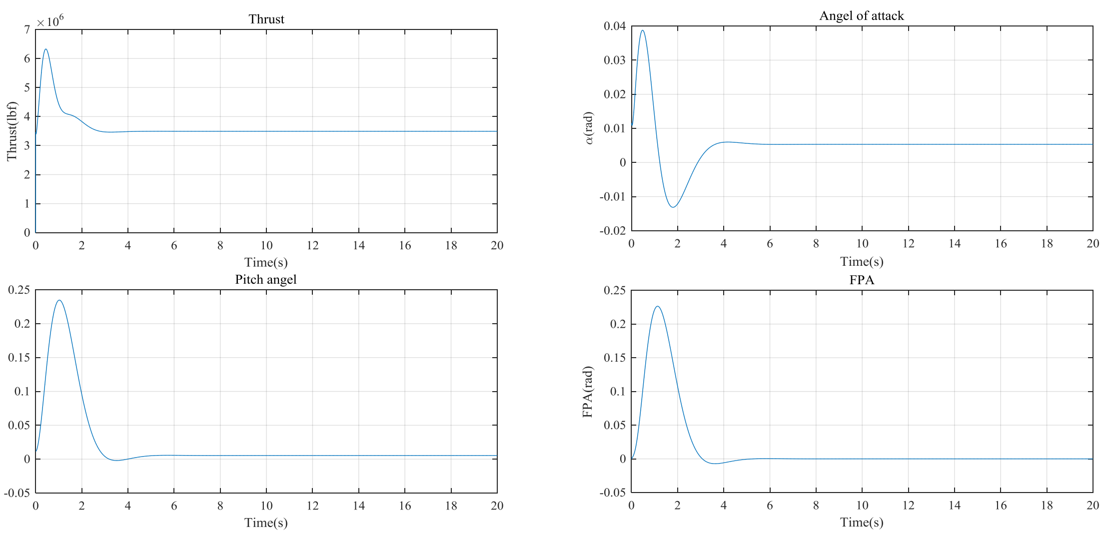

Fig. 3 Other variable curves

As seen in Fig. 1, the proposed controller provides a good tracking for the velocity and altitude commands in 5 seconds. The two control variables have a larger peak value in the process of acceleration and climbing, and tend to a smaller stable value after the system is stable. Other states also converged in a short time, and the change process did not exceed the expected limit.
Simulation results show that the proposed control method achieves finite-time high-accuracy tracking and has good robustness.

5. Conclusion

In this paper, a framework has been proposed for the development of a finite time sliding mode controller based on differential flatness for the longitudinal model of hypersonic vehicle. The salient features of the proposed approach consist in the obvious physical significance of derivation process and all the desired states and control inputs obtained in advance. The integration of the integral sliding mode solves the problem of system robustness caused by the exact model derivation, and the TD solve the problem of high order differential signal extraction. Current work is addressing the implementation of stable tracking of an accelerating and a climbing trajectory. The simulation results under multiple non nominal conditions will be given in the full text.

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