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Asymptotic Analysis of Steady Secondary Flow in a Turbulent Boundary Layer

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Abstract

Turbulent boundary layer of a viscous incompressible fluid past a flat plate is studied. The characteristic Reynolds number of the flow is assumed to be large with the boundary layer being thin. To analyze the problem, the method of multiple scales was applied, which allowed to investigate the steady secondary flow inside the turbulent boundary layer. At that, self-induced entrainment of fluid from the external flow is the main flow in this case, which ensures the supply of kinetic energy from the maximum speed zone to the turbulence generation zone near the streamlined wall. Secondary steady solutions were found analytically for the longitudinal velocity component. The approach obtained is applied to the flow in the channel and to free turbulence flows. The found solutions were compared with the available experimental data.

Keywords: turbulence, asymptotic analysis, boundary layers

1. Introduction

The purpose of this article is the use of the asymptotic analysis of the complete Navier-Stokes equations for describing the properties of turbulence without additional, even physically justified closure hypotheses. This article continues the work of Zametaev & Gorbushin (2016) and Gorbushin & Zametaev (2018). In the latter, an asymptotic investigation of two-dimensional viscous fluctuations in an incompressible turbulent boundary layer developing along a flat plate was carried out. It concerned only the local problem in the turbulence generation zone and in the viscous sublayer. However, in this paper, we describe a solution that is valid throughout the turbulent boundary layer and at lengths comparable to that of the streamlined plate. To analyze the complete nonsteady Navier-Stokes equations, the method of multiple scales is used when the Reynolds number tends to infinity, and the a priori unknown boundary layer thickness tends to zero. The paper suggests the existence of an analogy between the turbulent boundary layer and flows due to the rapid pulsating motion of the streamlined body itself or the presence of pulsations in the external flow. The so-called steady secondary flow, generated by a rapidly oscillating cylinder in a fluid at rest, described by Schlichting (1932), is well known. Detailed experiments with such a cylinder were performed by Tatsuno (1981), whose work is represented in Fig. 1 that depicts a cylinder oscillating along an arrow with a small amplitude and a high frequency and steady large-scale eddies generated by it.

Fig.1

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In spite of the fundamental difference between the given example of flow with an acting external periodic force and a turbulent boundary layer that generates self-sustaining fluctuations, it turns out that occurring of secondary steady flows is also possible in it. The equations are derived and solutions are sought for the secondary steady velocity normal to the streamlined surface. It is the main velocity that is an entrainment of fluid into the boundary layer from the external stream.

2. Main part of the boundary layer

We consider a steady three-dimensional flow of a viscous incompressible fluid past a flat plate of length L, Fig.2. The characteristic time, size, and velocity of the incoming stream equal $L/V_\infty, L, V_\infty$, respectively. The pressure is introduced by the formula $p' = p_\infty + \rho V_\infty^2 p$, where ρ the fluid density, μ is the dynamic viscosity coefficient. All hydrodynamic functions, lengths and times are nondimensioned in the traditional way, using the indicated flow parameters. Hereinafter in the paper all quantities and equations are assumed to be dimensionless. The Reynolds number is introduced as $\mathrm{Re} = \rho V_\infty L/\mu$ and is assumed to be large in this asymptotic study. In contrast to the external flow, the boundary layer developing along the plate is assumed to be turbulent, having in mind the presence of fluctuations of pressure and velocities with respect to some basic longitudinal flow velocity profile $u_0(x, y_1)$.

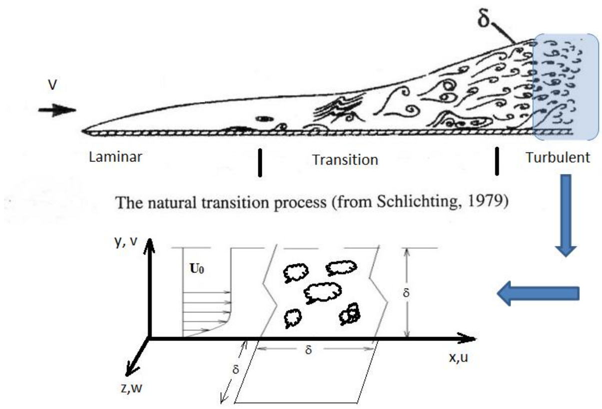

Fig.2

It is convenient to write the dimensionless full Navier-Stokes equations for the pressure increment p , velocity v normal to the plate, the transverse velocity w, and, as a result the longitudinal velocity component u can be found from the continuity equation.

$$\nabla^2 p = -2\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial x} - 2\left(\frac{\partial v}{\partial y}\right)^2 - 2\frac{\partial v}{\partial y}\cdot\frac{\partial w}{\partial z} - 2\left(\frac{\partial w}{\partial z}\right)^2 - 2\frac{\partial u}{\partial z}\cdot\frac{\partial w}{\partial x} - 2\frac{\partial w}{\partial y}\cdot\frac{\partial v}{\partial z}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\nabla^2 v$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{\mathrm{Re}}\nabla^2 w$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

The asymptotic expansions of the solution in the turbulent part of the boundary layer should be sought for $\delta \to 0,\ \mathrm{Re} \to \infty$ in the following form

$$\begin{aligned} u &= u_0(x, y_1) + \delta^{1/2} u_1(x_1, y_1, z_1, t_1, x) + \delta u_2 + \ldots, \\ v &= \delta^{1/2} v_1(x_1, y_1, z_1, t_1, x) + \delta v_2 + \ldots, \\ w &= \delta^{1/2} w_1(x_1, y_1, z_1, t_1, x) + \delta w_2 + \ldots, \\ p &= \delta^{1/2} p_1(x_1, y_1, z_1, t_1, x) + \delta p_2 + \ldots, \end{aligned}$$

$$y_1 = \frac{y}{\delta}, \quad x_1 = \frac{x}{\delta}, \quad z_1 = \frac{z}{\delta}, \quad t_1 = \frac{t}{\delta}$$

Substituting the asymptotic expansions in Navier-Stokes equations and deriving successively the principal terms in the equations, we obtain equations for first-order perturbations v_1, p_1 and w_1, u_1

$$O\left(\frac{\delta^{1/2}}{\delta^2}\right): \qquad \nabla^2 p_1 + 2\frac{\partial u_0}{\partial y_1} \cdot \frac{\partial v_1}{\partial x_1} = 0\,,$$

$$O\left(\frac{\delta^{1/2}}{\delta}\right): \qquad \frac{\partial v_1}{\partial t_1} + u_0 \frac{\partial v_1}{\partial x_1} + \frac{\partial p_1}{\partial y_1} = 0 + O\left(\frac{1}{\mathrm{Re}\cdot\delta}\right),$$

$$y_1 = 0: \quad v_1 = 0; \ \frac{\partial p_1}{\partial y_1} = 0, \quad y_1^2 + x_1^2 + z_1^2 \to \infty: \ p_1 \to 0$$

$$O\left(\frac{\delta^{1/2}}{\delta}\right): \qquad \frac{\partial w_1}{\partial t_1} + u_0 \frac{\partial w_1}{\partial x_1} + \frac{\partial p_1}{\partial z_1} = 0 + O\left(\frac{1}{\mathrm{Re}\cdot\delta}\right),$$

$$O\left(\frac{\delta^{1/2}}{\delta}\right): \qquad \frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} + \frac{\partial w_1}{\partial z_1} = 0,$$

The system contains derivatives only with respect to fast variables and in general the fluctuations v_1, p_1, u_1 must contain steady terms depending on the slow variable x. As for the fluctuation w_1 , if there are no physical reasons for the appearance of a steady component in the transverse flow, then there should be no such term either

$$\begin{aligned} & v_1 = v_{10}(x, y_1) + v_{11}(x_1, y_1, z_1, t_1, x), \quad p_1 = p_{10}(x) + p_{11}(x_1, y_1, z_1, t_1, x) \\ & w_1 = w_{11}(x_1, y_1, z_1, t_1, x), \quad u_1 = u_{10}(x, y_1) + u_{11}(x_1, y_1, z_1, t_1, x) \end{aligned}$$

Analysis of the equations for second order terms gives the necessary condition for the absence of secular terms:

$$\frac{1}{\mathrm{Re}\cdot\delta^{3/2}} \cdot v_{10}''' - v_{10} v_{10}'' + (v_{10}')^2 = 0, \qquad v_{10}(0) = 0$$

It was nice to find exact solution of this equation

$$v_{10} = \frac{V_e(x)}{\mathrm{Re}\cdot\delta^{3/2}} \cdot \left(1 - \exp(V_e \cdot y_1)\right), \quad V_e(x) < 0,$$

The solution which satisfies the impermeability condition on the wall is nothing else than a self-induced viscous steady distributed fluid entrainment into the turbulent boundary layer from the external flow. Longitudinal velocity can be found as a formula

$$u_0 = e^{V_e y_1} \int_0^{(-V_e) y_1} \exp(\eta - e^{-\eta}) d\eta,$$

$$u_0(y_1 \to 0) = \frac{(-V_e)}{e} y_1 + \ldots, \quad Y = (-V_e) y_1$$

Fig.3 shows a comparison of the solution obtained for the longitudinal velocity with the experimental data of Klebanoff & Diehl (1952) and Gorbushin *et al.* (2018).

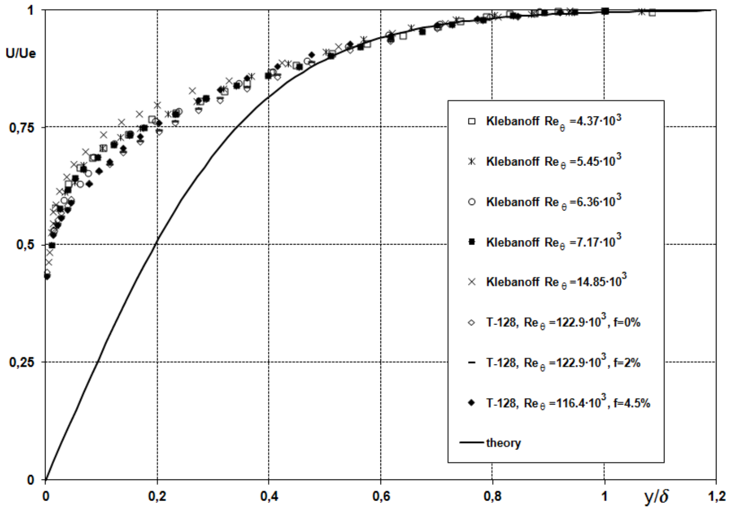

Fig.3

3. Conclusions

The asymptotic theory of secondary steady flow in wall turbulent boundary layers, in turbulent flows in a channel, as well as in thin turbulent mixing layers and jets is constructed. It is found that, in contrast to laminar flows, in the problems considered the primary phenomenon is fast fluctuations of normal velocity and pressure which generate steady fluid entrainment from high velocity regions into the turbulence generation zone (self-sustaining fluctuation zone). It is shown that the secondary steady flow is viscous throughout the entire turbulent layer thickness which allows speaking of large scale viscosity and justifies the well-known physical concept of "turbulent viscosity". In boundary layers and mixing layers, the fluid is entrained from the external flow having a maximum velocity, to the wall or to the main part of the layer. For the Poiseuille flow in a channel or for a free jet, fluid is entrained from the flow head at the symmetry center to the wall or to the peripheral flow region. The found steady solution can be called the 'mechanism of kinetic energy supply' to the turbulent generation zone, regardless of the type of generation. The known phenomenon of turbulent fluid entrainment into the mixing and jet layers from the rest region is explained by the pressure drop in the main part of the layer and is secondary. The solutions found reasonably agree with the experimental data obtained by various authors in a wide range of Reynolds numbers even if there is a mass exchange through the streamlined surface.

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