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Sparse Locally Linear Embedding for Modal Identification

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Abstract

In order to address false modal parameters identification caused by the uncertainty of the selection of adjacent points in manifold learning, a novel modal identification method for structural dynamics using sparse Locally Linear Embedding (LLE) method is proposed. Compared with conventional LLE algorithm, this method can adaptively find the neighbors and weight coefficients by solving a sparse optimization problem, which assumes the neighbors that lie in the same manifold and the low dimensional manifold embedding is extracted from observation space with high dimensional then. Numerical simulation and experiment results illustrate that the proposed method can effectively preserve the neighborhood structure of high dimensional response signals with small nonzero weights, and the modal parameters (modal shapes and modal frequencies) can be accurately identified in comparison to classical LLE algorithm or its various improvement strategies. In addition, the proposed method is more robust than various improvement strategies under different noise levels.

Keywords: *Modal Identification, Dimension Reduction, Manifold Learning, Locally Linear Embedding, Sparse Representation*

Nomenclature

SLLE – Sparse Locally Linear Embedding

MAC – Modal assurance criterion

$\mathbf{Q}_i$ – Proximity inducing matrix

$\mathbf{V}$ – Linear transformation matrix

$\mathbf{W}$ – The weighted matrix

$\mathbf{X}(t)$ – The structural response matrix

$\tilde{\mathbf{X}}_i$ – The normalized matrix

$\mathbf{Y}$ – Low dimensional embedding matrix

$\mathbf{\Phi}$ – Modal shape matrix

$\boldsymbol{c}_i$ – Sparsity coefficient vector

1. Introduction

The modal identification is the key basic issue to achieve precise design and safety assessment of high speed aircraft structures. In order to guarantee the quality and safety of engineering structures in the design and application, it is necessary to have a comprehensive understanding of the dynamic characteristics of structures, and the structural dynamics characteristics can be also obtained by modal parameters identification. In addition, for large complex engineering structures, especially for aerospace vehicles, the load is often difficult to measure. From the inverse problem point of view, only the operational modal parameters identification method based on output-only structural response data can be adopted. Therefore, it is of great significance to develop new operational modal parameter identification method only using the structural responses.

Over the past ten years, multivariate statistical signal processing techniques are popular and introduced into structural dynamics analysis, such as blind source separation. After that, the BSS techniques have received much attention in structural dynamics field. Therein, Independent component analysis (ICA) [1] and second order blind identification (SOBI) [2] are two popular approaches, which are based on fourth order and second order statistics, respectively. The manifold learning is a hot topic in machine

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learning. In recent years, manifold learning is also introduced into the modal identification of structural dynamics, which is also favored by some researchers. Wang et al. [3] proposed the operational modal identification method following Principal Components Analysis (PCA) for simple supported beam, in which the modal parameters identification is converted into principal component decomposition problem. The Locally Linear Embedding (LLE) is an important data dimensionality reduction algorithm, which has been widely applied in face recognition [4], fault diagnosis [5], image retrieval [6] and other related fields. A method based on LLE algorithm in time domain using output-only response data is proposed for modal parameters identification by Bai [7], which regards the response data as a high-dimensional data set and the modes of structure as the essential structure and the inherent characteristics of high-dimensional data set. The LLE algorithm even has better identification performance on nonlinear structures. However, the choice of primary parameter K of LLE algorithm is fixed and is artificially selected. In practice, the setting of the parameter K has great influence on the performance of the algorithm, and it is more difficult to appropriately choose. If K is chosen too large, the neighborhoods could no longer be locally linear. If K is chosen too small, local patches are unable to preserve the topological structure of the data set as a lower-dimensional embedding [8]. In 2011, Elhamifar [9] proposed a new manifold learning algorithm based on Sparse Manifold Clustering and Embedding (SMCE), which chooses the neighbors and weights automatically by solving sparse optimization problem and is more suitable for the solution of engineering problem.

In summary, to determine the appropriate neighbors, based on LLE algorithm and combined with the thought of SMCE, this paper proposed a novel sparse Locally Linear Embedding method for modal identification. Meanwhile, this study explores the modal identification results of several other different improvement strategies of LLE algorithm [10, 11]. Experiment results show that the proposed method can adaptively find the neighbors and weight coefficients than classical LLE algorithm. Compared with other improvement strategies, the proposed method has better recognition accuracy and robustness.

2. Modal parameters identification using Sparse LLE

2.1. The theory of modal identification

For an n-degree-of-freedom linear system, the governing equation of motion can be described as follows:

$$\mathbf{M}\ddot{\boldsymbol{x}}(t) + \mathbf{C}\dot{\boldsymbol{x}}(t) + \mathbf{K}\boldsymbol{x}(t) = \boldsymbol{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$ are the mass, damping and stiffness matrices, respectively. $\ddot{\boldsymbol{x}}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$ are the acceleration, velocity and displacement vector, respectively. $\boldsymbol{f}(t)$ is the external force vector. Based on the theory of modal expansion, the time domain responses $\boldsymbol{x}(t) = [x_1(t), \dots, x_n(t)]^T$ for a lightly damped system in modal coordinates can be decomposed as follows:

$$\boldsymbol{x}(t) = \mathbf{\Phi}\mathbf{q}(t) = \sum_{i=1}^{n} \boldsymbol{\varphi}_i\, q_i(t) \tag{2}$$

where $\mathbf{\Phi}$ is mode shape matrix which consists of mode shape vector $\boldsymbol{\varphi}_i \in \mathbb{R}^n$, and $\boldsymbol{q}(t) = [q_1(t), \dots, q_n(t)]^T$ is a modal response vector which composed of real value. In addition, when the natural frequency of each order is unequal, the modal shape vector and modal response vector meet the following relationship:

$$\boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_j = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \tag{3}$$

$$E\left(\mathbf{q}(t)\mathbf{q}^{\boldsymbol{T}}(t)\right) = \operatorname{diag}\left(q_1(t), \dots, q_n(t)\right) = \mathbf{\Lambda}_{n\times n} \tag{4}$$

where $\mathbf{\Lambda}$ represents a diagonal matrix.

2.2. Locally Linear Embedding algorithm

The LLE algorithm is an unsupervised dimensionality reduction algorithm that preserving the same topology structure from high dimensional to low dimensional data sets. The main idea is to use the local linearity of data to approximate the global linearity. The principle of LLE algorithm is summarized in [12]. The algorithm can be summarized as following three steps.

(I) Find neighbours in high dimensional space.

For a given high dimensional data $\mathbf{X} = [\boldsymbol{x}_1, \dots, \boldsymbol{x}_n] \in \mathbb{R}^{D\times N}$, the K neighbours data sets can be established by computing the Euclidean distances for each data point $\boldsymbol{x}_i$.

(II) Compute the reconstruction weight coefficients.

After establishing the neighbours data sets, the reconstruction weight coefficients w_{ij} can be resolved by the following optimization formula, where $\mathbf{W}$ is the weighted matrix consisting of w_{ij}.

$$\min_{w_{ij}} \sum_{i=1}^{N} \left\| \boldsymbol{x}_i - \sum_{j=1}^{K} w_{ij} \boldsymbol{x}_j \right\|^2$$
$$s.t. \sum_{j=1}^{K} w_{ij} = 1 \tag{5}$$

(III) Calculate the low embedding coordinates.

Finally, the low dimensional data matrix $\mathbf{Y}$ is solved by the following optimization formula in $\mathbb{R}^{d\times N}$, where $\mathbf{I}_d$ is unit matrix.

$$\min_{\boldsymbol{y}_i} \sum_{i=1}^{N} \left\| \boldsymbol{y}_i - \sum_{j=1}^{K} w_{ij} \boldsymbol{y}_j \right\|^2$$
$$s.t.\ N^{-1} \sum_{i=1}^{N} \boldsymbol{y}_i \boldsymbol{y}_i^T = \mathbf{I}_d,\ \ \sum_{i=1}^{N} \boldsymbol{y}_i = 0 \tag{6}$$

2.3. Sparse Locally Linear Embedding algorithm

The Sparse Locally Linear Embedding (SLLE) takes a full combination of the advantages of the LLE algorithm and SMCE model into account. For sparse LLE algorithm, the main principle is to find the nearest neighbors and weights automatically by the optimization model of SMCE, and the low dimensional embedding representation is computed by the sparse weight coefficient matrix then. The detail derivation process is illustrated as follow.

As described in Ref. [9], given a response data set $\mathbf{X}(t) = [\boldsymbol{x}_1, \dots, \boldsymbol{x}_n] \in \mathbb{R}^{D\times N}$ from n different manifolds, which are marked as $\{M_l\}_{l=1}^{n}$. Let the N_i be the neighborhood set of point $\boldsymbol{x}_i$. In general, the neighborhood N_i contains points from M_l as well as other manifolds. To exclude the points in other manifolds, for all points i there exists $\varepsilon \geq 0$ such that the nonzero entries of the sparsest solution of Eq. (7) corresponds to the neighbors of $\boldsymbol{x}_i$ from M_l.

$$\left\| \sum_{j\in N_i} c_{ij} \left(\boldsymbol{x}_i - \boldsymbol{x}_j \right) \right\|_2 \leq \varepsilon,\ \sum_{j\in N_i} c_{ij} = 1 \tag{7}$$

The Eq. (7) can be translated into a weighted quadratic programming problem as Eq. (8) described. Then, the sparsity coefficients $\{\boldsymbol{c}_i\}_{i=1}^{N}$ can be computed by Eq. (8) using linear programming technique [13].

$$\min \frac{1}{2} \left\| \widetilde{\mathbf{X}}_i \boldsymbol{c}_i \right\|_2^2 + \lambda \left\| \mathbf{Q}_i \boldsymbol{c}_i \right\|$$
$$s.t.\ \mathbf{1}^T \boldsymbol{c}_i = 1 \tag{8}$$

The parameter λ is a regularization parameter, which trades off the sparsity of the solution and the reconstruction error. The $\widetilde{\mathbf{X}}_i$ and $\mathbf{Q}_i$ are expressed as follows:

$$\widetilde{\mathbf{X}}_i = \left[\frac{x_1 - x_i}{\|x_1 - x_i\|_2}, \dots, \frac{x_n - x_i}{\|x_n - x_i\|_2} \right] \in \mathbb{R}^{D\times(N-1)},\ \mathbf{Q}_i = \frac{\|x_j - x_i\|_2}{\sum_{t\neq i} \|x_1 - x_i\|_2} \in (0,1] \tag{9}$$

Then, the weight coefficient $\boldsymbol{w}_i = [w_{i1}, \dots, w_{iN}]^T$ can be obtained by sparsity coefficients, and the low dimensional manifold embedding $\mathbf{Y}(t) = [\boldsymbol{y}_1, \dots, \boldsymbol{y}_N]$ are expressed as follows:

$$w_{ii} = 0, w_{ij} = \frac{c_{ij} / \|x_j - x_i\|_2}{\sum_{t\neq i} c_{it} / \|x_t - x_i\|_2}, j \neq i \tag{10}$$

$$\min_{\boldsymbol{y}_i} \sum_{i=1}^{N} \left\| \boldsymbol{y}_i - \sum_{j=1}^{K} w_{ij} \boldsymbol{y}_j \right\|^2$$
$$s.t.\ N^{-1} \sum_{i=1}^{N} \boldsymbol{y}_i \boldsymbol{y}_i^T = \mathbf{I}_d,\ \ \sum_{i=1}^{N} \boldsymbol{y}_i = 0 \tag{11}$$

where the indices of the nonzero elements of $\boldsymbol{w}_i$ correspond to the neighbors of $\boldsymbol{x}_i$ and the K indicates the number of neighbors of $\boldsymbol{x}_i$. $\boldsymbol{y}_i$ represents the output in low dimensional space. Finally, there is the following relationship between high dimensional responses and low dimensional embedding.

$$\mathbf{V} \approx \mathbf{X}(t) \mathbf{Y}(t)^{-1} \tag{12}$$

where $\mathbf{V}$ is a linear transformation matrix.

2.4. Modal identification based on Sparse LLE

Compared with Eq. (2) and Eq. (12), it is found that they have the same form of mathematical description. The linear transformation matrix is corresponding to modal shape matrix, while the low dimensional embedding is corresponding to modal shape vector. Therefore, the Spare LLE algorithm can be used for modal parameters identification based on matrix decomposition. The schematic diagram based on SLLE algorithm for modal identification is shown in Fig. 1.

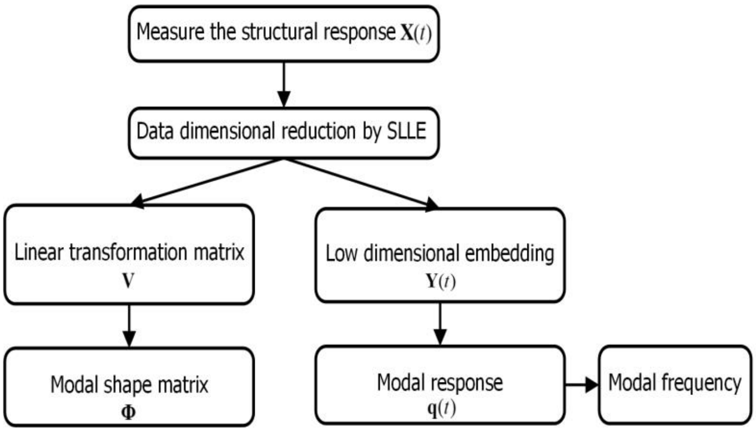


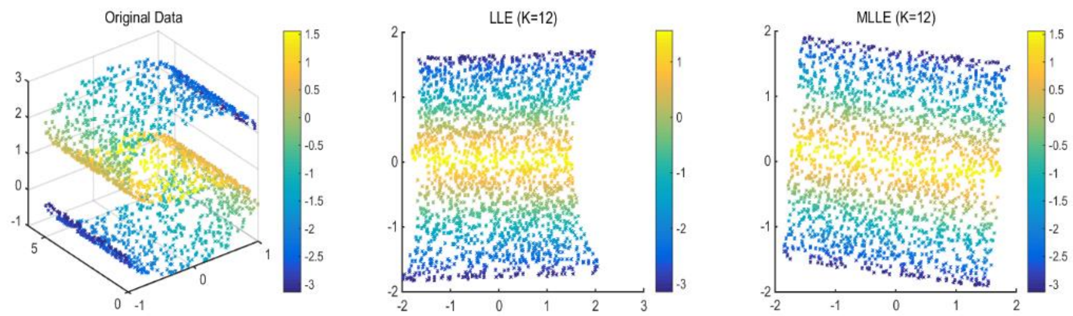
Fig.1 The flow chart of modal identification based on Sparse LLE algorithm

3. Numerical simulation and experiment verification

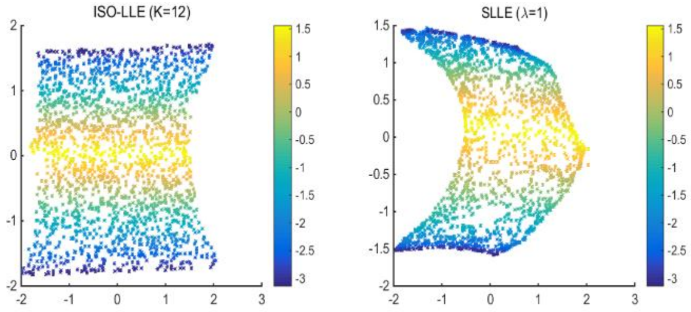
3.1. Numerical simulation

In this part, a general S-Curse data set [12] is utilized to elaborate the capacity of SLLE adaptively determining the neighbors and weights, and effectively preserving the neighborhood structure of high dimensional with small nonzero weights. What's more, the SLLE algorithm is analysed and compared with other improvement strategies of LLE algorithm.

As is shown from Fig. 2(b) to 2(e), the SLLE algorithm can also effectively preserve topology structure with $\lambda = 1$ from three-dimensional (3D) to two-dimensional (2D) in comparison to LLE algorithm and its different improved strategies with the number of optimal neighbors $K = 12$. Herein, to illustrate the SLLE method needs small nonzero weights, only the LLE algorithm is chosen to compare with SLLE. Moreover, the SLLE needs less nonzero weight coefficients than LLE from Fig. 3(a). The greater λ, the sparser the weights. Meanwhile, it is found that the average number of nonzero weights using SLLE with different K and λ tends to relatively stable state at the end. That is, the SLLE algorithm can adaptively find appropriate neighbors and weights only by setting parameter λ with a given larger $K_{\max}$.

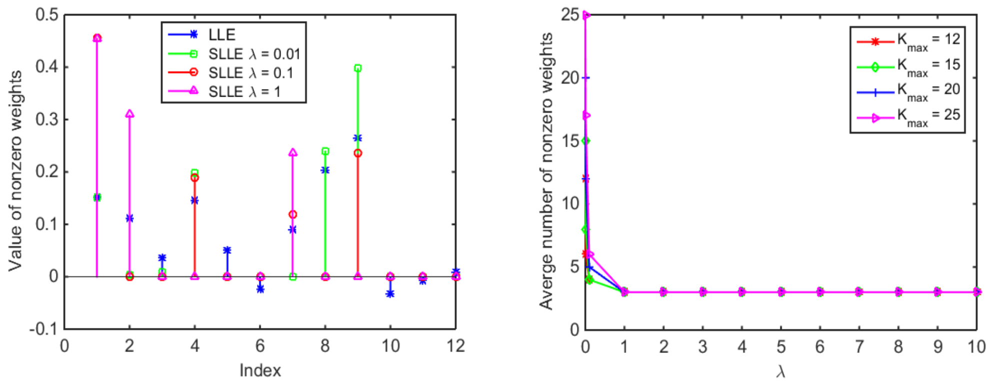


(a) (b) (c)



(d) (e)

Fig.2 LLE and its improved algorithm applied to S-Curse data set: (a) S-Curse in 3D; (b) LLE results in 2D; (c) MLLE results in 2D; (d) ISO-LLE results in 2D; (e) SLLE results in 2D



(a) (b)

Fig.3 (a) Distribution of nonzero weights using LLE and SLLE with $K = 12$ and different λ; (b) Number of nonzero weights using SLLE with different K and λ

3.2. Experiment verification

To validate the proposed algorithm, a cantilever steel plate is conducted with size $0.39 * 0.29 * 0.003\mathrm{m}^3$. The experimental setup is built as shown in Fig. 4. The entire measurement system includes a computer, Polytec scanning head (PSV-400), vibrometer controller (OFV-500), junction box (PSV-400-3D), smart shaker (K2007E01) and data management system. The spectral lines and frequency resolution are set as 1600 and 0.625Hz, respectively. In experiment, 20 measurement points are arranged in structure and the swept frequency excitation with a minimum frequency of 0 Hz and a maximum frequency of 2000 Hz is applied.

In addition, to investigate the robustness of algorithm, the responses are corrupted by white Gaussian noise, the root mean square (RMS) amplitude of which is set to three different noise levels (0, 5%, 10%) of the signal RMS value, respectively. Furthermore, the modal assurance criterion (MAC) which ranges from 0 to 1 is introduced to reflect the accuracy of modal shapes [14]. The larger the MAC value, the better the modal identification results. The modal shapes identification results (MAC values) based on different algorithms under different noise levels are shown in Table 1. The comparison of first 3 order modal shapes is shown in Table 2. The comparison of the first three order modal frequencies based on different algorithms under different noise levels are shown in Table 3. As is shown in Table 2, the modal shapes using different algorithms with appropriate preset can be well identified compared with experiment (Ex.). From Table 1, compared with other improved LLE algorithms with parameter $K = 20$, the identified modal shapes based on SLLE algorithm with parameter $\lambda = 0.01$ have higher MAC values under different noise levels. That is, the precision of modal parameters is higher by SLLE algorithm, which owns better robustness. As is shown in Table 3, the modal frequencies can be also be accurately identified by LLE and its improvement. Compared with other improved LLE, the precision of third order modal parameters is higher by SLLE from Table 3. Therefore, the proposed method can effectively identify the modal parameters (modal shapes and modal frequencies) and has strong anti-noise performance.

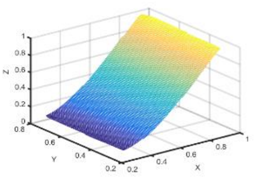
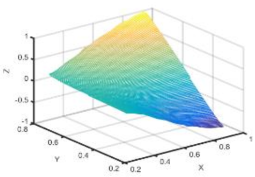
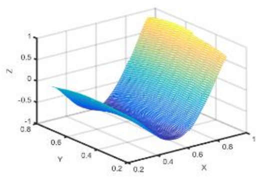
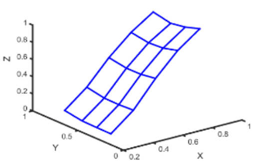
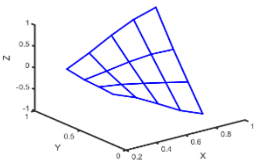
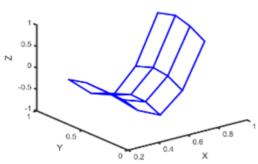
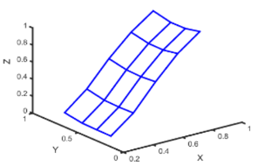
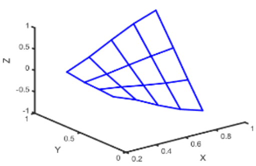
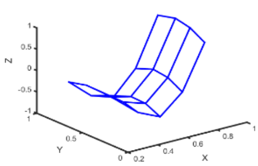
(a) (b)

Fig.4 The experimental setup: (a) Polytec scanning head; (b) Polytec vibrometer controller

Table 1. Comparison of MAC values

Noise level	0			5%			10%		
Mode	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
LLE ($K = 20$)	0.9818	0.9957	0.8468	0.9638	0.4250	0.8121	0.9548	0.0842	0.7550
MLLE ($K = 20$)	0.9542	0.9907	0.6325	0.9856	0.9144	0.8319	0.9861	0.9664	0.8465
ISO-LLE ($K = 20$)	0.9818	0.9957	0.8470	0.9638	0.4250	0.8121	0.9548	0.0842	0.7550
SLLE ($\lambda = 0.01$)	0.9543	0.9872	0.9402	0.9609	0.9883	0.9434	0.9991	0.9545	0.9849

Table 2. Comparison of the first 3 order modal shapes

Mode	**1st order**	**2nd order**	**3rd order**
Ex.			
LLE			
MLLE			

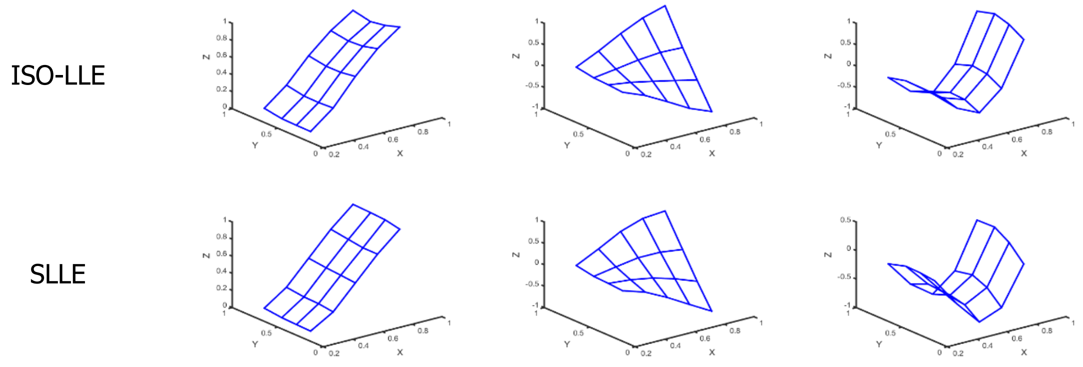


Table 3. Comparison of the first 3 order modal frequencies

Noise level	0			5%			10%		
Mode	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Ex.	23.13	61.88	130.00	23.13	61.88	130.00	23.13	61.88	130.00
LLE ($K = 20$)	22.96	61.55	129.0	22.96	61.55	129.0	22.96	61.55	129.0
MLLE ($K = 20$)	22.96	61.55	129.0	22.96	61.55	129.0	22.96	61.55	129.0
ISO-LLE ($K = 20$)	22.96	61.55	129.0	22.96	61.55	129.0	22.96	61.55	129.0
SLLE ($\lambda = 0.01$)	23.45	61.55	129.9	23.45	61.55	129.9	23.45	61.55	129.9

4. Conclusions

A novel method based on sparse LLE algorithm in time domain is proposed for estimation of modal parameters and is applied in typical plate structure in this paper. Compared with the classical LLE algorithm and its various improved strategies, the proposed method can adaptively find the neighbors and weights simultaneously, which can reduce the effect of false identification caused by the uncertainty of the selection of nearest neighbors K. Moreover, the proposed method has higher identification accuracy and better robustness. But this method is only suitable for overdetermined blind identification or determined blind identification, where the number of observed signals are greater than or equal to active modes. It will be of great significance to develop operational modal parameters identification approaches under underdetermined blind conditions, where the number of observed signals are less than active modes in the future.

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