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On global linear instability of laminar supersonic flow over planar and axisymmetric compression corners

Nicolas Cerulus [1], *Helio Quintanilha Jr* [2], *Vassilis Theofilis* [3]

Abstract

Linear modal global instability analyses of planar and axisymmetric, laminar compression corner flows are discussed at supersonic conditions. Steady basic flows have been computed with the OpenFOAM package and results were compared with existing literature and independently performed high-order direct numerical simulations. Global stability analysis reveals the leading two- and three-dimensional eigenmodes in both planar and axisymmetric compression corners and permits comparisons of the respective amplitude functions. The existence of increasingly complex viscous structures inside the laminar separation bubble, confined underneath the shock layer, is demonstrated, and the communication between these structures and the shock layer, recently also seen in kinetic theory simulations [40], is established. This qualitative behaviour is found to be consistently present in both planar and axisymmetric compression corners, the main quantitative difference between the two configurations being that, at the same nominal free-stream conditions, pressure relief makes the axisymmetric flow more stable than its planar counterpart.

1. Introduction

Prediction of linear instability and laminar-turbulent transition in high-speed flow is crucial to understanding the aerodynamic performance of components of vehicles traveling at supersonic and hypersonic speeds and, consequently, aid the efficient and effective design of such vehicles at all stages of their flight. On the compression ramp geometry the significance of laminar-turbulent transition has already been recognized in the seminal experimental work of Chapman *et al* [11]. Study of separation in compression ramps (and stability of this flow) has been the subject of intense investigation in the classic series of experiments of Ginoux [20, 21, 22] and continues to the present day [43, 2, 35, 15, 25, 26].

Direct numerical simulations performed at TsAGI [17, 18, 37] identified linear 2- and 3D instability mechanisms, and the nonlinear development thereof, in both attached and separated boundary layers. In the planar compression ramp, their work first revealed the existence of small-amplitude three-dimensional spanwise periodic structures forming downstream of the laminar separation region and amplifying exponentially [16, 41], thus establishing self-excited linear instability of the separation zone as a laminar-turbulent transition mechanism alternative to the imperfections of the leading edge, discussed by Simeonides and Haase [43] and Navarro-Martinez & Tutty [35]. This mechanism has been recently described in detail in the hypersonic regime using modal and nonmodal linear stability analyses [15, 25, 26].

In axisymmetric compression corners, the experiments of Gray [23] focused on instability of shock-induced laminar flow separation and showed that the separation length decreases as the unit Reynolds number increases, while both the separation length and the level of recirculation grow through an increase in Mach number or the flare angle. Early theoretical studies addressed compressible boundary layer instability on a cone [14, 32, 33, 50], while Horton [28] used integral boundary layer methods to perform calculations of flare-induced separation with an adiabatic wall and showed satisfactory agreement between theory and experiment. Chanetz *et al.* [10] have undertaken systematic experimental and numerical comparisons, using Navier-Stokes and Direct Simulation Monte Carlo (DSMC) methods, while

[1] *School of Engineering, University of Liverpool, United Kingdom, n.cerulus@liverpool.ac.uk*
[2] *School of Engineering, University of Liverpool, United Kingdom; now at Deloitte Berlin*
[3] *School of Engineering, University of Liverpool, United Kingdom, v.theofilis@liverpool.ac.uk*

Brown *et al.* [4], also using DSMC, have provided clear evidence of linear instability of axisymmetric amplified structures in the separation zone at the cylinder/flare junction.

The current effort employs an in-house global linear stability analysis code to expand on the existing literature on both the planar and axisymmetric compression corners, with a focus on comparing the leading eigenmodes in the two configurations and documenting their analogies and differences. The first concern in this context is reliability of the steady state base flow calculations, as convergence of the basic flows and their derivatives needs to be ensured in both time and space, in order for the instability physics at play to be adequately captured and discussed.

2. The base flows

Steady laminar two-dimensional basic states have been computed at $Ma = 3, Re' = 28000\ m^{-1}$ using the OpenFOAM module *rhoCentralFoam*, a density based solver using the central and upwind schemes of Kurganov and Tadmor [24, 31]. Conditions have been chosen to allow for comparison with benchmark planar compression ramp solutions of Carter [6] and Hung & MacCormack [29] and were kept the same in the planar and axisymmetric calculations.

The planar geometry, shown in Fig 1, comprises of a flat plate with a rounded leading edge, followed by a ramp with a sufficiently large downstream extent to fully capture the leading global modes, as will be discussed shortly. The rounded leading edge, magnified for clarity in this figure, is crucial in ensuring there is no singularity at the start of the flat plate. However, it also leads to a detached leading-edge shock and increases the spatial resolution demands; in the present simulations grids of up to $O(10^7)$ elements have been used. Fig 2 (left) shows a coarse representation of the grid used in the planar case. For both planar and axisymmetric flows the corner angle is set to $\alpha = 10°$ and the flat plate or cylinder streamwise extent are set to $L = 0.06\ m$.

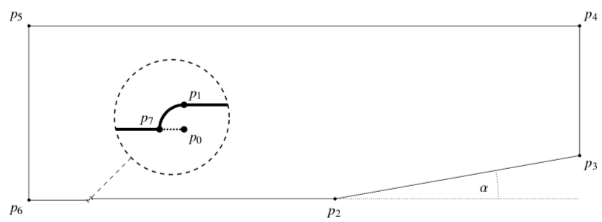

Fig 1. Schematic of a two-dimensional cut through the simulation domains; the inset provides details of the rounded leading edge. The coordinates of all points are shown in Table 1

Configuration	Resolution (Elements)	leading edge radius (mm)	Plate length (m)	Internal cylinder radius (m)
Planar	$6,949,543$	0.3	0.06	–
Axisymmetric	$7,018,034$			0.0254

Coordinates	p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7
x value	0	0	0.06	0.12	0.12	-0.015	-0.015	-0.0003
y value	-0.0003	0	0	0.0106	0.4232	0.4232	-0.0003	-0.0003

Table 1. Definition of the physical domain used in the calculation of base states and the coordinates required to define the domain.

Planar and axisymmetric basic states have been computed by extruding the configuration shown in Figure 1; in the axisymmetric case the intrinsic *wedge* function of OpenFOAM has been used. One difference between the planar and axisymmetric configuration is that in the latter case the domain has also been expanded to include a portion of flow within the internal cylinder, as shown in Fig 2 (right). Imposition of the internal cylinder radius, the value of which is stated in Table 1, completes the geometric description of the two configurations. The internal cylinder portion of the domain for the hollow cylinder-flare is defined by a streamwise extent of the cylinder equal to one third of the plate length. The mesh density is defined by imposing 50 elements across the boundary layer in both configurations, while the near-wall mesh of the internal cylinder in the axisymmetric case has the same grid density as that in the rounded leading edge.

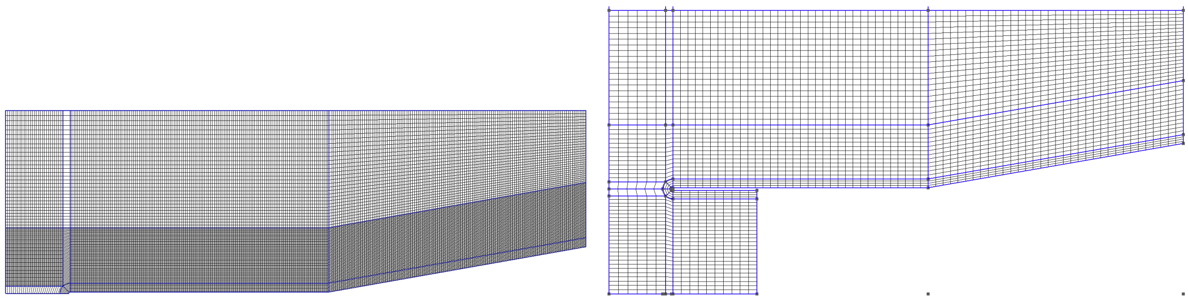

Fig 2. Visualisation of coarse meshes used for the planar compression ramp case (left) and the axisymmetric hollow cylinder-flare (right)

Fig 2 (left) shows a coarse mesh used in the planar case, while Fig 2 (right) presents the setup of the axisymmetirc mesh, with the understanding that the mesh is rotated around the axis of the cylinder to create the 3D volume elements required by the solver. Validation of the numerical procedures outlined here, as well as convergence studies of the base states have been presented elsewhere for the planar [1, 9] and for the axisymmetric [8, 39] configurations, respectively.

Both steady basic states computed contain a recompression shock associated to the compression corner section, underneath which laminar separation bubbles develop. The streamwise velocity component of the respective flows are shown in Fig 3. A quantitative difference arises on account of the pressure relief in the axisymmetric case, which leads to the separation region and the recirculation level in the axisymmetric configuration being substantially smaller than their planar compression ramp counterparts.

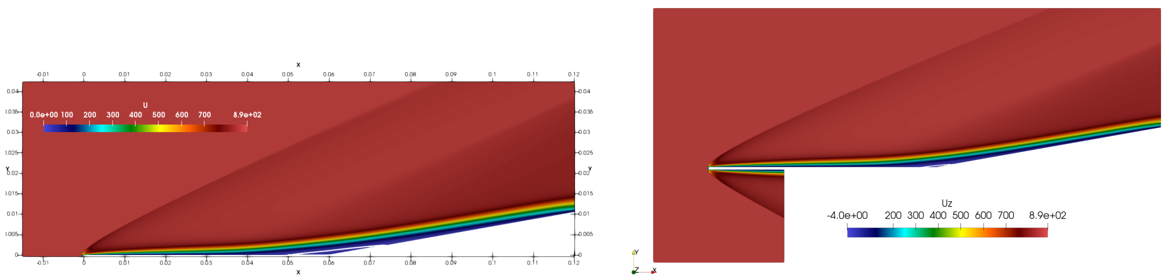

Fig 3. Basic flows in the planar compression ramp (left) and axisymmetric hollow cylinder-flare (right), depicting the streamwise velocity component. The (faint) white contour lines define the boundary of the separation region ($u = 0$)

3. Global Instability

The modal development of infinitesimal disturbances is studied using LiGHT (**Li**near **G**lobal stability analysis for **H**ypersonic **T**ransition) [38, 46], an in-house code for the parallel solution of the large complex non-symmetric generalised eigenvalue and singular value decomposition problems resulting in a matrix-forming context from coupled numerical discretization of the Linearised Navier-Stokes Equations in two- or three inhomogeneous spatial directions [45]. A key feature of the code is distribution of the discretized matrix and parallelization of its LU decomposition and subsequent operations within the Arnoldi algorithm. Spectral collocation methods [5] are used for the coupled discretization of the inhomogeneous spatial directions, following coordinate transformation of the domains of interest into the standard Chebyshev collocation domains $[-1,1]^2$ and $[-1,1]^3$ in two- and three spatial dimensions, respectively; here the BiGlobal planar and axisymmetric compressible eigenvalue problems have been solved. Details of the coordinate transformations have been provided elsewhere [7]. For the planar case the spanwise wavenumber β is real, while for the corresponding axisymmetric case the wavenumber m assumes non-negative integer values.

Fig 4 presents the least damped two dimensional ($\beta = 0, m = 0$) modes of the two configurations. Amplitude functions of the streamwise velocity perturbations of the planar and axisymmetric configurations are shown, using solutions of the respective eigenvalue problems; processing the corresponding unsteady base flow results using the residuals algorithm [44] delivers consistent results. A clear connection between the recompression shock and the boundary layer downstream of separation can be seen, with the perturbations peaking in both cases within the laminar separation bubble and inside the shock itself; this underlines the importance of including (and fully resolving) the shock in the stability analyses, as originally shown in the seminal works of Crouch and co-workers [12, 13]. Interestingly, the same conclusion has been reached in the related hypersonic two-dimensional ($m, \beta = 0$) analyses of shock / boundary layer global linear instability by Tumuklu *et al.* on the double cone [48] and the double wedge [49], while the recent work of Sawant *et al.* [40], which addressed the three-dimensional ($\beta \neq 0$) analog of [49], has also shown that spanwise periodic small-amplitude disturbances are present in the shock layer and their temporal evolution is synchronized with that of the well-known amplified 3D linear global mode of the underlying laminar separation bubble. However, both the 2D work of [48, 49] and its 3D extension [40] employed kinetic theory methods which fully resolve the shock layer, unlike the presently used computations which are based on the Navier-Stokes equations. Work using DSMC methods is underway [30] to address this issue in the present compression corner configurations.

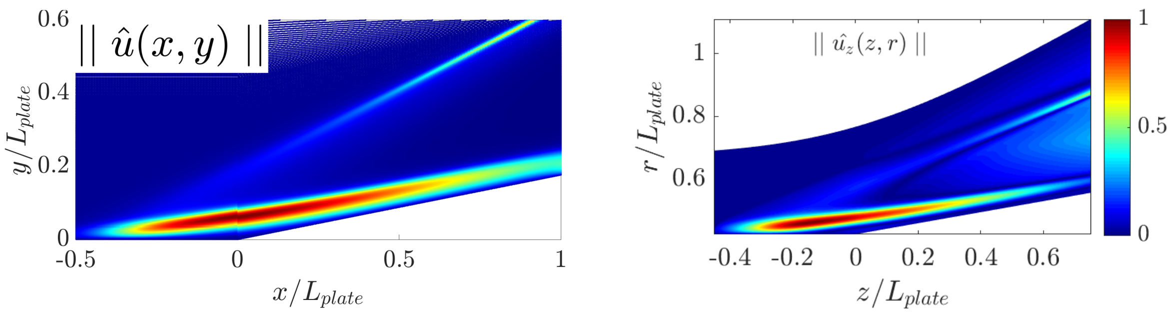

Fig 4. Amplitude function of streamwise velocity perturbation on the planar compression ramp (left) and on the hollow cylinder-flare (right). The separated region is highlighted by the white contour line

Fig 5 shows additional $\beta = 0$ global modes that can be found deeper in the eigenvalue spectrum of the planar configuration and are stronger damped than the leading mode shown in Fig. 4 (left). Viscous structures in the separation bubble are found to be confined underneath the shock layer. The complexity of the spatial structures increases in higher modes, while all modes are found to become progressively less stable as Re increases. Interestingly, recent work of Mustafa *et al* [34] post-processed turbulent compression ramp flow experimental data by Proper Orthogonal Decomposition techniques and shows that (POD) modes of increasing spatial complexity, confined underneath the separation shock, are also present in turbulent flow at essentially the same Mach number and orders-of-magnitude higher Reynolds numbers than those described by the present laminar flow global instability analysis.

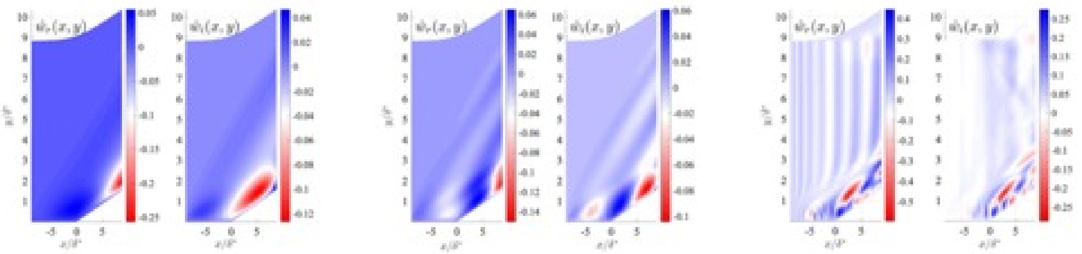

Fig 5. 2D modes of increasing spatial complexity in the compression ramp at $Ma = 3$, $Re' = 28,000m^{-1}$

3.1. Three dimensional analyses

Turning to the three dimensional ($\beta, m \neq 0$) cases, Fig 6 (left) shows the spanwise velocity component of the leading eigenmode in the planar case, obtained at $\beta\ \delta^* = 0.2$, scaled with the boundary layer displacement thickness at separation. The mode shape shows overlapping peaks around the separation region analogous to those found in the global instability analyses of the incompressible separated adverse pressure gradient boundary layer [47] and its compressible countepart [3, 36, 27], direct numerical simulations of the planar compression ramp [41, 16], as well as instabilities occurring on supersonic shock-induced laminar separation on a double wedge [42]. This is the leading mode, shown to become unstable at high enough Reynolds numbers and is associated directly to the laminar separated region; it will be discussed in detail during the conference.

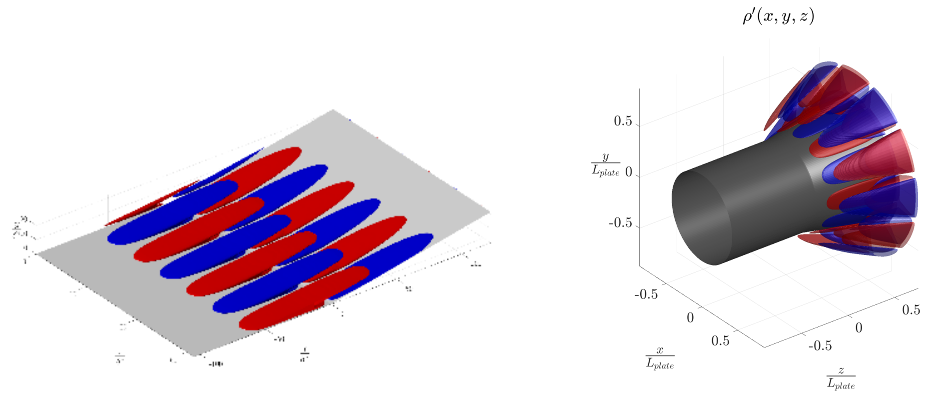

Fig 6. Left: Spanwise perturbation velocity component of the leading stationary mode on the planar compression ramp. Right: Density perturbation in the hollow cylinder / flare geometry

The leading stable stationary three dimensional global mode on the hollow cylinder-flare, obtained at $m = 6$, is shown in Fig 6 (right); it is seen to peak solely on the flare section at this set of parameters. Consistently with the two-dimensional case, perturbations extend in and couple both of the separated boundary layer and the separation shock, as clearly seen in the density amplitude function. This axisymmetric flow has been found to be stronger damped than its planar counterpart at the same free stream conditions. Instability of the hollow cylinder / flare flow is promoted by increasing the Reynolds number, e.g. by considering a longer cylindrical part of the geometry, or by increasing the flare angle. The relation of the present findings to experiments performed at UTSI [19] is presently being examined and further discussion will be provided at the time of the conference.

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